Receptivity of a Low Reynolds Number Bickley Jet to Harmonic Vortical Excitation

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Abstract

Receptivity of a Bickley jet to a time harmonic symmetric (S- class) and anti-symmetric (AS- class) vortical excitation is reported. Unlike wall bounded flows, the eigen-spectrum of jets reveal the presence of multiple dominant modes. The S- class displays the presence of upstream propagating disturbances. It is reasoned that due to limited streamwise extent of the domain, experiments and computations on round jets do not always correlate with the linear stability properties. For DNS, a new compact scheme (OUCS4), introduced in [6], along with RK_4 time stepping is used. A new filtering procedure is advocated in the radial direction, which removes the numerical instability at the core (due to a mathematical singularity) and allows us to study receptivity of round jets to different classes of excitations.

Key Words: Bickley jet, CAA, DNS, DRP, Eigen Spectrum, Filtering techniques, Viscous Instability.

1. Introduction

Our present interest is to study the viscous instability of jets by using the eigen-values (along with their directionality of propagation), as cataloged in [7]. In addition to usual classification (into S- class or varicose mode and the AS- class or the helical mode), such instabilities have also been labeled into shear layer and preferred modes [4]. Danaila *et al* [2] have reported from their DNS results that the disturbances switch from helical to varicose mode, when Re increases from 200 to 500. It is experimentally noted for plane jets [5], that the flow is strictly laminar when Re is less than 10. When Re exceeds 50, irregular turbulent fluctuations develop. In actual flow, simultaneous presence of multiple modes, decide the jet flow evolution. Hence, a 3-D DNS for a round jet is undertaken to relate the coherent structures with the eigen functions of the Bickley jet that is shown in Figure 1 for anti-symmetric excitation [7].

2. Governing Equations and Auxiliary Conditions

3-D, unsteady, compressible, non-dimensionalized NS- equations in conservation form and generalized curvilinear coordinate system is solved, which is given by:

$$\frac{1}{J}\frac{\partial Q}{\partial t}+\frac{\partial}{\partial \xi}(\frac{F-F_v}{J})+\frac{\partial}{\partial \eta}(\frac{G-G_v}{J})+\frac{\partial}{\partial \zeta}(\frac{H-H_v}{J})=0 \qquad (1)$$

where ξ, η and ζ are the generalized coordinates in the computational plane and J is the Jacobian of transformation from the physical to the computational plane. Ideal gas equation is used to relate pressure, density and temperature. The molecular viscosity is calculated using the Sutherland's law, with the Sutherland's constant chosen as 110K and the Prandtl number taken as 0.7. The radiation and outflow boundary conditions on the lateral and the outflow boundary respectively, are the same as given in [1]. At the inflow, all values are equal to the jet centerline values, except the axial velocity, taken as the Bickley jet profile parallel to the jet axis and which is given by,

$$U(z) = \cosh^{-2}(a\sqrt{(y-y_s)^2+ \quad (x-x_s)^2}) \text{ with } \quad a = 0.88136 \qquad (2)$$

3. Numerical Methods

The governing Equations (1) are solved using OUCS4 (for spatial discretization) and RK_4 temporal discretization [6]. The time step Δt, is taken as 2.5 X 10^{-3}. The radial and the stream-wise extent of the computational domain is 10R and 12R respectively, which is chosen using 71 points in (r-), 32 points in (θ-) and 52 points in the (z-) direction. The grid in θ- direction is equi-angular, while in the r- and z- directions it is stretched in an arithmetic progression. To avoid spurious reflections, an 8^{th} order filter is used in the axial (z-) and the azimuthal (θ-) direction. A spectral filter $F(k_r)$, which is the same transfer function of the 1^{st} derivative of OUCS4 for interior points, is used in the radial direction after every 100 time steps. The filter is shown in Figure 2. This function windows the Fourier transform of the numerical solution (Q) (where: Q(r, θ, z) = $\int \tilde{Q}(k_r,\theta,z)e^{ik_r r}dr$) that exhibits the presence of high wave number components at the jet core due to the numerical instability introduced by a mathematical singularity at $r = 0$. The unknowns at the core, if obtained by the interpolation formula given in [3] is used then the solution develops an inflexional velocity profile that suffers an inviscid instability and triggers spurious transition. Instead of that, if one filters the solution by the above mentioned filter in the spectral plane and the filtered solution is then inverse-transformed to obtain the physical variable, then no such spurious inflexional instability occurs.

4. Results and Discussion

4.1 Eigen-spectrum of a Bickley jet

Sengupta *et al* [7] reports the behaviour of the eigen-spectrum for two classes of disturbance. For the anti-symmetric disturbances (AS-class), the displayed mode in Figure 1(a) is violently unstable with spatial growth rate given by α_i = -0.1744721 and whose energy propagates with the group velocity, V_g = 0.7321 in the downstream direction. In contrast, the displayed eigen-vector in Figure 1(b) has the wave property given by α_r = 1.2340, α_i = 1.411685 and V_g = 0.5440. This mode is oscillatory across the middle of the shear layer, while it damps in the downstream direction.

In contrast, the symmetric disturbances are less unstable, with some modes moving upstream that only exist for low frequencies [7].

4.2 DNS of a Round Jet

3-D DNS of a Bickley jet (M_∞ = 0.5, Re = 500) is reported. OUCS4 scheme for spatial derivatives in r- and z- direction, and the DRP scheme of Tam & Webb [8] is used in the θ- direction. Two classes of time harmonic vortical excitation (with ω_0 = 0.2, implying that the excitation repeats itself after every time interval of 10π) are considered at the jet inflow plane. For S- class, a vortical disturbance (u_{ds}, v_{ds}) (The form of these disturbances are the same as given in [8], with a half-width equal to the jet-width) is imposed. For the AS- class, anti-symmetric vortical pulse (given by $u_{das}= u_{ds} \sin(2\theta)$, $v_{das}= v_{ds} \sin(2\theta)$) is used at the inflow plane.

In Figure 3, the axial velocity distribution in (*r*-*z*)-plane is shown for symmetric vortical excitation case at the indicated times. For the S- class excitation, the flow is highly stable. For this excitation, the flow is nearly symmetric as well as time-periodic – as seen in Figure 3, while the pressure and the axial velocity plots shown in Figure 4 for the AS- class excitation show loss of coherence away from the core due to spatial instability.

5. Conclusions

A time-harmonic case of vortical excitation at the inflow of a round jet at Reynolds number 500 and jet centerline Mach number of 0.5 has been computed by solving the full 3D Navier-Stokes equation, using a new compact scheme for spatial discretization [6] along with RK_4 time-stepping. A new filtering technique is advocated in the radial direction to remove the mathematical singularity that gives rise to spurious inflexional instability at the jet core. For the computed flow fields due to symmetric and anti-symmetric vortical excitation at the jet exit plane, display a stable flow for the former while the flow is unstable for the latter.

6. References

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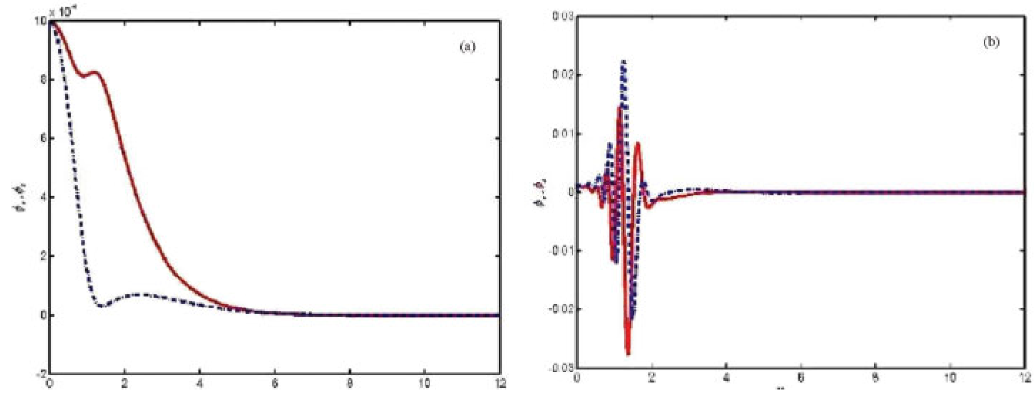

Figure 1. Eigen-vectors for AS- class disturbance field (Re=500 and ω_0 = 0.5) for (a) an unstable mode and (b) a very stable mode. Solid lines are for real part and chain-dotted lines are for imaginary part.

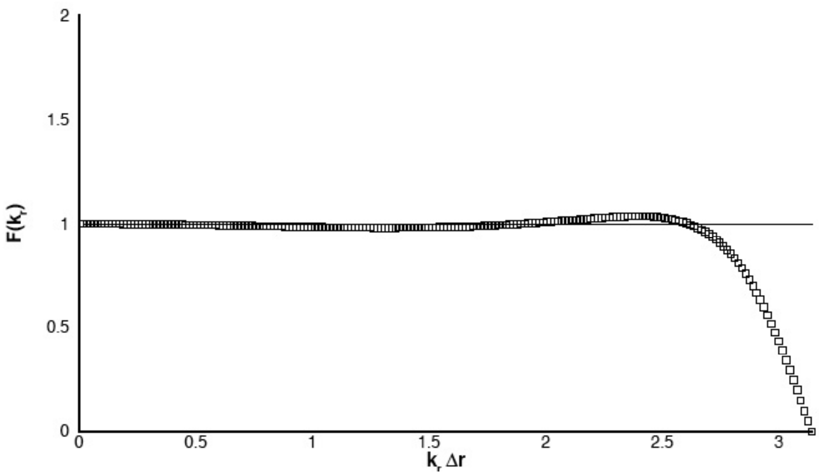

Figure 2. The spectral filter used in the radial direction for all the physical variables.

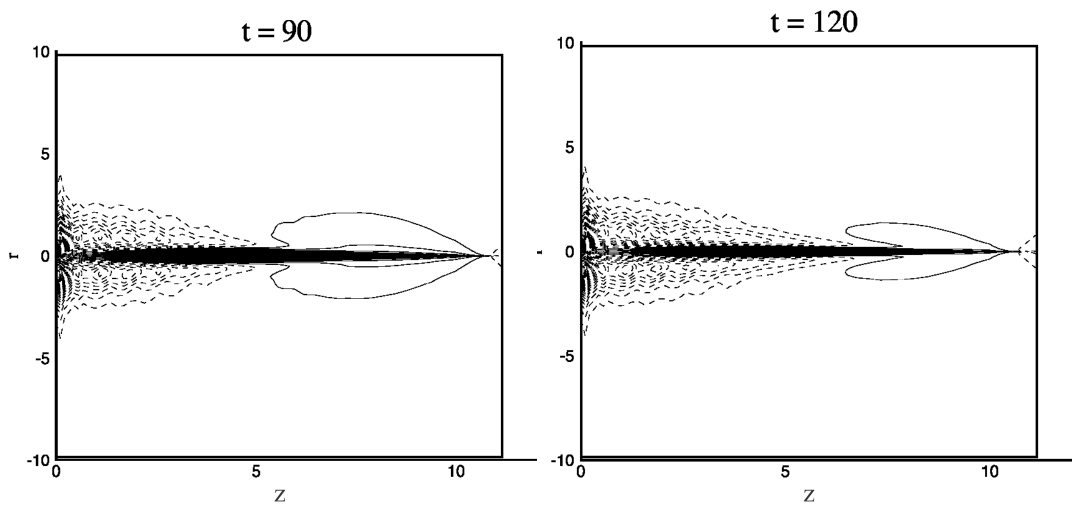

Figure 3. Disturbance axial velocity component for the symmetric vortical excitation at the indicated times, for Re = 500 and ω_0 = 0.2 in the (r-z) plane.

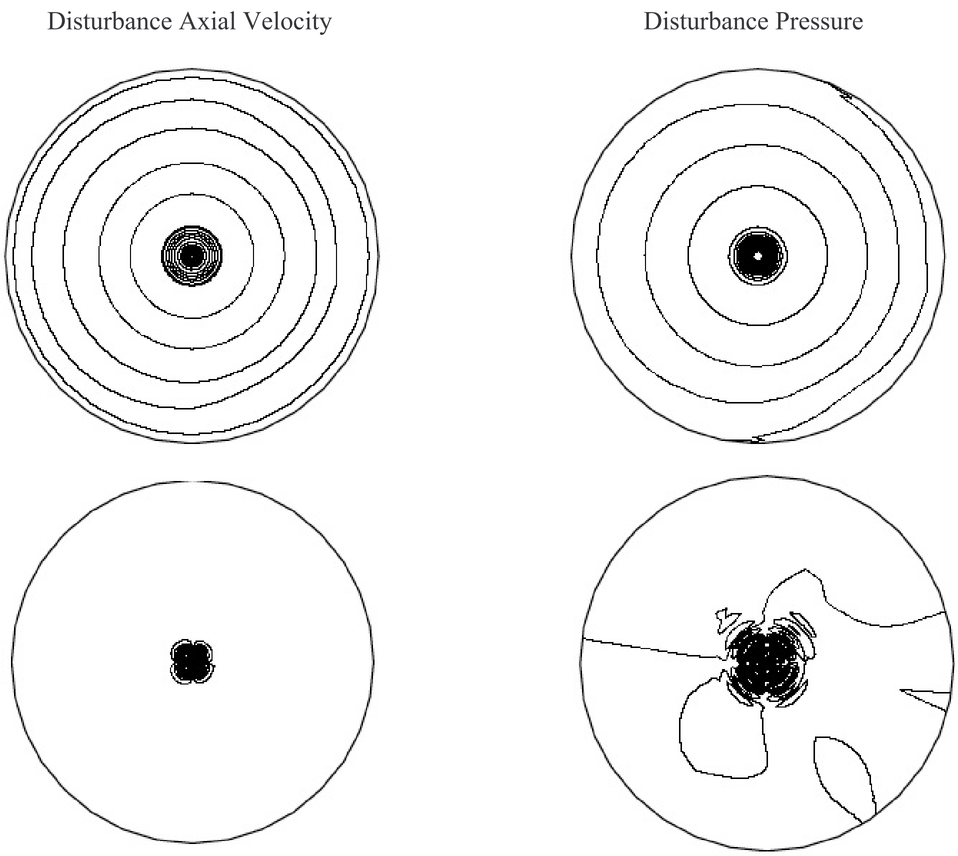

Figure 4. Disturbance axial velocity and disturbance pressure for S- (top) and AS- (bottom) vortical excitation in the (r-θ) plane at T=120, z = 4R, Re = 500 & ω_0 = 0.2.