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DESIGN SPECTRUM-BASED SCALING OF STRENGTH REDUCTION FACTORS

Arindam CHAKRABORTI[1] and Vinay K. GUPTA[2]

SUMMARY

The inelastic (design) spectra characterizing seismic hazard at a site are generally obtained by the scaling-down of the elastic (design) spectra via the use of response modification factors. These factors depend significantly on strength reduction factors (SRFs), where SRF represents the ratio of elastic strength demand to the inelastic strength demand of a single-degree-of-freedom oscillator, with the inelastic deformations limited to a specified ductility demand ratio. SRF spectrum gives the variation of this factor with the initial period of the oscillator. This study considers the scaling of SRF spectrum in case of an elasto-plastic oscillator undergoing strength and stiffness degradations. A new model is proposed in terms of the pseudo-spectral acceleration (PSA) values, when normalized to unit peak ground acceleration (PGA), and ductility demand ratio and a ductility supply-related parameter. Least-square estimates of the coefficients are obtained through linear regression analyses of the data for 956 recorded accelerograms in western USA. Parametric studies carried out with the help of the proposed model show that higher earthquake magnitude and/or alluvium site geology may result in higher SRFs for medium- to long-period structures.

INTRODUCTION

It is common to characterize seismic hazard at a site and to estimate the design forces or displacements of a linearly behaving single-degree-of-freedom (SDOF) structure, with specified period and damping, through elastic design spectrum. From economic point of view, however, structures need to be designed so as to permit dissipation of input energy by means of large inelastic deformations during severe ground shaking. Therefore, it is considered convenient to obtain inelastic design spectra as scaled-down forms of elastic design spectra. The scaling-down is achieved by the use of response modification factors, where a response modification factor is a product of (i) strength reduction factor (SRF), (ii) structural overstrength factor, and (iii) redundancy factor (ATC [1]). SRFs account for the non-linear characteristics of the structure, and thus play the most important role in the determination of response modifcation factors and in their parametric dependence on various structural and ground motion characteristics.

The study of SRF was initiated by Newmark [2]. They applied the equal-displacement, equal-energy, and equal-acceleration principles to estimate analytically the SRFs for long-period, short-period and zero-

[1] Formerly Graduate Student, IIT Kanpur, India
[2] Professor, IIT Kanpur, India

period structures, respectively, as functions of ductility (demand) ratio. This was followed by several studies based on actual computations of SRFs for elasto-plastic or more refined oscillators subjected to artificial or recorded ground motions (e.g., see Elghdamsi [3], Krawinkler [4], Miranda [5]). However, most of this research considered the effects of only one or two governing parameters simultaneously on SRFs, and thus, suffered from the limitation of the data-set not being large enough. For a data-set practically available for any study at present, various source and site parameters related to a ground motion have to be considered simultaneously along with the structural characteristics. Tiwari [6] proposed a comprehensive model in terms of earthquake magnitude, strong motion duration, predominant period of ground motion, geological site condition, and ductility demand ratio. However, some of these parameters may not always be conveniently available to the designer.

Ordaz [7] incorporated the effects of various governing parameters by expressing SRFs as a function of elastic spectral displacements and peak ground displacement. Use of peak ground displacement as an input parameter is however inconvenient since its value is determined by twice-integration of recorded accelerogram and is thus sensitive to the integration algorithm and missing out of ground motion in the beginning (due to delay in triggering of the accelerograph). In fact, peak ground displacement has never been considered important for seismic hazard characterization.

Since design spectra (normalized with respect to peak ground acceleration) form a more convenient input, this paper proposes an alternative scaling model in terms of the normalized pseudo-spectral acceleration spectrum. A stiffness-degrading and strength-degrading oscillator proposed by Gupta [8] has been used to model the non-linear behaviour of structures. Unlike earlier hysteretic models, this oscillator models the structural behaviour at global level, not at the elemental level. This considers the initial yield displacement level and ductility supply-related parameter as two additional input parameters.

Regression coefficients have been obtained in case of the proposed model for several sets of ductility (demand) ratio and ductility (supply) ratio parameters at several time periods. The regression analyses have been carried out for a database of 956 horizontal motion accelerograms corresponding to 106 earthquakes in western United States, between 1931 Long Beach earthquake, California and 1984 Morgan Hill earthquake, California (with details as in Lee [9]). The error estimates for different levels of confidence are presented along with the smoothed regression coefficients. The proposed model has been used to carry out a parametric study, and to see whether the dependence of SRFs on earthquake magnitude and site condition, as shown by the proposed model, is in agreement with the trends shown by earlier studies.

CALCULATION OF RAW R_μ DATA

SRF for a non-linear SDOF oscillator is defined as the ratio of elastic strength demand to inelastic strength demand such that the displacement ductility ratio is limited to a maximum value of μ, where ductility ratio is the ratio of the maximum inelastic displacement of the oscillator to its yield displacement. Thus, for a target ductility ratio, μ_i, $R_\mu|_{\mu=\mu_i}$ is defined as

$$R_\mu|_{\mu=\mu_i} = \frac{F_{y,1}}{F_{y,\mu_i}} \tag{1}$$

where, $F_{y,1}$ is the minimum strength for no yielding in the oscillator (i.e., when μ = 1), and F_{y,μ_i} is the minimum strength at first yield for inelastic deformations limited to the ductility ratio of μ_i. It is well known that $R_\mu(T) \to 1$, as $T \to 0$, and $R_\mu(T) \to \mu$ as $T \to \infty$.

For a given damping ratio of the SDOF oscillator, ductility ratio and earthquake ground motion, $R_\mu(T)$ is function of the type of non-linearity in the oscillator. This study uses a modified Clough-Johnston oscillator, proposed by Gupta [8]. This oscillator is elasto-plastic in nature, and undergoes stiffness and strength degradations. Here, the strength degradation is considered to be a function of initial yield displacement level, a ductility supply-related parameter, n, and accumulated plastic deformations. n is a measure of ductility supply ratio of the oscillator (estimated as $3.44n^{0.31}$ by Gupta [8]). The stiffness deterioration is characterized by the instantaneous values of yield displacement level and accumulated plastic deformations. The damping is assumed to be F-damping (i.e., with no effect of non-linear behaviour) with value equal to 5% of critical damping.

To compute the raw R_μ data for a given ground motion and initial time period, T, the oscillator has been subjected to the ground motion, and the lateral yield strength, F_{y,μ_i}, has been iterated until the calculated displacement ductlity (demand) ratio is within 1% of $\mu = \mu_i$. The non-linear time history analysis of the oscillator has been performed by using the fourth order Runge-Kutta method with an adaptive step size control scheme and with step size taken as $(1/\Delta t)\times$ ground motion duration, where $\Delta t = 0.01T$. During the iterations, if more than one values of F_{y,μ_i} are obtained for the same ductility ratio, the largest value has been considered for obtaining the minimum value of R_μ. The computation of R_μ for the given ground motion record has been repeated for 56 initial time periods from T = 0.1 to 4.0 s, in case of n = 6, 10 and μ = 2, 4, 6. The complete database for the raw R_μ data has been created by considering 956 horizontal accelerograms which were recorded during the 106 earthquake events in the western U.S.A. region from 1931 to 1984 (see Lee [9] for further details).

SCALING RELATIONSHIP AND REGRESSION ANALYSIS

It may be observed that normalized response spectrum, $PSA(T)/PGA$, for a ground motion shows quite similar trends as those shown by the SRF spectrum for the same ground motion at short and intermediate time periods. Both spectra approach unity as $T \to 0$. This similarity is however lost as $T \to \infty$ and normalized spectrum approaches zero value against SRF spectrum approaching the value of μ. Assuming that SRF spectrum attains the limiting value of μ at T = 10 s, $R_\mu(T)$ may be described by the following functional form:

$$R_\mu(T) = \beta(T)\left(\frac{PSA(T)}{PGA}\right)^{\alpha(T)} + \mu\frac{T}{10} \tag{2}$$

This form is obtained by superimposing a modified form of $PSA(T)/PGA$ curve over a line of $\mu/10$ slope and passing through origin. Based on Eq. (2), the scaling equation for $R_\mu(T)$ has been considered to be

$$\log_{10}\left(R_\mu(T) - \mu\frac{T}{10}\right) = b_1(T)\log_{10}\left(\frac{PSA(T)}{PGA}\right) + b_2(T) \tag{3}$$

where, $b_1(T)$ and $b_2(T)$ are (period-dependent) regression coefficients for a set of ductility (demand) ratio, μ, and ductility supply-related parameter, n. For convenience, the functional, $(R_\mu(T) - \mu T/10)$, will be referred to as $X_\mu(T)$ hereafter.

Linear regression analyses based on Eq. (3) have been carried out for 6 combinations of μ = 2, 4, 6, and n = 6, 10 at 56 periods. In order to remove bias on the values of the regression coefficients, which the uneven distribution of data among magnitude ranges, 3.0-3.9, 4.0-4.9, 5.0-5.9, 6.0-6.9, 7.0-7.9, may result in, data screening has been carried out, as in Tiwari [6]. Thus, for each regression analysis (for a set of T,

μ, and n), there are a maximum of 19 data points taken from each magnitude range. Let $\hat{b}_1(T)$ and $\hat{b}_2(T)$ denote the smoothed least-square estimates of the coefficients, $b_1(T)$ and $b_2(T)$, respectively. Those lead to the least-square estimate of $X_\mu(T)$ as

$$\log_{10} X_\mu(T) = \hat{b}_1(T)\log_{10}\left(\frac{PSA(T)}{PGA}\right) + \hat{b}_2(T) \tag{4}$$

The differences between the actual and the above estimates of $\log_{10} X_\mu(T)$ give the residuals which have been used to obtain mean, $m(T)$, and standard deviation, $\sigma(T)$, for all 6 combinations of n and μ. $m(T)$ and $\sigma(T)$ have been then smoothed along T. By assuming the normal distribution to describe the distribution of the calculated residuals, the error estimates at specified levels of confidence can be calculated from the smoothed values of $m(T)$ and $\sigma(T)$. Those estimates may then be added to the calculated value of $\log_{10} X_\mu(T)$ to obtain the value of $R_\mu(T)$ at the desired level of confidence. 'Goodness of fit' tests have also been performed to check the validity of normal distribution assumption, and it has been found that except for very short time periods, the normal distribution is a reasonable distribution for the residuals.

RESULTS AND DISCUSSION

The smoothed least-square estimates of regression coefficients, $\hat{b}_1(T)$ and $\hat{b}_2(T)$, along with the smoothed $m(T)$ and $\sigma(T)$ values, are shown in Tables 1 and 2 for n = 6, 10, and μ = 4. The probabilistic estimates of SRF spectra obtained from these estimates for p = 0.1, 0.5 and 0.9 have been obtained and compared with the actual spectra, in case of n = 6, μ = 4, for three recorded accelerograms. These accelerograms are (a) N75W component recorded at Coyote Lake dam during the 1984 Morgan Hill earthquake, (b) east component recorded at Stone Corral, Parkfield during the 1983 Coalinga earthquake, and (c) S65E component recorded at 6074 Park Drive (ground level), Wrightwood during the 1971 San Fernando earthquake. Figs. 1–3 show these comparisons for the Morgan Hill earthquake, Coalinga earthquake, and San Fernando earthquake motions, respectively. It is observed that the proposed model nicely reflects the trends in the actual R_μ spectrum; it captures the peaks in the low- to intermediate-period range fairly well. If we use a design spectrum with less fluctuations as $PSA(T)$, more smooth SRF spectra would be obtained.

Table 1 – Least Square Estimates of Regression Coefficients and Residual Parameters for μ = 4 and n = 6

Period, T (s)	Least Square Estimates		Residual Parameters	
	$10\hat{b}_1(T)$	$10\hat{b}_2(T)$	$100\,m(T)$	$10\,\sigma(T)$
0.10	8.119	0.784	-5.503	0.775
0.15	4.898	2.377	-6.661	1.286
0.20	2.775	3.469	-6.752	1.571
0.30	0.904	4.611	-5.644	1.733
0.40	0.361	5.183	-4.723	1.750
0.50	0.414	5.445	-4.331	1.743
0.60	0.571	5.604	-4.184	1.744
0.70	0.724	5.741	-4.067	1.748
0.80	0.864	5.859	-3.960	1.755
0.90	0.986	5.956	-3.850	1.762
1.00	1.087	6.033	-3.736	1.770
1.50	1.473	6.300	-3.149	1.816
2.00	1.741	6.476	-2.508	1.865
3.00	2.129	6.756	-0.937	1.948
4.00	2.445	7.009	-0.900	2.010

Table 2 – Least Square Estimates of Regression Coefficients and Residual Parameters for μ = 4 and n = 10

Period, T (s)	Least Square Estimates		Residual Parameters	
	$10\hat{b}_1(T)$	$10\hat{b}_2(T)$	$100\,m(T)$	$10\,\sigma(T)$
0.10	8.018	0.904	-5.575	0.755
0.15	4.929	2.471	-6.617	1.268
0.20	2.872	3.552	-6.668	1.556
0.30	1.018	4.695	-5.596	1.727
0.40	0.444	5.267	-4.736	1.752
0.50	0.457	5.523	-4.394	1.750
0.60	0.583	5.670	-4.290	1.754
0.70	0.708	5.795	-4.213	1.761
0.80	0.824	5.898	-4.146	1.769
0.90	0.925	5.981	-4.080	1.779
1.00	1.008	6.041	-4.014	1.790
1.50	1.318	6.215	-3.730	1.855
2.00	1.532	6.292	-3.513	1.942
3.00	1.878	6.401	-3.021	2.147
4.00	2.213	6.530	-2.400	2.360

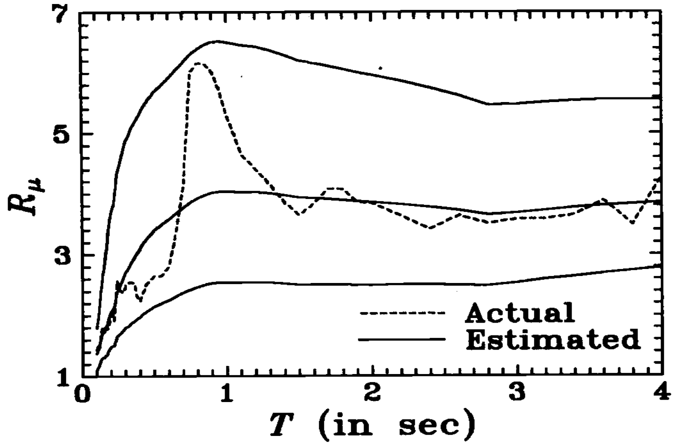

Figure 1 - Comparison of the Actual and Estimated SRF Spectra for Morgan Hill Earthquake Case with μ = 4 and n = 6.

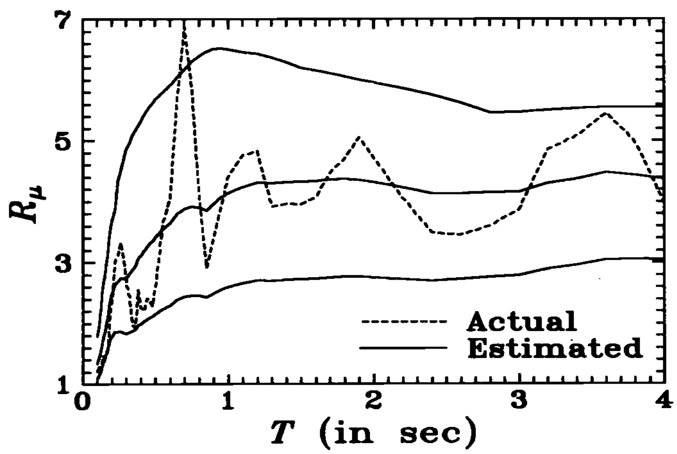

Figure 2 - Comparison of the Actual and Estimated SRF Spectra for Coalinga Earthquake Case with μ = 4 and n = 6.

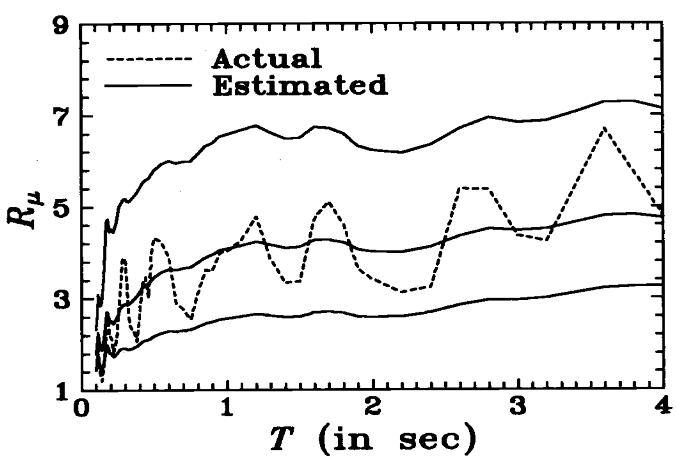

Figure 3 - Comparison of the Actual and Estimated SRF Spectra for San Fernando Earthquake Case with μ = 4 and n = 6.

According to the studies of Trifunac [10, 11], both $PSA(T)$ and PGA may be estimated in terms of the parameters, earthquake magnitude, M , geological site condition, and epicentral distance, R , for a given level of confidence. Therefore, it may be interesting to consider these models (say, by taking 0.5 confidence level and 100 km epicentral distance) together with the proposed model for $R_\mu(T)$, and to study the variation of SRF spectra due to parametric variations in M , μ, n and site condition. The following study has been carried out by considering M = 5.5, n = 10, μ = 4, p = 0.5, and alluvium site conditions, unless stated otherwise.

Fig. 4 shows the variations in $R_\mu(T)$ for M = 4.5, 5.5, and 6.5 in case of alluvium site conditions. It is observed that higher magnitude results in higher SRFs for medium- to long-period structures (T > 1.0 s), and in marginally lower SRFs for very stiff structures (T < 0.3 s). Intermediate and hard rock sites also show these trends. These observations contradict the findings of Miranda [5], who reported negligible effect of magnitude on SRF spectrum. Thus, it does not appear to be justified to neglect the effect of magnitude on SRFs. Further, as implied by Fig. 4, it may be conservative to estimate SRFs by using the proposed model with the design spectrum for a low-magnitude earthquake.

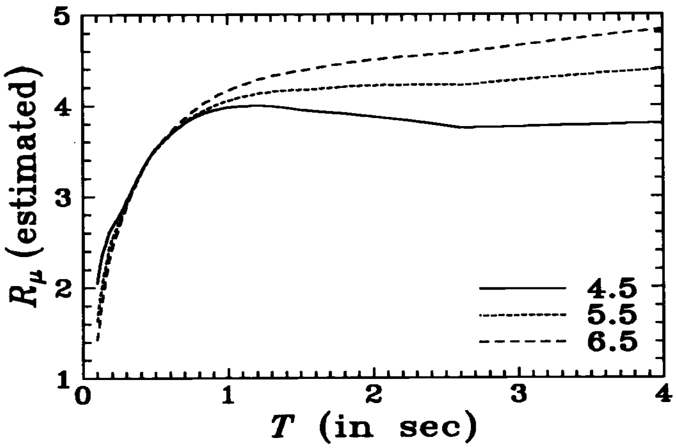

Figure 4 - Estimated SRF Spectra for Different Magnitudes in Case of μ = 4, n = 10, p = 0.5, and Alluvium Site Conditions.

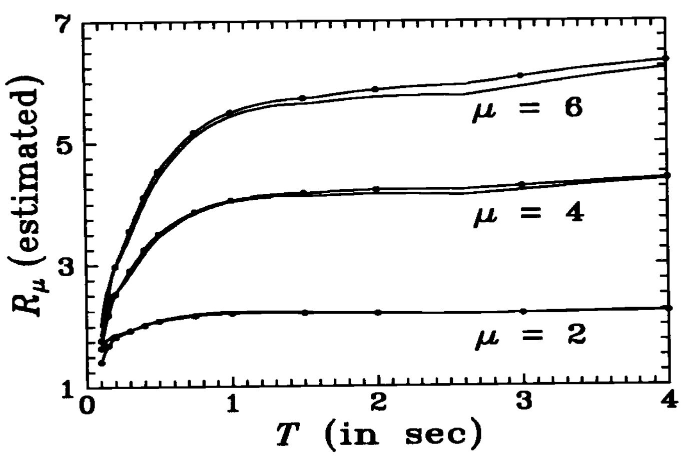

Figure 5 - Estimated SRF Spectra for Different Values of n in Case of M = 5.5, μ = 2, 4, 6, p = 0.5, and Alluvium Site Conditions.

The effects of n and μ on SRFs have been shown in Fig. 5 for alluvium site conditions. The curves with dots are for $n = 10$, while those without dots are for $n = 6$. It is observed, as expected, that higher ductility supply makes a difference only in case of high ductility demand ratios. Also, since there is lesser strength degradation in case of higher values of n, those cases are associated with higher SRFs. Fig. 5 also supports the bi-linear idealization of SRF spectrum, with the selection of proper slopes for both lines.
Fig. 6 shows the comparison of SRF spectra for different site categories. It is seen that except for stiff oscillators, hard rock conditions are associated with smaller SRFs. This contradicts the observation of Elghadamsi [3] that deamplification of elastic response is slightly more for a structure on rock than for a structure on alluvium site for most time-periods. Fig. 6 implies that it may be conservative to estimate SRFs by using the proposed model with a design spectrum for hard rock site geology.

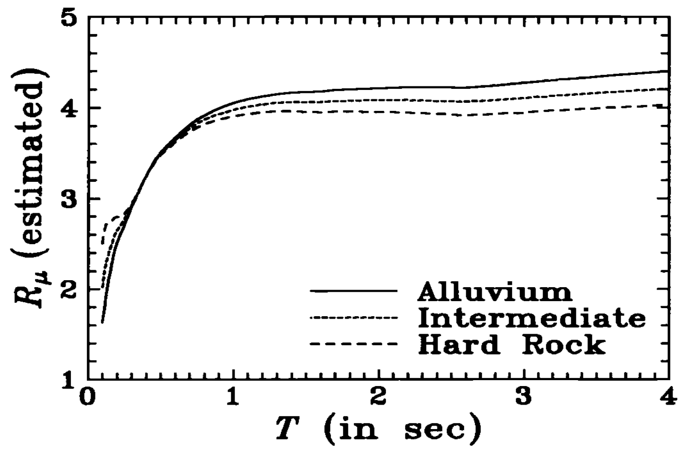

Figure 6 - Estimated SRF Spectra for Different Site Conditions with $M = 5.5$, $\mu = 4$, $n = 10$, and $p = 0.5$.

CONCLUSIONS

A new model has been proposed in this study for the scaling of SRF spectra, while considering pseudo-acceleration spectrum with unit value of zero-period-acceleration as the input data related to the ground motion. A recently developed elasto-plastic SDOF oscillator with specified strength and stiffness degradation characteristics and 5% F-damping has been considered for calculating SRFs from the recorded accelerograms. It has been found that the coefficients obtained from the linear regression analyses are able to predict the average trends of SRFs with (initial) time periods of the oscillators fairly well, and thus, the proposed model may be convenient when design spectrum is the only input available to the designer.
A parametric study carried out with the help of the proposed model shows that higher earthquake magnitudes and alluvium site conditions may be associated with greater SRFs, unless the oscillator is stiff.

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