Robins–Magnus effect: A continuing saga

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The experimental observation by Robins, that a projectile spinning about its axis of travel experiences a transverse force (lift) due to a cross-flow was refuted by Euler purely as a contradiction to expected symmetry of fluid flow. This undoubtedly had taken away the precedence of finding this effect by Robins and subsequently the same was credited to Magnus—a testimony of intuition overtaking physical observation. In the last century, Prandtl looked at this problem once again and came up with a maximum lift value that a section of a spinning projectile (cylinder) experiences due to cross-flow. This proto-typical model is extensively used to explain the aerodynamic phenomenon of lift generation. However, in recent times experimental and numerical investigations have identified a new temporal instability for this flow at high rotation rates that sets the lift generation process for spinning axi-symmetric bodies exceeding the above maximum limit. In this note, we trace the origin of this particular effect to its present day status with respect to flow past a rotating cylinder.

The beginning

Benjamin Robins' contribution[1] to fluid mechanics and aerodynamics has received less recognition than it deserves due to various reasons. One of the major reasons is that he propounded too many new ideas in a short span of time and he was also busy defending Newton's contribution to calculus. He was largely a self-taught person with a desire to take up teaching profession. Upon proving Newton's 'Treatise of quadratures', he was elected Fellow of the Royal Society, London at the early age of twenty. However, he switched his attention to engineering by constructing bridges, mills, harbours, making rivers navigable and draining fens. That he had multifaceted talent is evident, when one notes that he is now acknowledged as the father of science of ballistics[2,3] (introducing the concept of rifling the bore of guns, holding the importance of air-resistance in deciding the range of artillery shots and improving the accuracy of projectiles by spinning them), credited for fundamental contributions to aerodynamics[4] (the complex relationship between drag, shape of the body, its angle of attack and air–velocity could not be explained by the then simple theory propounded by Newton and he suggested that ground testing of vehicle is a prerequisite for a successful design), experimental fluid mechanics[1,4] (developing the whirling arm, the predecessor of present day wind tunnels, the only experimental device at that time and ballistic pendulum for the measurement of velocity of projectiles) and his many contributions to mathematics. He also noted the drag rise at transonic flight regime almost 200 years ago, before its importance was re-discovered around the Second World War[4]. Additionally, he dabbled in contemporary politics and also got involved in controversies related to writing the accounts of Lord Anson's voyage around the world. To this, one must add the fact that he left the centre-stage of England, when he was appointed the engineer-general of the East India Company to improve the fortifications at St. David, Madras, where he died of fever at an early age of forty-four. In writing the book on ballistics[2], he recorded the experimental observation (with ballistic–pendulum and whirling arms) that a spinning projectile experiences a normal force due to cross-wind. It is clear that in the absence of cross-flow, a projectile at zero incidence will not experience any side-force or lift. It was, however, not clear why a lift force will be experienced due to cross-flow. Existence of such a lift force due to cross-flow was not supported by Euler, the leading hydrodynamicist of the time, and this effect was rediscovered by Magnus[5], almost a century later. In this note, we will discuss this particular effect with respect to flow past a rotating cylinder starting with Robins' work to its present day status.

It is incorrectly stated in some references[6] that Robins was responsible for finding the lift force acting on a rotating sphere. Undoubtedly, he experimented on spherical shots used for artillery purposes, but he was also first to suggest that a teardrop or egg-like shape of projectile with a centre of gravity near the front of it. The observation of Robins' for spinning projectiles was made using the whirling arm – not a very satisfactory experimental device by the present day standard. A whirling arm was used to measure aerodynamic forces at low speed, where the tested body was used to be hung at the end of a long arm that was free to rotate. This arm was rotated by a falling weight via a shaft with cable–pulley arrangement. The rotation of the arm produces the relative motion in air, the same principle that is even used today in wind tunnels to measure forces for steady flight. However, sustained rotation of whirling arm will impart angular momentum to the surrounding air, thereby making the accuracy of such measurements a point of concern. It is with this equipment that he reported his findings[2] in 1742. It is noted[4] that, *Euler was so excited about Robins' book that he personally translated it into German in 1745 adding some commentary.... Euler's interest in Robins' work was both a hindrance and a help. The hindrance concerned Robins' observation of the side force exerted on a spinning projectile moving through the air. Euler considered that to be a spurious finding, due to manufacturing irregularities in the projectile. Recognized as the dominant hydrodynamicist of the eighteenth century, Euler far overshadowed Robins, and thus Robins' finding was not taken seriously for another century, until Gustav Magnus (1802–1870) verified the phenomenon as a real aerodynamic effect.* This book[2] was also translated into French in 1751, the year of Robins' death. It is to be noted that Napoleon read the latter translation from Euler's German translation of the original book[5], while he was a young artillerist at Auxonne, France.

It must be pointed out that both Euler and Robins had mutual admiration for each other's work. For example, Robins published in 1739 ***Remarks on M. Euler's***

Treatise of Motion. So the interpretation of Robins' work was based truly upon the intuitive observation of Euler – not based on his famous equations of fluid motion that were enunciated later in 1752, the first mathematical model of flows. Euler's observation that a spinning axisymmetric body in cross-flow (with top–down symmetry) cannot experience a lift force in the symmetric direction, was heuristic in nature. The occurrence of transverse force experienced by a spinning body was explained by Prandtl[7] based on a steady irrotational flow model. He also advanced a maximum limit to this transverse force. This was an interesting development in this subject area and is discussed next.

Maximum principle for lift

The first qualitative explanation of lift force experienced by an aerodynamic shape was made possible by using Kutta–Jukowski theorem[6] that assumes a specific flow behaviour in the near vicinity of the body with a sharp trailing edge. For an airfoil (the quintessential cross-section of a flying wing), the accepted solution is given by an application of this theorem that forces the flow to stagnate at the trailing edge. This theorem is not applicable for flow past bodies without sharp trailing edges – as in the case of a rotating cylinder. Prandtl[7] explained the flow past a rotating cylinder heuristically by considering the flow to be inviscid and irrotational.

In putting forward his results, Prandtl[7] estimated the maximum lift a rotating cylinder experiences when the rotation rate is increased beyond a critical limit. This can be readily explained with the help of Figure 1. If one defines a non-dimensional rotation rate by $\Omega = \Omega^* D / 2U_\infty$, where the cylinder of diameter D rotates at Ω^* while being placed in a uniform stream of velocity U_∞, then one can define a non-dimensional number, called the Reynolds number, by $Re = U_\infty D/\nu$ for this flow field. In Figure 1 *a*, the steady inviscid irrotational flow field is depicted when the cylinder does not rotate and one can note a top–down and fore-aft symmetry of the flow field. In Figure 1 *b*, a case is depicted for $\Omega < 2$, where both the front and rear stagnation points (half-saddle points) are deflected downwards, causing the flow to exert an upward force on the cylinder. With increase of Ω to 2, these stagnation points move towards each other and merge at the lowermost point on the cylinder, as shown in Figure 1 *c*. For this location of stagnation point, it is easy to show that the corresponding non-dimensional lift value is given by the coefficient $C_{L_{max}} = 4\pi$. Prandtl heuristically reasoned this lift as maximum, because with further increase in Ω, the half-saddle point of Figure 1 *c* would move in the flow as a full saddle-point on a closed streamline that demarcates the flow field into two parts, as shown in Figure 1 *d*. The region inside the closed streamline is insulated from the region outside permanently for steady inviscid flow. This fixes the vorticity at the critical rotation rate for the case of Figure 1 *c*. In a real flow, vorticity created at the solid wall is convected and diffused according to the governing Navier–Stokes equation. A steady flow model, presupposes equilibrium between the creation of vorticity and its viscous diffusion for all rotation rates. It was argued by Prandtl that the equilibrium at $\Omega = 2$, decides the lift value when the rotation rate is increased further. This model appeared realistic in the absence of any counter-examples and is used in textbooks to explain lift generation and limiting mechanism. However, some recent experimental and numerical observations provide counter-examples where lift is found to exceed Prandtl's maximum (8–10).

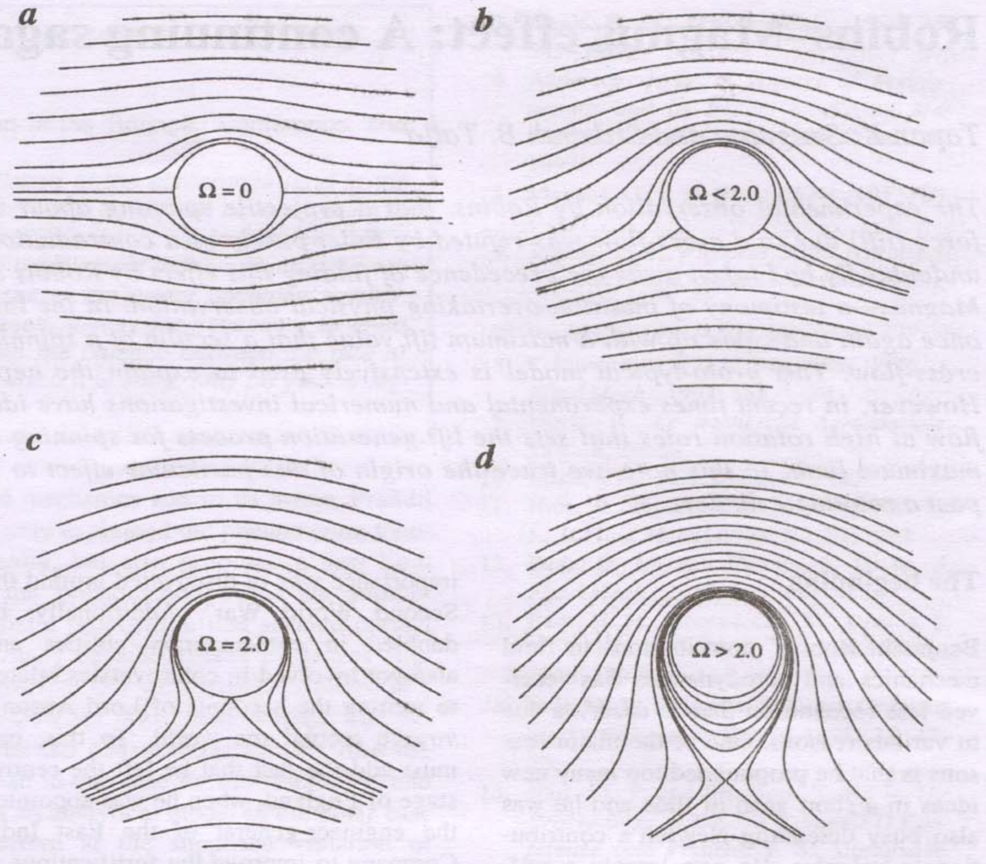

Figure 1. Inviscid irrotational flow past a rotating cylinder for (*a*) zero, (*b*) subcritical, (*c*) critical and (*d*) supercritical rotation rates.

Tokumaru and Dimotakis[8] have observed that the maximum lift limit was violated by 20% for $Re = 3800$ and $\Omega = 10$. The authors considered diffusion, unsteady flow processes as the main contributor in violating the maximum limit, while three-dimensional end-effects will tend to reduce the mean spanwise lift. For $\Omega > 2$, the vorticity will be generated at a larger rate at the solid wall than it is dissipated by viscous action, thereby showing a monotonic increase in lift value, if the vorticity remains confined within the recirculating streamline. The role of diffusion is thus to peg the net circulation at a lower level. However, we will see in the next section that viscous diffusion also plays a subtle role in supporting enhanced lift when it interferes with physical instability processes. This is clearly seen in computations that use excessive numerical dissipation to *stabilize computations*. It should be noted that for super-critical rotation rates, three-dimensionality of the flow is suppressed due to Coriolis force predominating over convection and viscous diffusion[6,11]. Thus, it is instructive to compute the flow by solving time dependent two-dimensional Navier–Stokes equation, as is reported in refs 12–14. The results in refs 12, 13 are particularly noteworthy for the accuracy, but are reported for low rotation rates and short period of integration times. The high order method used in ref. 14 for a range of Reynolds numbers and lower rotation rates was extended for high Reynolds numbers and rotation rates in refs 15–17. The computed lift coefficients in refs 15–17 matched with the experimental results in ref. 8. For the first time, numerical calculation revealed the violation of the maximum limit from two-dimensional flow model. The numerical results apart from validating experimental observation, also provide detailed time accurate account of the physical events, that is otherwise difficult to track experimentally. In doing so, the computational results also revealed a new physical instability that limits the monotonically increasing lift in an aperiodic manner.

A new instability uncovered

Computations in refs 15–17 revealed a series of temporal instabilities at early stages of flow evolution. It is important to discuss the feasibility of such physical instabilities. In an experiment, Werle[9] noted a layer of co-rotating fluid, in contact with the cylinder surface, suffering *aperiodic* instabilities for supercritical rotation rates for Re = 3300, a value similar to that in ref. 8.

In refs 15–17, a high order accurate method was used to capture these temporal instabilities. In numerical computations of physical instabilities, one must ensure that the discretization should not alter the physical processes, specially the physical dissipation process. In the method used in refs 15–17, this has been particularly ensured. However, there are methods used in refs 18, 19 for this problem that did not report such instabilities. In ref. 18, the author used a 'Streamline-Upward/Petrov–Galerkin' (SUPG) method, along with Pressure–Stabilizing/Petrov–Galerkin (PSPG) numerical stabilization terms for a case where the cylinder rotated eccentrically, as according to the author: *in a real situation it is almost certain that the rotating cylinder will be associated with a certain degree of wobble*. This was prompted by the author's earlier attempt[19] in computing the flow for Re = 3800 and $\Omega = 5$ that resulted in four times the measured lift value. One of the reasons for the failure of SUPG/PSPG method is due to the added massive artificial dissipation term that increases physical dissipation by orders of magnitude and does not allow the physical instability to be computed.

A typical set of computational results are shown in Figure 2 where time variation of force and moment coefficients have been shown for Re = 3800 and $\Omega = 5$. Here, an accurate compact difference scheme has been used[20] in solving the vorticity transport equation. The instability displayed in this case consists of sharp discontinuous jump in the values of force and moment coefficients at discrete times. Such instabilities were computed and reported earlier in refs 15–17. The physical mechanism behind the instability has been explained in ref. 17 by an equation based on energy principle derived from full Navier–Stokes equation

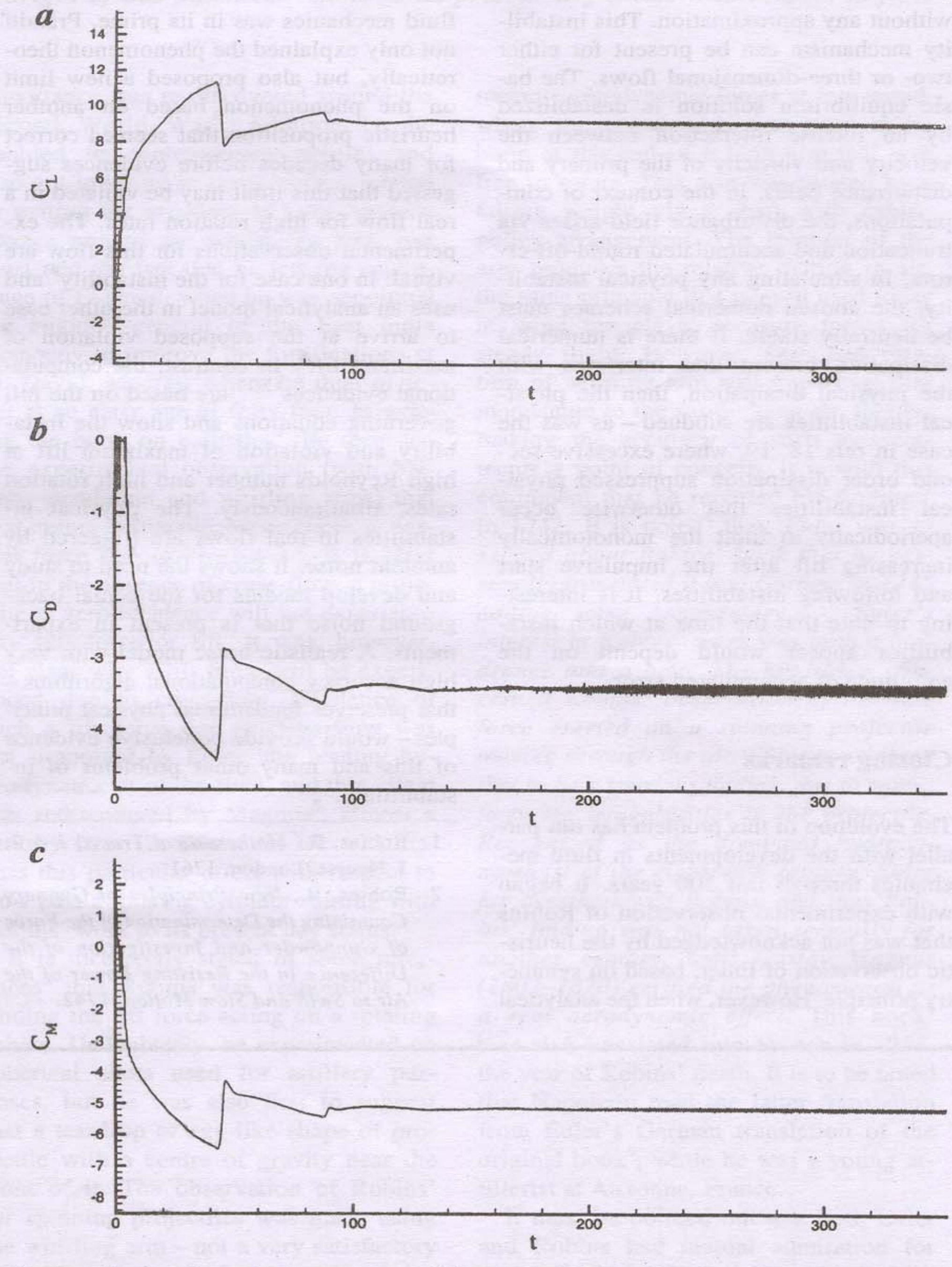

Figure 2. Loads and moment co-efficients variation with time for Re = 3800 and $\Omega = 5.0$: (***a***) lift co-efficient, (***b***) drag co-efficient and (***c***) pitching moment co-efficient.

Box 1. Excerpts from the fax V. Modi sent to Sengupta on 4 October 1998.

Dear Dr Sengupta

Thank for your kind words about presentation at the Götingen Conference, and the paper which has just arrived . . . .

I did give thought to the point you had mentioned at the conference (and in the letter) about the periodic shedding of 'puffs' of vorticity. It may not have anything to do with the gap or the stability of the computing process . . . . It is possible that a part of the momentum injection leads to vorticity entrainment in the wake cavity with an increase in momentum injection, surface area of the cavity becomes smaller. Although the vortex shedding frequency increases, it may be necessary to shed transient (periodic?) puffs of vorticity to maintain the balance between the rate at which the vorticity is generated and the rate at which it is shed. May be your analysis and our video show the same effect . . . .

without any approximation. This instability mechanism can be present for either two- or three-dimensional flows. The basic equilibrium solution is destabilized by an intense interaction between the velocity and vorticity of the primary and disturbance fields. In the context of computations, the disturbance field arises via truncation and accumulated round-off errors. In simulating any physical instability, the chosen numerical schemes must be neutrally stable. If there is numerical dissipation present that interferes with the physical dissipation, then the physical instabilities are subdued – as was the case in refs 18, 19, where excessive second order dissipation suppressed physical instabilities that otherwise occur aperiodically to limit the monotonically increasing lift after the impulsive start and following instabilities. It is interesting to note that the time at which instabilities appear would depend on the amplitude of accumulated error.

Closing remarks

The evolution of this problem has run parallel with the developments in fluid mechanics through last 300 years. It began with experimental observation of Robins that was not acknowledged by the heuristic observation of Euler, based on symmetry principle. However, when the analytical fluid mechanics was in its prime, Prandtl not only explained the phenomenon theoretically, but also proposed a new limit on the phenomenon based on another heuristic proposition that seemed correct for many decades before evidences suggested that this limit may be violated in a real flow for high rotation rates. The experimental observations for this flow are visual: in one case for the instability[9] and uses an analytical model in the other case to arrive at the supposed violation of maximum lift[8]. In contrast, the computational evidences[15–17] are based on the full governing equations and show the instability and violation of maximum lift at high Reynolds number and high rotation rates, simultaneously. The physical instabilities in real flows are triggered by ambient noise. It shows the need to study and develop models for the actual background noise that is present in experiments. A realistic noise model with very high accuracy computational algorithms – that preserves fundamental physical principles – would provide conclusive evidence of this and many other problems of instabilities.

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