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NUMERICAL ANALYSIS OF PERFORMANCE OF SOIL NAIL WALLS IN SEISMIC CONDITIONS

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ABSTRACT

Evidences from the field and full-scale laboratory tests suggest that soil nail walls perform remarkably well under seismic conditions. In this study an attempt has been made to study the performance of a soil nail wall supporting a vertical cut of 8 m height under seismic conditions. The wall is designed in conventional manner by using the allowable stress design procedure. The response of the wall is then simulated numerically by using a finite element analysis. Seismic data from Bhuj and Uttarkashi earthquakes is used for the pseudo-static and dynamic analyses. To assess the performance of the soil nail wall, parameters such as maximum lateral displacements, development of nail forces, important failure modes of soil nail walls, have been studied under both static and seismic conditions. Results of the numerical analyses indicate that the use of soil nail walls is desirable to impart stability to the retaining systems under seismic conditions.

KEYWORDS: Soil Nailing, Conventional Design, Seismic Performance, Numerical Analysis

INTRODUCTION

In areas of high seismic activity, earthquake effects on the stability of retaining walls assume considerable importance, and soil nail walls are considered as useful options in this context. A review of literature shows that soil nail walls have performed well during strong ground motions in contrast with the generally poor performance of gravity retaining structures. After the 1989 Loma Prieta, 1995 Kobe, and 2001 Nisqually earthquakes, it was reported that soil nail walls showed no sign of distress or significant permanent deflection, despite having experienced, in some cases, ground accelerations as high as 0.7*g* (Felio et al., 1990; Tatsuoka et al., 1996). These observations indicate that soil nail walls appear to have an inherent satisfactory seismic response. This has been attributed to the intrinsic flexibility of soil-nailed systems (which is comparable to that of other flexible retaining systems, such as MSE walls) and possibly to some levels of conservatism in the existing design procedures. Similar trends have been obtained from centrifuge tests performed on reduced-scale models of soil nail walls (e.g., Vucetic et al., 1993; Tufenkian and Vucetic, 2000). However, any detailed analysis on the performance of soil nail walls is not brought out in the above studies. It is also desirable that a detailed numerical analysis is conducted with regard to the application of the technique in Indian context using appropriate earthquake data. The results presented in this paper fulfill this requirement.

In this study, a typical soil nail wall of 8 m height, that is representative of typical heights of retaining walls, is designed in conventional manner by using the allowable stress design approach presented in the Federal Highway Administration report for analyses, design and construction of soil nail walls (Lazarte et al., 2003). The external failure modes of soil nail walls namely, global stability and sliding stability, are studied under static, pseudo-static, and dynamic conditions. Plaxis (2002), a two-dimensional finite element based computational tool, is used for numerical simulation of the (conventionally designed) soil nail wall and for studying its stability under static and seismic conditions. Additionally, influence of seismicity on nail forces, maximum horizontal displacements, and important internal failure modes such as nail pullout failure and nail tensile strength failure, is also studied and compared under static and seismic conditions. Table 1 shows the recommended minimum factors of safety, modified after Byrne et al. (1998), for design of permanent soil nail walls based on the allowable stress design (ASD) method, where loads are unfactored.

Table 1: Minimum Recommended Factors of Safety for Permanent Soil Nail Walls

Failure Mode	Resisting Component	Symbol	Minimum Recommended Factors of Safety	
			Seismic	Static
External stability	Global stability	FS_G	1.10	1.5
	Sliding	FS_{SL}	1.10	1.5
Internal stability	Pull-out resistance	FS_P	1.50	2.0
	Nail bar tensile strength	FS_T	1.35	1.8
Facing failure	Facing flexure	FS_{FF}	1.10	1.5
	Facing punching failure	FS_{FP}	1.10	1.5

MATERIAL PROPERTIES AND DESIGN PARAMETERS

It is assumed that the in-situ ground constitutes a deposit of sandy soil. Terrain is considered to be generally flat and the elevation of ground water table is significantly below the bottom of the soil nail wall. Table 2 shows the material properties and other design parameters adopted for the study. For dynamic simulations, the January 26, 2001 Bhuj earthquake record at Ahmedabad and October 20, 1991 Uttarkashi earthquake record at Uttarkashi (Shrikhande, 2001) are used.

Table 2: Material Properties and Other Parameters Adopted for the Conventional Design

Parameter	Value
Vertical height of wall, H (m)	8.0
Face batter, α' (degree)	0.0
Slope of backfill, β (degree)	0.0
Wall-soil interface friction angle, δ (degree)	15.0
Cohesion, c (kPa)	1.0
Friction angle, $\varphi(=\varphi')$ (degree)	30.0
Unit weight, γ (kN/m^3)	16.0
Modulus of elasticity of soil, E_s (MPa)	20.0
Yield strength of nail, f_y (MPa)	415
Modulus of elasticity of nail, E_n (GPa)	200.0
Nail spacing, $S_V \times S_H$ (m×m)	1.0×1.0
Nail inclination (with horizontal), i (degree)	15.0
Drill hole diameter, D_{DH} (mm)	100.0
Compressive strength of grout, f_{ck} (MPa)	20.0
Ultimate bond strength, q_u (kPA)	100.0
Modulus of elasticity of grout, E_g (GPa)	22.0
Horizontal seismic coefficient, k_h	0.106
Vertical seismic coefficient, k_v	0.0

CONVENTIONAL DESIGN

The conventional design of the soil nail wall is carried with reference to the allowable stress design procedure stated in the report of Federal Highway Administration for analyses, design and construction of soil nail walls (Lazarte et al., 2003). The design is carried out in two stages: (i) preliminary design, and (ii) final design.

1. Preliminary Design

The preliminary design involves use of simplified design charts and tables to arrive at the design nail length and diameter. Following are the general steps in the preliminary design:

a) For the specific project application, obtain general parameters such as face batter α, back slope β, effective friction angle φ', and ultimate bond strength q_u, and calculate the normalized allowable pullout resistance μ using Equation (1):

$$\mu = \frac{q_u D_{DH}}{FS_P \gamma S_H S_V} \tag{1}$$

b) From the relevant design chart, obtain the normalized length *L/H* and normalized force $t_{\max-s}$.

c) Evaluate and apply suitable correction factors to the *L/H* ratio and $t_{\max-s}$ values obtained in the previous step for the drill hole diameter other than 100 mm, normalized cohesion value c^* other than 0.02, and global factor of safety other than 1.35.

d) Determine the maximum design load in the nail, $t_{\max-s}$, (in kN) using the value of corrected $t_{\max-s}$ as in Equation (2), and calculate the required cross-sectional area of the nail bar, A_t, from Equation (3):

$$T_{\max-s} = t_{\max-s} \gamma H S_H S_V \tag{2}$$

$$A_t = \frac{T_{\max-s} FS_T}{f_y} \tag{3}$$

e) Finally select the closest commercially available bar size that has a cross-sectional area at least equal to A_t, as evaluated in the previous step.

2. Final Design

The final design includes analysis of external failure modes (such as global stability and sliding stability), analysis of internal failure modes (such as nail pullout failure and nail tensile strength failure), design of permanent facing and verification of important facing failure modes (such as facing flexure failure and facing punching shear failure), and influence of other site-specific considerations, such as seismic loading, on global and sliding stability. In the present study only the important external and internal failure modes are considered to assess and compare the performance of the (conventionally designed) soil nail wall under static and seismic conditions.

According to the conventional design procedure used in this study, whenever seismic considerations are involved, a pseudo-static approach is adopted for determining the factor of safety for global stability, FS_G, and factor of safety for sliding stability, FS_{SL}. However, the global stability analysis (for FS_G) is generally carried out by using a computational tool for slope stability, and major emphasis is laid on the selection of suitable seismic acceleration coefficients (k_h and k_v). While analyzing the sliding stability (for FS_{SL}) of a soil nail wall under seismic loads, the total active thrust P_{AE} during an earthquake due to the earth pressures behind the soil block must be considered. This force is a combination of the static and dynamic active lateral earth pressures that are induced by the inertial forces. The lateral earth force, including the seismic effects, is evaluated using the Mononobe-Okabe (M-O) method, which is an extension of the Coulomb theory (Kramer, 2005). Table 3 presents a summary of important design variables determined for the soil nail wall based on the conventional design procedure.

Table 3: Summary of Conventional Design Results

Design Variable	Value
Nail length, L_N (m)	4.70
Nail diameter, d (m)	16.0
Maximum axial force in nail, $T_{\max-s}$ (kN)	32.56
Axial force at nail head, T_o (kN)	19.53
Pullout capacity of nail per unit length, Q_u (kN/m)	31.41
Maximum axial tensile load capacity of nail, R_T (kN)	83.44
Factor of safety against pullout (on ultimate bond strength), FS_P	2.00
Factor of safety against nail tensile strength, FS_T	2.56
Factor of safety against global stability, FS_G	1.56 (1.11)
Factor of safety against sliding stability, FS_{SL}	2.21 (1.74)
Facing	Permanent cast in-place R.C.C. facing 200 mm thick
Note: Figures in bracket indicate the corresponding values from seismic considerations (with k_h and k_v taken as 0.106 and 0.0, respectively)	

In Table 3, the maximum axial force $T_{\max-s}$ is calculated from Equation (2), whereas the axial force at the nail head, T_o, is given by Equation (4). Based on the measurements of forces in nails at the head, the nail head force, T_o, is expressed as

$$T_o = T_{\max-s}\left[0.6+0.2\left(S_v\left[m\right]\right)-1\right] \tag{4}$$

The pullout capacity per unit length, Q_u (also referred to as the load transfer rate capacity), is given by Equation (5):

$$Q_u = \pi q_u D_{DH} \tag{5}$$

The maximum axial tensile load capacity of a nail, R_T, is given by Equation (6):

$$R_T = A_t f_y \tag{6}$$

The factor of safety against the nail pullout failure, FS_P, is calculated as the ratio of pullout capacity R_P of the nail to the maximum axial force developed in the nail, i.e.,

$$FS_P = \frac{R_P}{T_{\max-s}} = \frac{Q_u L_P}{T_{\max-s}} \tag{7}$$

where, L_p is the pullout length of the nail. The factor of safety against the nail tensile strength failure, FS_T, is calculated as the ratio of maximum axial tensile load capacity of the nail to the maximum axial force developed in the nail, i.e.,

$$FS_T = \frac{R_T}{T_{\max-s}} \tag{8}$$

The factor of safety against the global failure, FS_G, is expressed as the ratio of the sum of resisting forces, ΣR, and the sum of driving forces, ΣD, which act tangent to the potential failure plane:

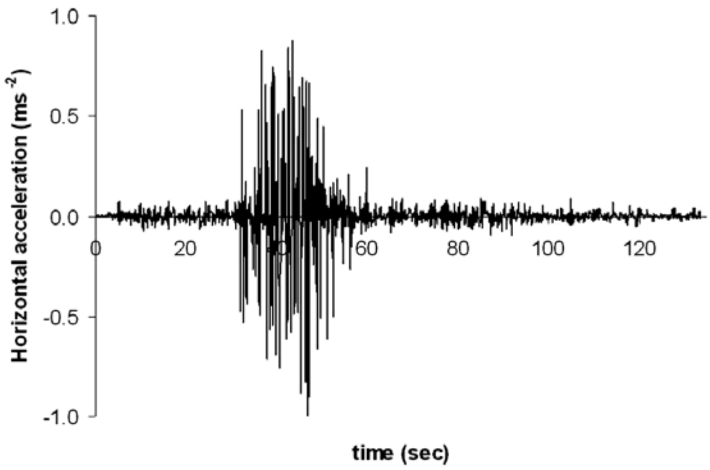
$$FS_G = \frac{\sum R}{\sum D} \tag{9}$$

Similarly, the factor of safety against the sliding failure, FS_{SL}, is expressed as the ratio of the sum of the resisting forces, ΣR, and the sum of the driving forces, ΣD, which act along the potential horizontal sliding failure plane at the base of the soil nail wall:

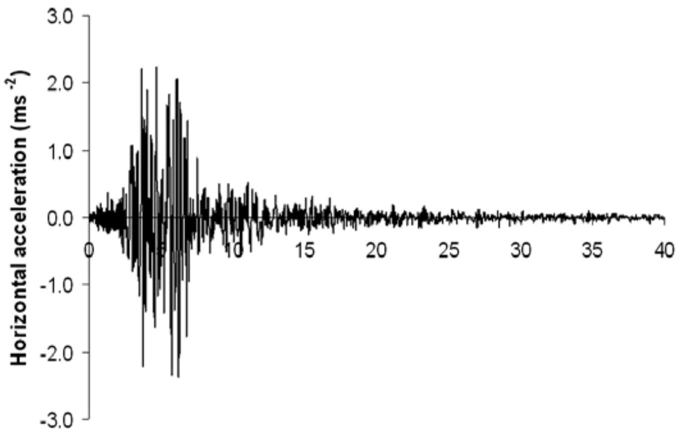
$$FS_{SL} = \frac{\sum R}{\sum D} \tag{10}$$

NUMERICAL SIMULATION

As stated earlier, Plaxis (2002) is used for simulating and analyzing the response of soil nail wall under static and seismic conditions. Numerical modelling is carried out assuming the plane strain state of stresses. The 15-node triangular elements with medium mesh density are used for the finite element discretization. The in-situ soil is simulated as Mohr-Coulomb (MC) material for the static and pseudo-static analyses and as hardening soil with small-strain stiffness (HSS) for the dynamic analyses. Various inputs parameters for the HSS model are judiciously adopted from the manual of Plaxis (2002) after calibration for the Houston sand. The advantage of using HSS model is that it accounts for the increased stiffness of soils at small strains (Benz, 2007). For the dynamic analyses, strong motion records, as shown in Figures 1(a) and 1(b) for the 2001 Bhuj and 1991 Uttarkashi earthquakes respectively, are used. Soil nails and wall facing are simulated as the linear elastic materials. Plate elements are used to model the nails and facing. Figures 2(a) and 2(b) show the outline and finite element model of the 8-m high vertical soil nail wall. Construction sequences are simulated as the staged construction with 2-m excavation lift in each stage.



(a)



(b)

Fig. 1 Strong motion records for the Bhuj and Uttarkashi earthquakes: (a) Bhuj earthquake record at Ahmedabad; (b) Uttarkashi earthquake record at Uttarkashi

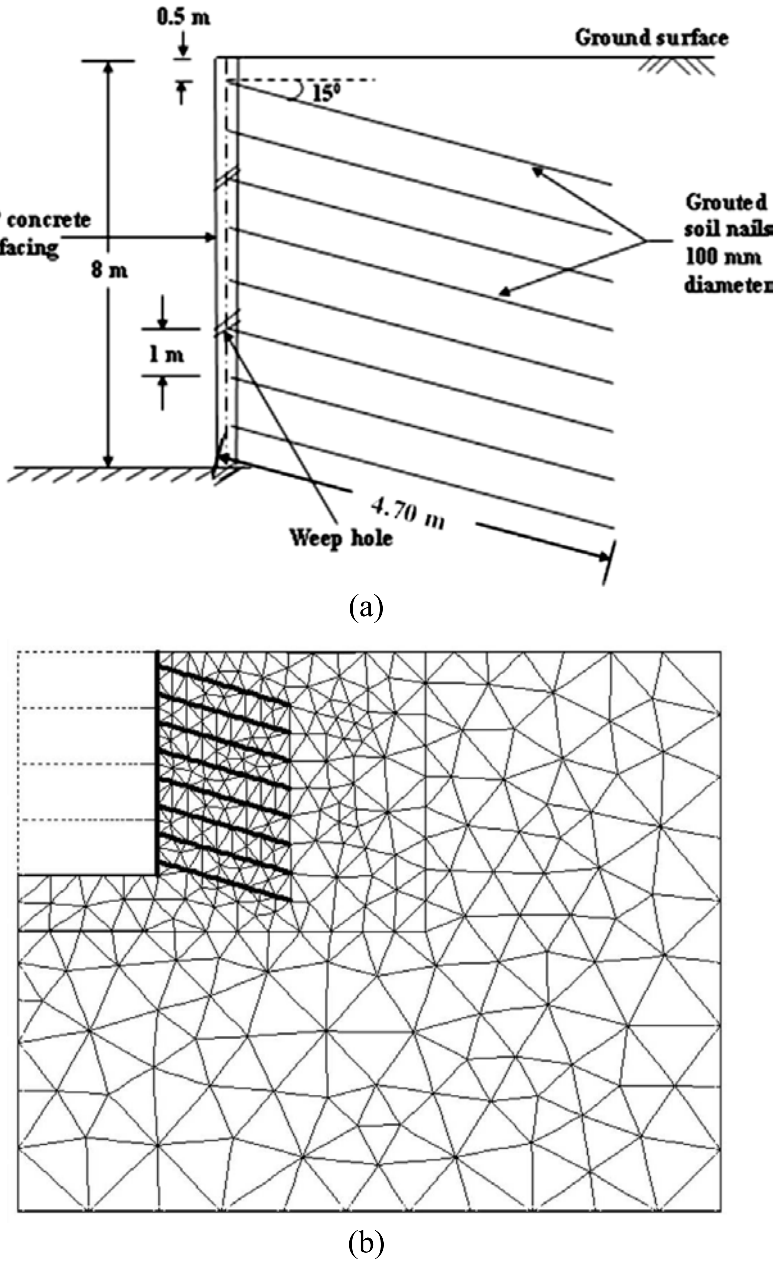


Fig. 2 Soil nail wall supporting the 8-m high vertical cut: (a) outline of the designed soil nail wall; (b) the numerically modelled state

Numerical simulations are conducted for various parameters to assess the performance of soil nail wall system under static, pseudo-static, and dynamic conditions. Table 4 presents a summary of some of the important results obtained. For determination of factors of safety against the internal failure modes (i.e., the nail pullout failure and the nail tensile strength failure), the corresponding values of $T_{\max-s}$ obtained from the numerical simulations are used in Equations (7) and (8).

The strength reduction technique (also known as the phi-c reduction technique (Matsui and San, 1992)) is typically used for the determination of factor of safety in numerical analysis in geotechnical engineering. In this technique, the strength parameters "tan φ" and "cohesion c" of the soil are successively and simultaneously reduced until failure of the structure occurs. Though this method is advantageous (Dawson et al., 1999) as it identifies the critical failure mechanism automatically, which is generally assumed in the conventional analysis, it excludes the stress-dependent stiffness behavior and hardening effects of the soil. To circumvent the above difficulty and to obtain the factors of safety corresponding to the results of dynamic analyses, a regression analysis procedure is used. To obtain the relationship between the global factor of safety and the maximum displacement (for the determination of global factor of safety), several numerical simulations of the soil nail wall are carried out for the static case by varying the values of cohesion c and friction angle φ. The results are presented and discussed in the following sections.

Table 4: Summary of the Results of Numerical Simulations

Analysis Parameter	Analysis				
	Static	Pseudo-Static		Dynamic	
		Bhuj	Uttarkashi	Bhuj	Uttarkashi
Horizontal seismic coefficient, k_h	--	0.106	0.241	--	--
Vertical seismic coefficient, k_v	--	0.00	0.00	--	--
Maximum axial force in nail, $T_{\max-s}$ (kN)	22.34	34.50	40.73	34.98	33.06
Maximum axial force at nail head, T_o (kN)	19.96	27.80	32.46	29.17	28.86
Maximum horizontal displacement of soil nail wall (%)	0.61	3.29	5.24	1.13	0.82
Factor of safety against global stability, FS_G	1.23	0.95	0.81	1.10	1.17
Factor of safety against nail pullout failure, FS_P	1.78	1.15	0.98	1.14	1.21
Factor of safety against nail tensile strength failure, FS_T	3.73	2.42	2.05	2.38	2.52

RESULTS AND DISCUSSION

Important results of the conventional design and the numerical simulations (for the static and seismic conditions) of the soil nail wall are summarized in Tables 3 and 4, respectively. It is evident from the factor of safety values for the important failure modes considered in the present study that the soil nail wall is stable under the static as well as seismic conditions used for the analyses.

To study the response of dams under dynamic conditions, Sivakumar Babu and Rao (2005) obtained a regression relationship between the global factor of safety and the maximum displacement obtained from the dynamic analyses. To obtain this relationship, the use of phi-c reduction technique was made. A similar procedure is adopted in this paper. For the static case with actual soil strength parameters (i.e., cohesion c = 1 kPa, friction angle φ = 30^o) a factor of safety value of 1.23 is obtained. This value according to the phi-c reduction technique corresponds to reduced values of strength parameters (i.e., cohesion c = 0.83 kPa, friction angle φ = 25.51^o), resulting in a factor of safety value equal to unity. Hence, by varying the soil strength parameters between the actual and reduced values, factor of safety values and the corresponding horizontal displacements are obtained. Finally, a power law is fitted, as shown in Figure 3, between the factor of safety and the maximum horizontal displacement, with an r-square value of 0.9714 indicating a good correlation, and is given by Equation (11):

$$FS_G = 2.7346\left(h_x\right)^{-0.2013} \tag{11}$$

where, FS_G denotes the global factor of safety and h_x the maximum horizontal displacement (in mm).

It may be noted that the values of global factor of safety FS_G (see Table 4) obtained from the pseudo-static analyses are less in comparison to those obtained from the dynamic analyses using actual time history data for the earthquakes. This is due to the limitation of the pseudo-static approach, which assumes that the earthquake force acts continuously on the wall, whereas time history data of the earthquake shows that the earthquake force acts for a short duration. The duration of strong ground motion can have a strong influence on the damage caused by an earthquake. A motion of short duration may not produce enough load reversals for damaging response to build up in a structure, even if the amplitude of the motion is high. On the other hand, a motion with moderate amplitude but long duration can produce several load reversals to cause substantial damage (Kramer, 2005).

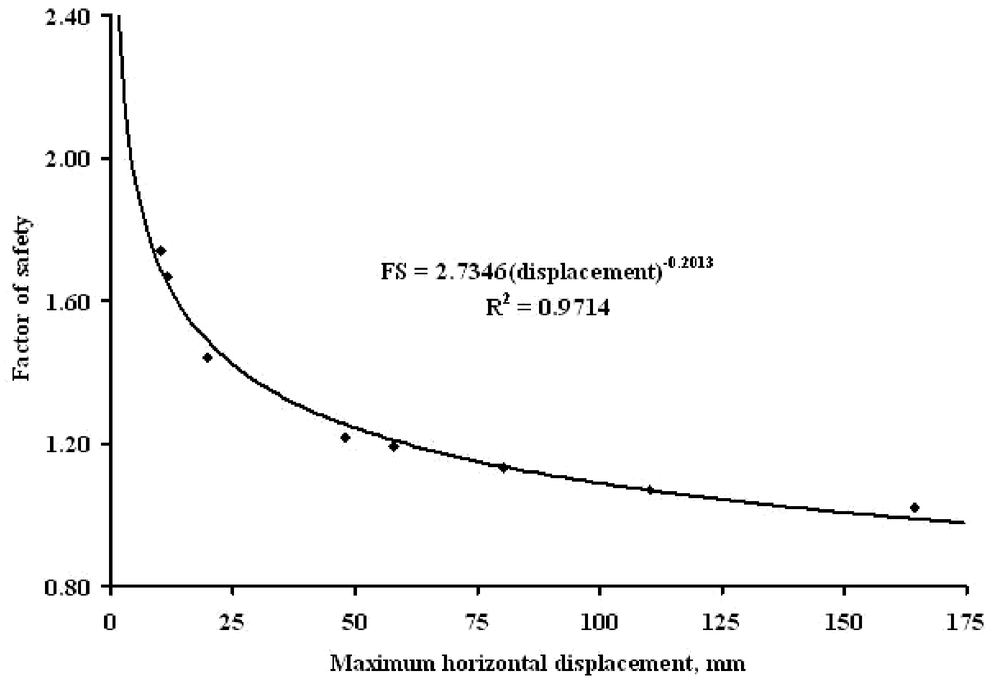


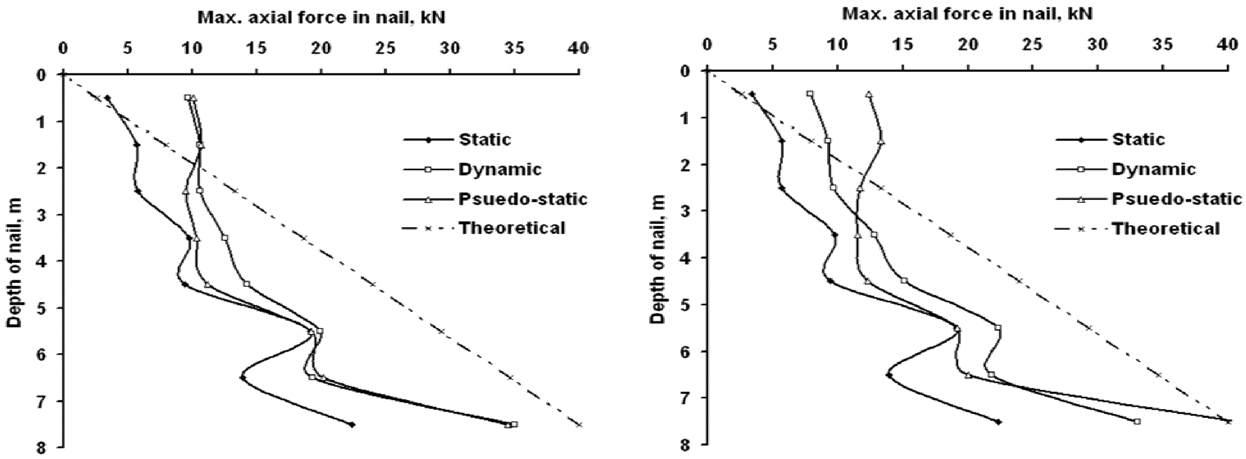
Fig. 3 Relation between the global factor of safety and the displacement of the soil nail wall

The factors of safety values in Table 4 show higher values of factor of safety for the Uttarkashi earthquake motion that has higher ground accelerations and shorter strong motion duration (equal to 8 s) in comparison to the smaller factor of safety values for the Bhuj earthquake motion with lesser amplitudes of ground accelerations and longer strong motion duration (equal to 25 s). This trend in the factor of safety values is in accordance with the above explanation and hence justifies the factor of safety values given in Table 4. Thus, it may be stated that the soil nail wall is stable under static and seismic conditions and fulfills the minimum recommended values (see Table 1), as necessary for the stability of the permanent soil-nailed structures.

Theoretically, the average maximum tensile force in the nails of the wall is calculated as $T_{\max} = K_a \gamma H S_V S_H$. However, the tensile force in the lower portion of the wall decreases considerably to approximately 50 percent of the value in the upper part. Alternatively, Briaud and Lim (1997) suggest that the average maximum in-service tensile force in the top row of the soil nails can be calculated as $T_{\max} = 0.65 K_a \gamma H S_V S_H$. For the subsequent soil nail rows, Briaud and Lim (1997) also suggest that the maximum in-service tensile force is only half of the force in the upper nails. Figures 4(a) and 4(b) show similar trends in the variation of maximum axial force in the nails with the depth of inclusion. Those illustrate that the average in-service nail force is smaller than that calculated by considering the full active earth lateral pressure distribution.

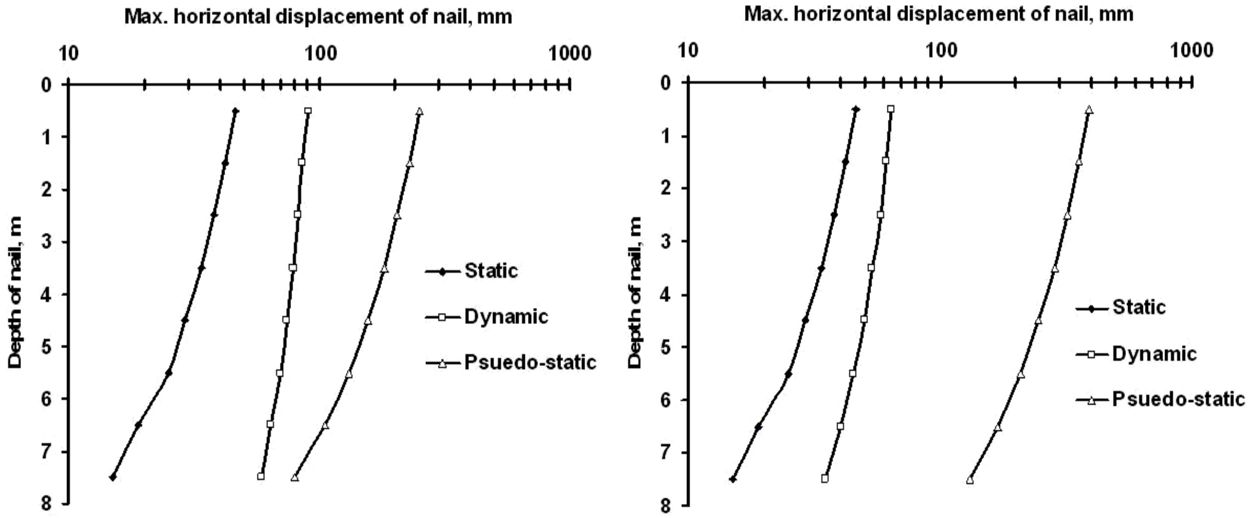
According to Juran (1985), the maximum lateral displacement of the soil nail walls does not generally exceed 0.2% of the vertical height. Additionally, according to the conventional design procedure adopted in present study, for a vertical soil nail wall with sandy soil behind, the maximum horizontal displacements at the top of the wall should not exceed 1/500 of the wall height. From the numerical simulations under the static and seismic conditions, it is observed that the displacement values of soil nail wall do not satisfy the maximum displacement limitations. Figures 5(a) and 5(b) show a comparison of the maximum horizontal displacement of the soil nails with the depth of embedment, as obtained from the static, pseudo-static and dynamic simulations of the soil nail wall. It is evident that the pseudo-static analyses predict very conservative estimates of the displacements.

The trends in the development of bending moments and shear forces in nails obey the established research findings. Compared to the axial forces, development of the bending moments and shear forces in nails is not significant. Elias and Juran (1991) have found that the shear and bending nail strengths contribute less than 10 percent to the overall stability of the wall. Due to this relatively modest contribution, the shear and bending strengths of the soil nails are conservatively disregarded in the conventional design procedure.



(a) (b)

Fig. 4 Development of the maximum (tensile) axial force in nails with the depth of embedment for (a) the Bhuj earthquake record at Ahmedabad, and (b) the Uttarkashi earthquake record at Uttarkashi



(a) (b)

Fig. 5 Variation of the maximum horizontal displacement of nails with the depth of embedment for (a) the Bhuj earthquake record at Ahmedabad, and (b) the Uttarkashi earthquake record at Uttarkashi

CONCLUSIONS

In the present study, the stability of a typical soil nail wall under seismic conditions has been examined using the conventional FHWA procedure and via numerical simulations. An overview of the various important failure modes suggests that the factor of safety values obtained conventionally as well as from the numerical simulations for different cases (i.e., static, pseudo-static and dynamic) satisfy the corresponding minimum recommended values. This implies that the soil nail wall under consideration is stable under the seismic conditions similar to those used in the present study. This is in accordance with the previous research findings about the performance of soil nail walls in seismic conditions. It has been also observed that the pseudo-static analyses result in conservative estimates of displacements and factor of safety values compared to those obtained from the time histories of the considered earthquake motions. The results show that the soil nailing technique provides a feasible, efficient, and economical alternative to the conventional retaining structures under seismic conditions, particularly for supporting vertical or near vertical cuts made in soil for various slope stability applications in geotechnical engineering.

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