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A NEW MODAL COMBINATION RULE FOR THE BUILDING RESPONSE TO EARTHQUAKES

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SYNOPSIS

A new modal combination rule has been proposed to estimate the linear design response of a multistoried building subjected to earthquake excitations. This considers the modal cross-correlation more accurately without requiring the calculation of spectral moments, and is thus shown to lead to improved estimates over those from the SRSS and CQC methods. It is also more generalized as it explicitly considers the computations of the peak factors in the modal as well as the overall responses.

INTRODUCTION

In the current earthquake-resistant design practice, it is common to characterize the design ground motion in form of a set of design spectra and to design the structures to behave elastically for the corresponding lateral loads. These lateral loads may be reduced by appropriate reduction factors if the design spectra are obtained for the most critical ground motion expected at the site of the structure. By definition, design spectra are measure of the mean maximum response of the single-degree-of-freedom (SDOF) systems to an ensemble of the possible critical ground motions, and thus, those do not directly characterize these motions. A suitable modal combination rule is therefore required to estimate the design response of the multi-degree-of-freedom (MDOF) systems. Considerable effort has been devoted by several research workers [e.g., Goodman et al. (1953), Rosenblueth and Elorduy (1969), Singh and Chu (1976), Der Kiureghian (1981), Wilson et al. (1981), Singh and Mehta (1983), Der Kiureghian and Nakamura (1993)] to the problem of estimating the largest peak response in a MDOF system accurately from the available design spectra. Most of these studies have given simple rules of modal combination [e.g. the SRSS rule by Goodman et al. (1953), and CQC rule by Wilson et al. (1981)] while keeping the constraints of the design engineer in view. However, none of these is both simple and accurate at the same time, in all practical situations. For example, the popular CQC method is not applicable when the input excitation is narrow-banded and/or flexible to the structural system.

In this study, a new, simple, and reasonably accurate modal combination rule has been presented for predicting the largest response peak amplitude in a multistoried building response directly from the prescribed design spectra. The approach of Gupta and Trifunac (1989) which required the knowledge of the power spectral density function (PSDF) of the ground motion has been reformulated with the purpose of integrating the earlier modal combination rules. The need of computing the power spectral density function (PSDF) of the system response has been obviated by making necessary simplifications regarding i) the modal cross-correlation terms in the total response, ii) the total number of peaks in the system response, iii) the peak factors for the response peaks, and iv) the spread of energy among the excitation and response frequencies. One example building and two example excitations have been considered to show the illustration of the proposed rule and its comparison with the popular CQC method.

BASIC FORMULATION

Let us consider a n-degrees of freedom building with the mass matrix, $[m]$ and stiffness matrix, $[k]$. For ground acceleration, $\ddot{z}(t)$ at its base, the n-coupled undamped equations of motion for this system can be written as

$$[m]\{\ddot{x}(t)\} + [k]\{x(t)\} = -\ddot{z}(t)[m]\{\Gamma\} \tag{1}$$

where $\{\Gamma\}$ is the ground displacement influence vector and $\{x(t)\}$ is the relative displacement vector. These equations can be decoupled into n equations, each describing the motion in a specific mode, by premultiplying Eq. 1 with the transpose of the modal matrix, $[A]^T$ $(= [\{\phi^{(1)}\}\{\phi^{(2)}\}....\{\phi^{(n)}\}]^T)$ and by expanding $\{x(t)\}$ in terms of the normal coordinate vector, $\{\xi(t)\}$, as

$$\ddot{\xi}_j + \omega_j^2 \xi_j = -\alpha_j \ddot{z}\ ; \qquad j = 1, 2, ..., n \tag{2}$$

where, $\alpha_j = \{\phi^{(j)}\}^T[m]\{\Gamma\}/\{\phi^{(j)}\}^T[m]\{\phi^{(j)}\}$ and ω_j respectively are the modal participation factor and natural frequency in the jth mode. Introducing damping through damping ratios, ζ_1, ζ_2,..., ζ_n into these equations and solving for the normal coordinates in frequency domain, the ith relative displacement, $X_i(\omega)$ (in frequency domain) can be expressed as

$$X_i(\omega) = \sum_{j=1}^{n} \phi_i^{(j)} \alpha_j H_j(\omega) Z(\omega) \tag{3}$$

where

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + 2i\zeta_j\omega_j\omega} \tag{4}$$

is the transfer function relating the relative displacement of the equivalent SDOF oscillator in the jth mode to the input base excitation, $\phi_i^{(j)}$ is the ith element of the jth mode shape vector, $\{\phi^{(j)}\}$, and $Z(\omega)$ is the Fourier transform of $\ddot{z}(t)$. On assuming stationarity in the excitation and the response, the expression of the PSDF, $S_{x_i}(\omega)$, of $x_i(t)$ response process becomes [Gupta and Trifunac (1989)]

$$S_{x_i}(\omega) = S_{\ddot{z}}(\omega) \sum_{j=1}^{n} |H_j(\omega)|^2 \left[(\phi_i^{(j)})^2 \alpha_j^2 + \sum_{k=1,\ k\neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k \left\{ C_{jk} + \left(1 - \frac{\omega^2}{\omega_j^2}\right) D_{jk} \right\} \right], \tag{5}$$

where, $S_{\ddot{z}}(\omega)$ is the PSDF of the input base excitation process, $\ddot{z}(t)$, and C_{jk} and D_{jk} are the coefficients given in terms of ζ_j, ζ_k and $\varrho = \omega_k/\omega_j$ as

$$C_{jk} = \frac{1}{B_{jk}}\left[8\zeta_j(\zeta_j + \zeta_k\varrho)\{(1-\varrho^2)^2 - 4\varrho(\zeta_j - \zeta_k\varrho)(\zeta_k - \zeta_j\varrho)\}\right] \tag{6}$$

$$D_{jk} = \frac{1}{B_{jk}}\left[2(1-\varrho^2)\{4\varrho(\zeta_j - \zeta_k\varrho)(\zeta_k - \zeta_j\varrho) - (1-\varrho^2)^2\}\right] \tag{7}$$

and

$$B_{jk} = 8\varrho^2\left[(\zeta_j^2 + \zeta_k^2)(1-\varrho^2)^2 - 2(\zeta_k^2 - \zeta_j^2\varrho^2)(\zeta_j^2 - \zeta_k^2\varrho^2)\right] + (1-\varrho^2)^4 \,. \tag{8}$$

Generalizing Eq. 5 for any response, $r(t)$ of our linear system such that $r(t) = \sum_{j=1}^{n}\rho_j\xi_j(t)$, we can write

$$S_r(\omega) = S_{\ddot{z}}(\omega)\sum_{j=1}^{n}\left[\left\{\rho_j^2\alpha_j^2 + \sum_{k=1,\ k\neq j}^{n}\rho_j\rho_k\alpha_j\alpha_k(C_{jk} + D_{jk})\right\}|H_j(\omega)|^2 - \frac{|\omega H_j(\omega)|^2}{\omega_j^2}\sum_{k=1,\ k\neq j}^{n}\rho_j\rho_k\alpha_j\alpha_k D_{jk}\right] \tag{9}$$

as the spectral density function of $r(t)$. Here, ρ_j is the normalized amplitude of response, $r(t)$ in the jth mode of vibration and is expressed as a linear combination of the elements of the jth mode shape, $\{\phi^{(j)}\}$. For example, it is equal to $\phi_i^{(j)}$ for the displacement response at the ith floor level.

Taking moments of $S_r(\omega)$ about the origin leads to [see Gupta (1994) for details]

$$\lambda_p = \sum_{j=1}^{n}\rho_j^2\alpha_j^2\lambda_{p,j}^{D}(1+\delta_{p,j}), \quad p = 0, 1, 2, \ldots \tag{10}$$

where, $\lambda_{p,j}^{D}$ is the pth moment of the PSDF of displacement response of a SDOF oscillator oscillator with ω_j frequency and ζ_j damping ratio and subjected to the base acceleration, $\ddot{z}(t)$, and

$$\delta_{p,j} = \sum_{k=1,\ k\neq j}^{n}\frac{\rho_k\alpha_k}{\rho_j\alpha_j}(C_{jk} + D_{jk}\gamma_{p,j}) \tag{11}$$

is the correction term accounting for the cross-correlation of jth mode with the remaining $(n-1)$ modes. $\gamma_{p,j}$ is the multiplying factor for D_{jk} and it is expressed as

$$\gamma_{p,j} = 1 - \frac{\lambda_{p,j}^{V}}{\omega_j^2\lambda_{p,j}^{D}} \tag{12}$$

where, $\lambda_{p,j}^{V}$ is the pth moment, similar to $\lambda_{p,j}^{D}$, for relative velocity response. For $p = 0$, $\delta_{p,j}$ is a measure of the deviation of the rate of zero crossings of the displacement response of the SDOF system from what would have been, had this system been subjected to an ideal white noise. For most excitation processes, this continuously decreases with the increasing natural period of the oscillator, remaining negative when the SDOF system is more flexible compared to the excitation process, and positive when the SDOF system is stiffer. It becomes zero near the dominant frequency of the ground motion.

By calculating the required moments from Eq. 10 for $p = 0$, 2 and 4, and then multiplying the root-mean-square (r.m.s.) value, $r_{\rm rms}$ with the peak factor, the largest peak amplitude of the response, $r(t)$ with the desired level of confidence can be estimated. Since the response process is nonstationary, $r_{\rm rms}$ may be modified by using the response spectrum amplitudes. If SD_j represents the spectral displacement corresponding to the natural frequency, ω_j and damping ratio, ζ_j, and β_j represents the 'nonstationarity' factor by which the r.m.s. value in the jth mode is modified, $\sqrt{\lambda^D_{0,j}}$ may alternatively be represented as $\mathrm{SD}_j/\beta_j\eta_j$ where η_j is the peak factor for the largest peak displacement response of the jth mode oscillator. Hence, it is possible to express the largest peak response of $r(t)$ as [see Gupta (1994) for details]

$$\bar{r}^2_{\rm peak} = \sum_{j=1}^{n} \frac{\beta^2}{\beta_j^2}\frac{\eta^2}{\eta_j^2}\rho_j^2\alpha_j^2\mathrm{SD}_j^2(1+\delta_{0,j}) \tag{13}$$

where, η is the peak factor corresponding to the largest peak of response, $r(t)$, and β represents the 'nonstationarity' factor by which $r_{\rm rms}$ is modified. Let $\bar{r}_{j,\max} = \rho_j\alpha_j\mathrm{SD}_j$ represent the maximum value of the response, $r(t)$ in the jth mode as obtained from the response spectrum curves. Then, $\bar{r}_{\rm peak}$ may alternatively be expressed in the familiar SRSS form as

$$\bar{r}^2_{\rm peak} = \sum_{j=1}^{n} \bar{r}^2_{j,\rm peak} \tag{14}$$

with

$$\bar{r}_{j,\rm peak} = \frac{\beta}{\beta_j}\frac{\eta}{\eta_j}\bar{r}_{j,\max}(1+\delta_{0,j})^{\frac{1}{2}} \tag{15}$$

representing the contribution of the jth mode to the largest peak response. It has been customary in the formulations of the earlier modal combination rules to implicitly assume that $\beta_j = \beta$ and thus to consider

$$\bar{r}_{j,\rm peak} \simeq \frac{\eta}{\eta_j}\bar{r}_{j,\max}(1+\delta_{0,j})^{\frac{1}{2}} \tag{16}$$

as the peak modal contribution. In fact, higher modes are associated with faster convergence to the state of stationarity by the response due to greater number of cycles per unit time. This implies that the effective damping as defined by Rosenblueth and Elorduy (1969) is greater in the lower modes. Due to this, β/β_j is likely to be greater than unity in case of lower modes, and less than unity for the higher modes. Assuming β/β_j uniformly equal to unity for all the modes is thus equivalent to decreasing the lower mode response and increasing the higher mode response. Due to this, the resultant response as calculated may deviate only slightly from the exact value. If η_j is also assumed same as η, Eq. 16 becomes

$$\bar{r}_{j,\rm peak} \simeq \bar{r}_{j,\max}(1+\delta_{0,j})^{\frac{1}{2}} \tag{17}$$

and then, it can be shown that this with Eq. 14 leads to the popular modal combination rules, depending on how the calculations for $\delta_{0,j}$ are simplified through different approximations [Gupta (1994)]. In the absence of any further approximation, this becomes same as the Generalized CQC method as proposed by Der Kiureghian and Nakamura (1993). If $\gamma_{0,j}$ is approximated by $(1 - \mathrm{SV}_j^2/\mathrm{PSV}_j^2)$ in the calculation of $\delta_{0,j}$, this leads to the formulations proposed by Singh and Chu (1976), and Singh and Mehta (1983) where SV_j and PSV_j ($= \omega_j SD_j$) respectively represent the spectral velocity and pseudo spectral velocity ordinates for the given base excitation. By this approximation, $\gamma_{0,j}$ may become zero at several oscillator periods, not just at the dominant period of ground motion, and thus, may sometimes lead to highly erroneous estimates of $\delta_{0,j}$ in case of wide-band ground motion processes [Gupta (1994)]. If $\gamma_{0,j}$ is assumed to be identically zero for all modes, Eq. 17 instead gives an alternative form of the Original CQC method (hereafter referred to as the CQC method) proposed by Der Kiureghian (1981), and Wilson et al. (1981). Implicit behind this simplification is the assumption that $\lambda^V_{0,j} \simeq \omega_j^2 \lambda^D_{0,j}$ for all the modes which may actually be valid for the truly narrow band processes only. In case of the usually found levels of damping, if the oscillator frequency is not close to the dominant frequency of ground motion, particularly in the cases of the excitation energy distributed close to a single frequency, this assumption may give quite unacceptable results [see Singh and Mehta (1983) for illustrative examples]. For very lightly damped systems and when the excitation energy is distributed in a wide band of frequencies, this assumption is reasonable for those modes which are not too stiff to the ground motion. In the formulation for the CQC method, if C_{jk} is also assumed to be zero (thus effectively assuming the contribution of modal cross-correlation denoted by $\delta_{0,j}$ to be negligibly small for all the modes), the SRSS method proposed by Goodman et al. (1953) is obtained. It is obvious that the SRSS method has all the limitations of the CQC method. Besides, this method has further narrow range of applicability since C_{jk} can be zero only for those modes whose frequencies are far apart from each other, and therefore, this cannot be applied to those structures which have closely spaced modes e.g., those with torsional coupling. This method is undoubtedly the simplest method for a practising engineer, and that is the reason why it is still popular with several working professionals world-wide despite its serious limitations. Several other methods proposed in the past, e.g., those by Rosenblueth and Elorduy (1969), Kennedy (1979), and Gupta and Chen (1983) can also be shown to be simpler forms of Eq. 15.

There is an implicit assumption in all the previous modal combination rules that the peak factors for the system and the modal responses can be taken as same. Further, the modal cross-correlation is accurately accounted for only by the Generalized CQC method. This method however requires the computation of the spectral moments which may not be an attractive proposition to many practising engineers. A simplified alternative approach is obtained in the following section by making new approximations in Eq. 16 regarding $\gamma_{0,j}$ and η/η_j.

APPROXIMATIONS FOR THE PROPOSED APPROACH

In order to evolve a simplified yet accurate modal combination rule, it is desirable that calculations for the spectral moments are completely avoided without significantly sacrificing accuracy. As mentioned earlier, the value of $\gamma_{0,j}$ may be taken as negative for the periods longer and positive for the periods shorter than the dominant period in the ground motion. Hence, it is proposed to make the SV/PSV approximation of Singh and Mehta (1983) which closely follows the exact calculations of $\gamma_{0,j}$ based on spectral moments, more reasonable for the wide-band excitations. If the approximate value of $\gamma_{0,j}$ calculated on the basis of SV/PSV approximation is found to be negative for the periods shorter than the dominant period, it may be made equal to $\bar{\gamma}$. If this value is found to be positive for the longer periods, it may be made equal to $-\bar{\gamma}$. This correction may be required only at the small values of $\gamma_{0,j}$, and an appropriate value of the parameter, $\bar{\gamma}$ may be fixed by studying the exact $\gamma_{0,j}$ curves for different ground motions with the wide-band characteristics. Qualitatively, this parameter increases with the damping ratio, and based on the El Centro excitation, this may be taken as 0.08, 0.10 and 0.22 respectively for the 2%, 5% and 10% damping ratios. As regards the factor, η/η_j, it can be shown that

$$\frac{\eta}{\eta_j} = 0.95\frac{2.0 + 0.35\ln(N/5)}{2.0 + 0.35\ln(N_j/5)} \tag{18}$$

provides a good approximation [Gupta (1994)]. N and N_j here represent the number of peaks respectively in the total response and in the jth modal response. Exact determination of these parameters will require the calculation of spectral moments, thus making the proposed procedure computationally unattractive. However, it is possible to approximate N_j by its lower bound estimate while assuming that the modal response is a narrow-band process, and thus, $N_j \simeq (T/2\pi)\omega_j$, where T is the stationary duration of the ground motion. For approximating N, following relationship may be used:

$$\frac{1}{N^2} \simeq \frac{1}{n+1}\left[\frac{1}{N_1^2} + \frac{1}{N_2^2} + \dots + \frac{1}{N_n^2} + \frac{1}{N_g^2}\right] \tag{19}$$

where N_g $(= T/T_g)$ is the number of peaks corresponding to the ground period, T_g. T_g may be taken as that period at which the Fourier spectrum of the ground motion becomes maximum. It has been found that the total number of peaks computed by using Eq. 19 correlates well with the exact number of peaks in several example cases of response time histories for the wide-band as well as the narrow-band earthquake processes [Gupta (1994)].

ILLUSTRATIONS OF THE PROPOSED APPROACH

A fixed-base multistoried building has been considered here for the illustration of the approach presented in the previous section. This is a 7-story building having a non-uniform floor dimensions, 26×33 m for the bottom three stories and 20×25 m for the remaining four at the top. The bottom three floors are of 1.15×10^6 kg mass and 12.015 m radius of gyration each, and the top four floors are of 0.8×10^6 kg mass and 9.242 m radius of gyration each. The story stiffnesses in the X-direction (i.e. in the direction of the ground motion) are taken respectively as 2.212, 2.696, 3.1, 3.424, 4.998, 5.24, 5.361×10^6 kN/m for the top to bottom stories. The corresponding values in the Y-direction respectively are 1.392, 2.179, 2.697, 3.112, 4.792, 5.083, 5.229×10^6 kN/m. The story torsional stiffnesses have been computed by neglecting the contributions of the torsional stiffnesses of various supporting structural elements about their longitudinal axes. Further, with $r = 9.242$ m, the static eccentricities in the X and Y-directions respectively are $0.2r$ and $0.26r$ for the top three floors, $0.24r$ and $0.26r$ for the next lower floor, and $0.34r$ and $0.50r$ for the bottom three floors. Thus, the centres of stiffness are not lying on a vertical straight line. The centres of mass are assumed to lie on a vertical axis. The natural frequencies are computed to be 12.49, 14.63, 18.87, 29.21, 34.99, 42.74, 45.81, 55.81, 63.11, 67.53, 75.27, 77.27, 88.52, 91.23, 96.62, 100.39, 108.54, 113.41, 122.41, 129.43, and 156.98 rad/sec. The critical damping ratio has been assumed to be equal to 0.05 in all the modes of the example building.

The input ground motion has been characterized by two example excitations of widely different characteristics. These are: i) recorded S00E component at El Centro site during Imperial Valley Earthquake, 1940, and ii) synthetically generated motion for Mexico Earthquake, 1985 at Mexico City site as in Gupta and Trifunac (1989, 1990). The first excitation is of broad-band nature with the distribution of energy in the range of 0.2–5.0 sec, while the second excitation is of narrow-band nature with heavy concentration of energy at the periods close to 2.5 sec. The pseudo spectral acceleration response spectra for these excitations in case of 5% damping are shown in Fig. 1. The value of ground period, T_g for these spectra is taken as 0.17 and 0.97 sec respectively for the Imperial Valley and Mexico City earthquakes. Thus, the example building is flexible compared to the Imperial Valley excitation, while in case of the Mexico City earthquake, it is stiffer. Since the Imperial Valley excitation is of broad-band nature, all significant modes of the building are almost uniformly excited as in the case of ideal white noise. This is however not true for Mexico City excitation. To compute the total number of peaks, values of the stationary duration, T for these excitations have been taken as 24.44 and 46.44 sec following the definition given by Trifunac and Brady (1975).

The example building has been subjected to the two example excitations and analyzed for the story shear response in the direction of earthquake ground motion by i) exact time history analysis with step-by-step integration, ii) CQC method, and iii) the proposed approach. The results of these methods have been compared in Fig. 2 and Fig. 3 by plotting the envelopes of the largest peak shears. In each figure, the envelope values have been normalized to the corresponding overall maximum shear force. It may be observed from these figures that the CQC method gives poor or very poor estimates, particularly in the case of Mexico City excitation where the estimates by this method may be in error by as much as 40%. On the other hand, the estimates given by the proposed approach are always better than those by the CQC method and also have good agreement with the exact values. Thus, the proposed approach offers a better alternative to the CQC method.

CONCLUSIONS

A new modal combination rule has been proposed for estimating the largest 'expected' peak response of a multistoried building when it is subjected to an earthquake excitation specified through a set of design spectra. The formulation of this rule is based on more realistic computations of the peak factors. Through the example cases, the proposed rule has been found to work reasonably well in those critical cases also where the earlier methods, e.g., the SRSS and CQC methods gave significant errors.

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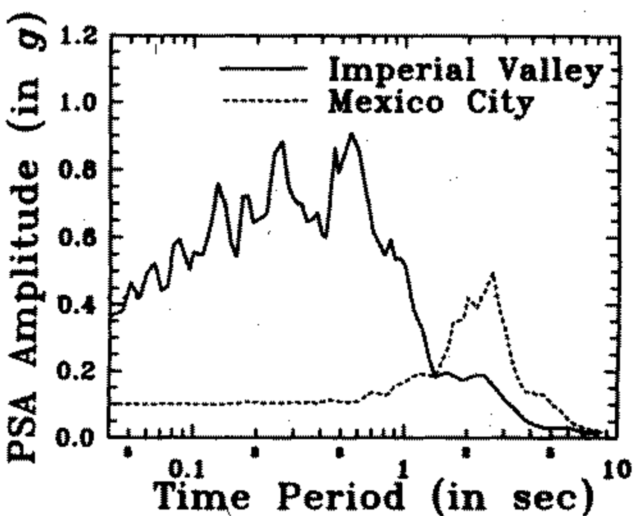

Figure 1 5% damping pseudo spectral acceleration response spectra for the example excitations.

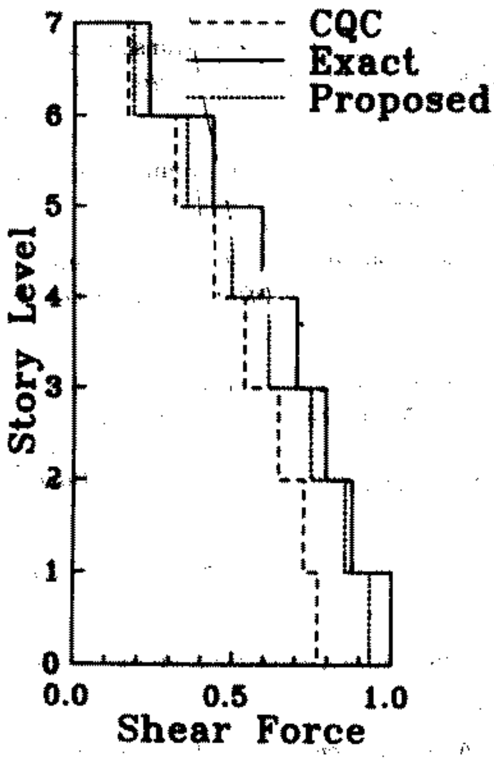

Figure 2 Comparison of shear force envelopes for Imperial Valley Earthquake.

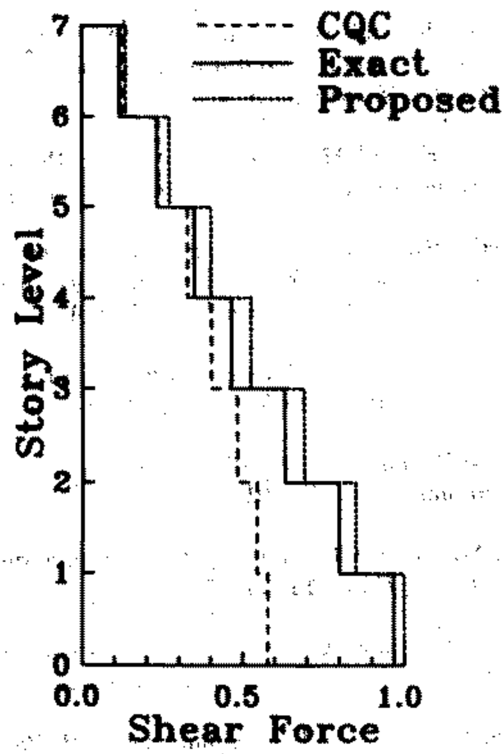

Figure 3 Comparison of shear force envelopes for Mexico Earthquake.