Equated Monthly Installment and Amortization Schedule

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In this article we derive the formula used to compute EMI (Equated Monthly Installment) and what part of EMI gets deducted for principal and interest.

Let us suppose we borrow L INR[1] amount as loan at the rate of interest $i\%$ per annum for a period of n months. Then what should be the E, an equal amount (called EMI), that we agree to pay every month to clear the loan in n months. Since we shall work in terms of month, let us convert the rate of interest per annum to per month. Thus, the rate of interest per month is $\frac{i}{12}\%$. This means for every 100 INR of the loan amount the lender charges an extra of $i/12$ per month. Equivalently, for each 1 INR of your loan L, the lender charges you an extra $\frac{i}{12\times 100}$ per month. This means that at the end of the first month you owe the lender an amount which is the sum of the original loan L and the interest accrued in a month, i.e.,

$$L + L\left(\frac{i}{12\times 100}\right) = L\left(1 + \frac{i}{12\times 100}\right) = Lr$$

where we have set $r = 1 + \frac{i}{12\times 100}$ to simplify our notation. You will pay E at the end of first month and hence will owe an amount $L_1 = Lr - E$. At the end of second month you will owe

$$L_1 + L_1\left(\frac{i}{12\times 100}\right) = L_1\left(1 + \frac{i}{12\times 100}\right) = L_1 r.$$

[1] the unit of currency is not an issue

After paying E, you owe

$$L_2 = L_1 r - E = (Lr - E)r - E = Lr^2 - E(1 + r).$$

Continuing this argument, we notice that at the end of n-th month, after paying E, we will owe

$$L_n = Lr^n - E(1 + r + r^2 + \ldots + r^{n-1}).$$

Let us set $S = 1 + r + r^2 + \ldots + r^{n-1}$ and get a formula for S in terms of r and n. In fact, S is the sum of the first n terms of a geometric series. Every term in the sum is a r multiple of its predecessor. Note that

$$\begin{aligned} rS - S &= r + r^2 + r^3 + \ldots + r^{n-1} + r^n - (1 + r + r^2 + \ldots + r^{n-1}) = r^n - 1 \\ S &= \frac{r^n - 1}{r - 1} \end{aligned}$$

and hence

$$L_n = Lr^n - ES = Lr^n - E\left(\frac{r^n - 1}{r - 1}\right).$$

If we want to finish our loan at the end of n-th month, we expect $L_n = 0$. This gives that

$$E = \frac{Lr^n(r-1)}{r^n - 1} = \frac{L\left(1 + \frac{i}{1200}\right)^n \frac{i}{1200}}{\left(1 + \frac{i}{1200}\right)^n - 1}.$$

This is the formula for the EMI that you pay for any kind of loan.

Those who have taken loan might have noticed that some part of EMI is deducted from principal and the remaining as interest. A natural question is, on any given month, how much from your EMI is deducted towards principal payment. Let us deduce this formula. Recall that at the end of first month the interest accrued is

$$L\left(\frac{i}{12 \times 100}\right) = L(r - 1).$$

This interest accrued, at the end of first month, is deducted from your EMI E and the balance is used as payment towards principal. Therefore the amount that goes as payment towards your principal loan amount is $E - L(r - 1)$ at the end of first month. Continuing this way, one notices that out of your k-th month EMI E, an amount of $r^{k-1}\{E - L(r - 1)\}$ is deducted towards principal payment and the rest is towards interest. Notice that $r > 1$ and hence your principal payment increases with each month. Also, as derived above, the principal loan you owe after k-th month EMI is $r^k L - E\left(\frac{r^k - 1}{r - 1}\right)$.