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MAKSIMALNA UBRZANJA NA SPRATOVIMA VIŠESPRATNIH ZGRADA

Rezime

Za obezbeđenje sigurnost krutih nenosećih komponenti u konstrukcijama zgrada važno je tačno i jednostavno odrediti najveća ubrzanja na spratovina gde se ove komponente nalaze, za odredjen seizmički hazard. Nedavno predloženo pravilo, na bazi superpozicije karakterističnih funkcija konstrukcije, je dalje usavršeno čime se postiže tačnija ocena u slučajevima kad silno pomeranje sadrži duge periode. Numeričke analize pokazuju da predloženo pravilo daje tačnije ocene u poređenju sa popularnom CQC metodom, kada se uzimaju u obzir samo nekoliko osnovnih tonova i kada je perioda konstrukcije kraća od periode silnog pomeranja. Za konzervativnije ocene, verzije predložene metode bazirane na SRSS varijanti, dobijene zanemarivanjem međusobnih korelacija, mogu se koristiti kad zgrada ima kraći period od periode silnog omeranja.

Ključne reči: Krute nenoseće komponente, maksimalne akceleracije na spratovima, pravila za kombinaciju tonova vibracija, pseudo-spektralna ubrzanja.

PEAK FLOOR ACCELERATIONS IN MULTISTORIED BUILDINGS

Summary

To ensure the safety of rigid nonstructural components in structural systems it is important to correctly estimate largest peak accelerations of the floors to which those are attached, for the specified seismic hazard, in a simple manner. A recently proposed modal combination rule is modified here for more accurate estimation in the case of long-period ground motions. A numerical study shows that the proposed rule gives more accurate estimates than the popular CQC rule when only the first few modes are considered and/or the structural system is stiff to the ground motion. For more conservative estimates, the SRSS-type variants of the proposed method obtained after ignoring cross-correlation terms may be used, provided the building is not flexible with respect to the ground motion.

Key words: rigid nonstructural components, peak floor accelerations, modal combination rule, pseudo spectral acceleration spectrum.

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1 INTRODUCTION

Safety of nonstructural components in a building, like masonry panels, parapets, chimneys, storage tanks, escalators, and pipes, against seismic hazard has received considerable attention of the earthquake engineering profession in the last 10–15 years. There have been numerous cases of large damage to these components, even when the damage to the main skeletons was not significant. Damage to nonstructural components poses serious threat to the lives of the building occupants besides causing heavy financial losses.

Nonstructural components are subjected to the (absolute) accelerations of the floors on which those are supported, and thus to amplified ground motions, depending on the building characteristics and the location of the floor. If nonstructural components are sufficiently stiff to vibrate in phase with their attachment points, it is desirable for their design to properly estimate the largest peak values of the floor accelerations consistent with the specified seismic hazard.

Despite the efforts made in the past 10 years to improve the code provisions to avoid damage to the nonstructural components, much still remains to be done. The present code provisions (see, for example, [1]) have been shown by Taghavi and Miranda [2] and Singh et al. [3] to be leading to too conservative estimates. Those have yet to become rigorous enough despite significant research efforts, e.g. those by Singh et al. [3, 4], Villaverde [5], and Soong et al. [6]. Estimation of linear response of nonstructural components and the supporting structure from the elastic pseudo-spectral acceleration (PSA) spectrum of the input excitation continues to form the basis of codal provisions, and thus there is a case for the future research to focus on the response-spectrum based estimation of largest peak in (linear) floor acceleration response and on developing appropriate modal combination rules.

Except for the modal combination rule proposed recently by Kumari and Gupta [7], no modal combination rule has been derived till date to predict the peak floor accelerations in a structural system by directly using the response spectrum ordinates. The modal combination rules proposed in the past, e.g. those by Goodman et al. [8], Rosenblueth and Elorduy [9], Wilson et al. [10], Singh and Mehta [11] can be used for this purpose, because absolute acceleration response of a floor can be described by a linear superposition of (absolute) acceleration responses in different modes, but this is true only when all modes are considered. Peak floor accelerations may be considered as zero-period ordinates of floor response spectra, which are PSA ordinates corresponding to the floor motions, and thus the response spectrum-based formulations by Singh [12], Der Kiureghian et al. [13], Singh and Sharma [14], Igusa and Der Kiureghian [15], Suarez and Singh [16], Singh et al. [3] can be used to estimate the peak floor accelerations (see [7] for further details). However, availability of a modal combination rule is always desirable for estimating the peak floor accelerations in terms of the PSA ordinates and modal properties of a linear structural system. The modal combination rule proposed by Kumari and Gupta [7], along with its simpler variants on the lines of SRSS (Square-Root-of-Sum-of-Squares) rule [8], address this need, but this rule is suitable only when the structural system is not too stiff or flexible to the given ground motion.

This study considers modification of the modal combination rule by Kumari and Gupta [7] to make it applicable for those structures that are stiff with respect to the input ground motion. A numerical study is also carried out to evaluate the relative performances of the modified rule and its simpler variants (on the lines of SRSS rule) over other approximate methods in estimating peak floor accelerations from the input response spectrum. The other approximate methods considered are: (i) the method by Singh et al. [3], due to its simplicity and demonstrated superiority over previous code-based methods, and (ii) the CQC rule [10], due to its greater popularity among the earlier modal combination rules. The numerical study is carried out by considering three example buildings and six example ground motions.

It may be noted that the majority of the methods developed so far for the estimation of floor response analyses have neglected the role of soil-structure interaction (SSI). In this paper also the role of SSI is assumed to be negligible and examples are given for the floor motions of buildings with fixed base. This approach will lead to accurate results only for the buildings founded on geological basement rock, when the system frequency f_{system} is approximately equal to the fundamental frequency f_1 of the fixed-base building [17, 18]. For the buildings founded on typical soils, found in many urban areas, $f_{\text{system}} < f_1$, and hence the formulation presented in this paper will give only approximate estimates of the floor accelerations. It is possible to formulate the floor accelerations based on superposition principles within the theoretical framework discussed by Gupta [19], and considering the effects of SSI, as shown by Ray Chaudhuri and Gupta [20] in the case of floor response spectra based on mode acceleration approach. However, the discussion of those methods is beyond the scope of this paper.

2 FORMULATION OF THE PROPOSED RULE

2.1 FORMULATION BY KUMARI AND GUPTA [7]

We consider a symmetric shear building, where masses are lumped at the floors as m_i, $i = 1, 2, ..., n$, and massless columns provide the lateral stiffness with story stiffnesses as k_i, $i = 1, 2, ..., n$ (see Figure 1). The building is classically damped with interstory viscous dampers of damping constants c_i, $i = 1, 2, ..., n$, and is subjected to ground acceleration $\ddot{z}(t)$ at its base. Let ω_j and ζ_j denote the natural frequency and damping ratio, respectively, of this system in the *j*th mode. The modal participation factor in this mode becomes $\alpha_j = \sum_{i=1}^{n} m_i \phi_i^{(j)} \Big/ \sum_{i=1}^{n} m_i \left(\phi_i^{(j)}\right)^2$, where $\phi_i^{(j)}$ is the *i*th element of the *j*th mode shape vector.

On assuming stationarity in the excitation and response, the power spectral density function (PSDF) of a response may be obtained by multiplying the PSDF of the excitation with the squared modulus of the corresponding transfer function. On computing moments of the PSDF of the process, root-mean-square (r.m.s.) value of the process and peak factors for the largest peak amplitude may be estimated, which lead to the largest peak of the response process on multiplication. The largest peak amplitude so obtained is multiplied with a nonstationarity factor [19] in order to account for the fact that the response process is not a stationary process. Thus, the largest peak amplitude of the absolute acceleration response of the *i*th floor may be expressed as [7]

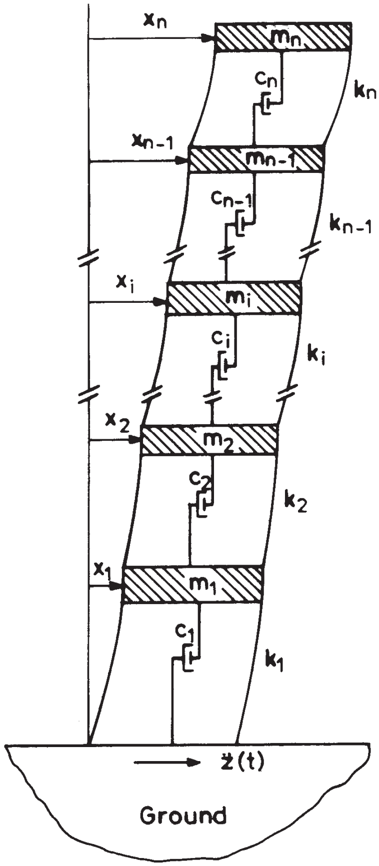

Figure 1- Shear Building Model of n-Storied Building.

$$a_{i,\max} = \left[\left(\frac{\eta^{a_i}}{\eta^{G}}\right)^2 \left(\frac{\beta^{a_i}}{\beta^{G}}\right)^2 PGA^2 + \sum_{j=1}^{n} \left\{ \left(2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k D_{jk} \right) \phi_i^{(j)} \alpha_j \omega_j^2 \left(\frac{\eta^{a_i}}{\eta_j^{V}}\right)^2 \left(\frac{\beta^{a_i}}{\beta_j^{V}}\right)^2 SV_j^2 \right.\right.$$
$$\left.\left. + \left(\phi_i^{(j)} \alpha_j - 2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k \left(C_{jk} - D_{jk} \right) \right) \phi_i^{(j)} \alpha_j \left(\frac{\eta^{a_i}}{\eta_j^{A}}\right)^2 \left(\frac{\beta^{a_i}}{\beta_j^{A}}\right)^2 RSA_j^2 \right\} \right]^{\frac{1}{2}} \tag{1}$$

where, C_{jk} and D_{jk} are the coefficients given in terms of ζ_j , ζ_k and $f_{kj} = \omega_k / \omega_j$ as

$$C_{jk} = \frac{1}{B_{jk}} \left[8 f_{kj} \zeta_j \left(\zeta_k + \zeta_j f_{kj} \right) \left\{ \left(1 - f_{kj}^{\;2} \right)^2 - 4 f_{kj} \left(\zeta_j - \zeta_k f_{kj} \right) \left(\zeta_k - \zeta_j f_{kj} \right) \right\} \right] \tag{2}$$

$$D_{jk} = \frac{1}{B_{jk}} \left[2 \left(1 - f_{kj}^{\;2} \right) \left\{ 4 f_{kj} \left(\zeta_j - \zeta_k f_{kj} \right) \left(\zeta_k - \zeta_j f_{kj} \right) - \left(1 - f_{kj}^{\;2} \right)^2 \right\} \right] \tag{3}$$

with

$$B_{jk} = 8 f_{kj}{}^2 \left[\left(\zeta_j^2 + \zeta_k^2 \right) \left(1 - f_{kj}{}^2 \right)^2 - 2 \left(\zeta_k^2 - \zeta_j^2 f_{kj}{}^2 \right) \left(\zeta_j^2 - \zeta_k^2 f_{kj}{}^2 \right) \right] + \left(1 - f_{kj}{}^2 \right)^4 \tag{4}$$

Further, in Equation (1) η^{a_i} and β^{a_i} respectively denote the peak factor and nonstationarity factor for $a_{i,\max}$; η^G and β^G respectively denote the peak factor and nonstationarity factor for the largest peak amplitude PGA of the ground acceleration process $\ddot{z}(t)$; η_j^V and β_j^V respectively denote the peak factor and nonstationarity factor for the largest peak amplitude SV_j of the relative velocity response process of the single-degree-of-freedom (SDOF) oscillator (with ω_j frequency and ζ_j damping ratio) in the *j*th mode, in response to $\ddot{z}(t)$; and η_j^A and β_j^A respectively denote the peak factor and nonstationarity factor for the largest peak amplitude RSA_j of the relative acceleration response process of the single-degree-of-freedom (SDOF) oscillator in the *j*th mode, in response to $\ddot{z}(t)$. It may be mentioned that $a_{i,\max}$ is for the same level of confidence [19] to which η^{a_i} corresponds, PGA for the confidence level to which η^G corresponds, SV_j for the confidence level to which η_j^V corresponds, and RSA_j is for the same confidence level to which η_j^A corresponds.

Assuming that $a_{i,\max}$, PGA, SV_j, and RSA_j are estimated for the same level of confidence, various η and β ratios can be taken as unity in Equation (5) [7] and Equation (1) may be rewritten as

$$a_{i,\max} \approx \left[PGA^2 + \sum_{j=1}^{n} \left\{ \left(2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k D_{jk} \right) \phi_i^{(j)} \alpha_j \omega_j^2 SV_j^2 + \left(\phi_i^{(j)} \alpha_j - 2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k \left(C_{jk} - D_{jk} \right) \right) \phi_i^{(j)} \alpha_j RSA_j^2 \right\} \right]^{\frac{1}{2}} \tag{5}$$

It is possible to use this expression to estimate the largest floor acceleration at the *i*th floor consistent with the seismic design levels at a site characterized by the available PGA, Spectral Velocity (SV), and Relative Spectral Acceleration (RSA) curves. On using Pseudo-Spectral Velocity (PSV) curves in place of the SV curves as per the existing engineering practice and PRSA (Pseudo-Relative Spectral Acceleration) curves in place of the RSA curves [21], the modal combination rule proposed by Kumari and Gupta [7] becomes

$$a_{i,\max} \approx \left[PGA^2 + \sum_{j=1}^{n} 2\phi_i^{(j)} \alpha_j \left\{ PSA_j^2 - PRSA_j^2 \right\} + ra_{i,\max}^2 \right]^{\frac{1}{2}} \tag{6}$$

or, depending on how the mean period T_c of ground motion (corresponding to the centre of gravity of the Fourier spectrum of ground motion [21]) compares with the natural periods of the system, T_j (= $2\pi/\omega_j$), $j = 1, 2, ..., n$,

$$a_{i,\max} \approx \left[PGA^2 \left(1 + \sum_{j=1}^{n} 2\phi_i^{(j)}\alpha_j \right) + ra_{i,\max}^2 \right]^{\frac{1}{2}} \qquad ; \; T_1 < T_c$$

$$= \left[PGA^2 \left(1 - \sum_{j=1}^{n} 2\phi_i^{(j)}\alpha_j \right) + ra_{i,\max}^2 \right]^{\frac{1}{2}} \qquad ; \; T_n > T_c \tag{7}$$

$$= \left[PGA^2 \left(1 - \sum_{j=1}^{\hat{n}} 2\phi_i^{(j)}\alpha_j + \sum_{j=\hat{n}+1}^{n} 2\phi_i^{(j)}\alpha_j \right) + ra_{i,\max}^2 \right]^{\frac{1}{2}} ; \; T_{\hat{n}+1} < T_c < T_{\hat{n}}$$

where

$$ra_{i,\max}^2 = \sum_{j=1}^{n} \left[\sum_{k=1,k\neq j}^{n} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k D_{jk} PSA_j^2 + \left(\phi_i^{(j)}\alpha_j + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)}\alpha_k \left(C_{jk} - D_{jk} \right) \right) \phi_i^{(j)}\alpha_j PRSA_j^2 \right] \tag{8}$$

or 0, whichever is greater

A simpler variant, called as Quasi-SRSS rule, is obtained by ignoring the cross-correlation between the ground acceleration and the relative floor acceleration [7]:

$$a_{i,\max} \approx \left[PGA^2 + ra_{i,\max}^2 \right]^{\frac{1}{2}} \tag{9}$$

On ignoring the cross-correlation of the jth mode with the remaining $n-1$ modes (in the relative acceleration response), a further simpler variant, called as SRSS rule, is obtained as [7]:

$$a_{i,\max} \approx \left[PGA^2 + \sum_{j=1}^{n} \left(\phi_i^{(j)} \right)^2 \alpha_j^2 PRSA_j^2 \right]^{\frac{1}{2}} \tag{10}$$

2.2 SV APPROXIMATION BY GUPTA [22]

The SV values for short periods (i.e. periods shorter than the mean period T_c of the ground motion) may be approximated more accurately as [22]

$$SV(T) \approx \frac{1}{\omega_n} \sqrt{\left\{ PSA(T) \right\}^2 - PGA^2} \tag{11}$$

in place of

$$SV(T) \approx \frac{PSA(T)}{\omega_n} \tag{12}$$

The approximation for the SV values at periods longer than the mean period T_c may be considered same as in Equation (12).

2.3 PROPOSED RULE AND ITS VARIANTS

On considering Equation (5) together with the RSA approximation (in form of PRSA curves) of Trifunac and Gupta [21], and on using Equation (11) for periods shorter than T_c and

Equation (12) for periods longer than T_c, the proposed modal combination rule (for the peak floor acceleration response) becomes for first $p(\leq n)$ modes and with $1 \leq \hat{n} < p$ as

$$a_{i,\max} \approx \begin{cases} \left[PGA^2 + ra_{i,\max}^2\right]^{\frac{1}{2}} & ; \; T_1 < T_c \\ \left[PGA^2\left(1 - 2\sum_{j=1}^{\hat{n}} \phi_i^{(j)}\alpha_j\right) + ra_{i,\max}^2\right]^{\frac{1}{2}} & ; \; T_{\hat{n}+1} < T_c < T_{\hat{n}} \\ \left[PGA^2\left(1 - 2\sum_{j=1}^{p} \phi_i^{(j)}\alpha_j\right) + ra_{i,\max}^2\right]^{\frac{1}{2}} & ; \; T_c < T_p \end{cases} \tag{13}$$

where

$$ra_{i,\max}^2 = \begin{cases} \sum_{j=1}^{p}\left\{\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 - PGA^2\right) + \sum_{k=1,k\neq j}^{p} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k\left(PSA_j^2 - PGA^2\right)C_{jk}\right\}; \; T_1 < T_c \\ \sum_{j=1}^{\hat{n}}\left\{\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 + PGA^2\right) + \sum_{k=1,k\neq j}^{p} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k\left(C_{jk}PSA_j^2 + \left(C_{jk} - D_{jk}\right)PGA^2\right)\right\} \\ \quad + \sum_{j=\hat{n}+1}^{p}\left\{\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 - PGA^2\right) + \sum_{k=1,k\neq j}^{p} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k\left(PSA_j^2 - PGA^2\right)C_{jk}\right\} \\ \qquad ; \; T_{\hat{n}+1} < T_c < T_{\hat{n}} \\ \sum_{j=1}^{p}\left\{\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 + PGA^2\right) + \sum_{k=1,k\neq j}^{p} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k\left(C_{jk}PSA_j^2 + \left(C_{jk} - D_{jk}\right)PGA^2\right)\right\} \\ \qquad ; \; T_c < T_p \end{cases} \tag{14}$$

or 0, whichever is greater

The Quasi-SRSS variant of the proposed rule is again described by Equation (9), with $ra_{i,\max}$ obtained by using Equation (14). The SRSS variant of the proposed rule is also described again by Equation (10). This may be alternatively expressed as

$$a_{i,\max} \approx \begin{cases} \left[PGA^2 + \sum_{j=1}^{p}\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 - PGA^2\right)\right]^{\frac{1}{2}} & ; \; T_1 < T_c \\ \left[PGA^2 + \sum_{j=1}^{\hat{n}}\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 + PGA^2\right) + \sum_{j=\hat{n}+1}^{p}\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 - PGA^2\right)\right]^{\frac{1}{2}} & ; \; T_{\hat{n}+1} < T_c < T_{\hat{n}} \\ \left[PGA^2 + \sum_{j=1}^{p}\left(\phi_i^{(j)}\alpha_j\right)^2\left(PSA_j^2 + PGA^2\right)\right]^{\frac{1}{2}} & ; \; T_c < T_p \end{cases} \tag{15}$$

3 OTHER APPROXIMATE METHODS

3.1 CQC RULE

According to the CQC rule, the largest peak amplitude of the absolute acceleration response of the *i*th floor may be approximated as [10]

$$a_{i,\max} \approx \left[\sum_{j=1}^{p} \sum_{k=1}^{p} \rho_{jk} a_{ji,\max} a_{ki,\max} \right]^{1/2} \tag{16}$$

with

$$a_{qi,\max} \approx \alpha_q \phi_i^{(q)} PSA_q \tag{17}$$

denoting the largest peak amplitude of the contribution of the *q*th mode to the total floor acceleration and

$$\rho_{jk} = \frac{8\sqrt{\zeta_j \zeta_k}\left(\beta_{jk}\zeta_j + \zeta_k\right)\beta_{jk}^{3/2}}{\left(1-\beta_{jk}^2\right)^2 + 4\zeta_j\zeta_k\beta_{jk}\left(1+\beta_{jk}^2\right) + 4\left(\zeta_j^2 + \zeta_k^2\right)\beta_{jk}^2} \tag{18}$$

denoting the correlation coefficient between the *j*th and *k*th modes. In Equation (18), β_{jk} ($= \omega_j / \omega_k$) is the frequency ratio between the *j*th and *k*th modes.

3.2 METHOD BY SINGH ET AL. [3]

According to the simple method proposed by Singh et al. [3], the largest peak amplitude of the absolute acceleration response of the *i*th floor is approximated as

$$a_{i,\max} \approx C_i \times PGA \tag{19}$$

where C_i is the acceleration coefficient for the *i*th floor. For $n \leq 8$ this coefficient is defined as

$$C_i = 1 + \frac{z_i}{h}\left(C_n - 1\right) \tag{20}$$

with z_i denoting the height of the *i*th floor and *h* the height of the building above the base. C_n is the acceleration coefficient for the roof level defined as

$$C_n = \alpha_1 \phi_n^{(1)} \sqrt{\left(1 + 1.03\left(PSA_1 / PGA\right)^2\right)} \geq 1.0 \tag{21}$$

For $n > 8$, the acceleration coefficient is defined as

$$C_i \approx \begin{cases} 1 + \dfrac{z_i}{0.2h}\left(C_l - 1\right) & ;\ z_i \leq 0.2h \\ C_l & ;\ 0.2h < z_i \leq 0.8h \\ C_l + \dfrac{z_i - 0.8h}{0.8h}\left(C_n - C_l\right); & 0.8h < z_i < h \end{cases} \tag{22}$$

with $C_l = C_n / T_1^{1/3}$.

4 NUMERICAL EXAMPLES

4.1 EXAMPLE BUILDINGS AND EXCITATIONS

To illustrate the proposed rule and its variants and compare those with the other approximate methods, six earthquake ground motions and three (fixed-base) example buildings considered by Kumari and Gupta [7] are considered. Details of the example motions are listed in Table 1, and the values of floor masses and story stiffnesses for the example buildings are given in Table 2. The natural periods of the example buildings are given in Table 3.

The example ground motions cover a wide range of energy distributions, with the dominant period varying from about 0.48 s in the Parkfield motion to about 5.5 s in the Borrego Mountain and San Fernando motions. The Michoacan motion is also a long-period motion with the dominant period of about 2.6 s. It also has a narrow band of 1.8–3 s of significant energy. The Imperial Valley and the Kern County motions are medium-period motions with dominant periods as 0.85 and 0.65 s, respectively. The Kern County motion has significant energy over a large band of 0.2–5 s. Among the remaining motions, the energy is concentrated in a narrow band of periods in the case of the San Fernando motion, fairly wide band in the case of the Imperial Valley motion, and in a medium band in the cases of Borrego Mountain and Parkfield motions.

Table 1—Details of the Example Ground Motions

Motion No.	Earthquake	Site	Component	Mean Period T_c (s)
1	Borrego Mountain Earthquake, 1968	Engineering Building, Santa Ana, Orange County, California	S04E	0.38
2	Imperial Valley Earthquake, 1940	El Centro Site, Imperial Valley Irrigation District, California	S00E	0.17
3	Kern County Earthquake, 1952	Taft Lincoln School Tunnel, California	N21E	0.25
4	Michoacan Earthquake, 1985	Mexico City	Synthetic	0.96
5	Parkfield Earthquake, 1966	Array No. 5, Cholame, Shandon, California	N05W	0.19
6	San Fernando Earthquake, 1971	Utilities Building, 215 West Broadway, Long Beach, California	N90E	0.39

Table 2—Mass and Stiffness Properties of the Example Buildings

i	Floor Mass m_i (t)			Story Stiffness k_i (kN/mm)		
	BD1	BD2	BD3	BD1	BD2	BD3
1	7,426	280	166	6650	525	290
2	7,426	200	166	6260	536	290
3	6,918	200	166	5880	536	290
4	6,970	200	166	5880	536	290
5	5,849	200	141	5510	536	290
6	5,587	200		5480	536	
7	5,569	200		5480	536	
8	4,063	200		5100	536	
9	3,678	200		5010	536	
10	3,678	200		5010	536	
11	3,678	200		4960	536	
12	3,415	200		4920	536	
13	3,415	200		4920	536	
14	2,855	200		4720	536	
15	2,469	200		4670	536	
16	2,469			4670		
17	2,329			4610		
18	1,769			4220		
19	1,769			4220		
20	1,524			4260		
21	1,278			4240		
22	1,261			4260		
23	928			4250		
24	771			4420		

Table 3—Natural Periods of the Example Buildings

Mode No.	Periods (s)		
	BD1	BD2	BD3
1	2.002	1.200	0.514
2	0.804	0.402	0.177
3	0.501	0.244	0.113
4	0.360	0.177	0.089
5	0.285	0.141	0.078
6	0.233	0.118	
7	0.201	0.102	
8	0.175	0.090	
9	0.158	0.082	
10	0.143	0.075	
11	0.132	0.070	
12	0.124	0.067	
13	0.116	0.064	
14	0.112	0.062	
15	0.107	0.061	
16	0.102		
17	0.095		
18	0.090		
19	0.086		
20	0.081		
21	0.076		
22	0.069		
23	0.061		
24	0.052		

The example buildings cover the range of fundamental periods typically found in multistoried buildings to a large extent. On one extreme, the first example building, BD1, is very stiff to the San Fernando motion, stiff to the Michoacan and Borrego Mountain motions, flexible to the Imperial Valley and Kern County motions, and very flexible to the Parkfield motion. On the other extreme, the third example building, BD3, is very stiff to the Borrego Mountain, Michoacan and San Fernando motions, little stiff to Imperial Valley and Kern County motions, and is in near resonance with the Parkfield motion. The example buildings are assumed to be classically damped with the damping ratio of 0.05 in all modes.

4.2 RESULTS AND DISCUSSION

The performances of the proposed rule and its variants (SRSS and Quasi-SRSS) and the other two approximate methods are compared by subjecting the example buildings to the example ground motions. Estimates of peak floor accelerations are obtained from the (exact) time-history analyses and compared with the approximate estimates from: (i) the proposed rule (see Equations (13) and (14)), (ii) the SRSS variant (see Equation (15)), (iii) the Quasi-SRSS variant (see Equations (9) and (14)), (iv) the method by Singh et al. [3], and (v) the CQC rule. Table 4 shows the percentage absolute error averaged over all floors for all 18 combinations of example buildings and ground motions in the cases of these approximate methods. The average absolute errors are also shown for the modal combination rule proposed by Kumari and Gupta [7] (see Equations (7) and (8)). The maximum error figure with each of the approximate methods is underlined. The combinations with error figures in bold represent the worst cases (i.e. the cases of maximum error) for the proposed rule in the case of respective ground motions. The envelopes of floor accelerations for the five approximate methods are compared in Figures 2(a)–2(f) with the exact envelope for these cases. Figures 2(a), 2(c), 2(e) and 2(f) show the comparisons for BD1 in the cases of Borrego Mountain, Kern County, Parkfield and San Fernando motions, respectively. Figure 2(d) shows the comparisons for BD2 in the case of Michoacan motion, and Figure 2(b) shows the comparisons for BD3 in the case of Imperial Valley motion.

It is clear from Table 4 that the performances of the proposed rule and the rule by Kumari and Gupta [7] are comparable for most cases. The proposed rule, however, reduces the error in the rule by Kumari and Gupta [7] for the combination of BD3 and Michoacan motion due to improved SV estimates at short periods. The performance of the proposed rule is also the best with the average error being less than 10% in most cases. However, the maximum average error of 25.59% observed in the case of BD1 subjected to the Parkfield motion indicates that the proposed rule may not work so well, like the rule by Kumari and Gupta [7], when the structural system is very flexible with respect to the ground motion. The performance of the CQC rule is also quite close to that of the proposed rule, with the maximum average error being 25.37% (in the case of BD1 subjected to the Parkfield motion). The notable difference between the performances of the two rules is in the case of BD3 subjected to the Michoacan motion, where the proposed rule is associated with 1.23% error, compared to 19.83% error for the CQC rule. Since BD3 is very stiff with respect to the Michoacan motion, the proposed rule may perform significantly better than the CQC rule for relatively very stiff systems. Table 4 also shows that the simple method by Singh et al. [3] performs significantly better than the SRSS and Quasi-SRSS variants of the proposed rule, with the maximum average error being about 50% (in the case of BD2 subjected to the Michoacan motion). The Quasi-SRSS variant performs marginally better than the SRSS variant due to modal cross-correlation being weak in the example buildings considered. However, cross-correlation between the ground acceleration and the relative floor acceleration is strong and both variants perform poorly due to this having been ignored. The maximum average errors for both variants exceed 80%. It is noteworthy that the simple SRSS variant (and the not-so-simple Quasi-SRSS variant) works better than the method by Singh et al. [3] for relatively stiff systems (see, for example, the results for the Michoacan motion) and may thus be a better alternative to this method, provided the structural system is not flexible with respect to the ground motion.

Table 4 — Comparison of the Averaged Percentage Absolute Errors in Peak Floor Acceleration from Different Methods

Example Motion	No. 1 (BM)	No. 2 (IV)	No. 3 (KC)	No. 4 (MX)	No. 5 (PK)	No. 6 (SF)
Example Building			*BD1*			
Proposed*	**11.63**	7.89	**11.88**	3.51	**<u>25.59</u>**	**9.88**
Kumari and Gupta^{+}	12.84	7.86	11.76	3.45	24.37	11.51
SRSS**	28.77	59.00	39.69	5.02	56.31	24.82
Quasi-SRSS***	27.18	52.68	34.58	5.09	48.01	25.13
Singh et al.$^{++}$	15.76	17.93	16.90	36.67	16.91	26.90
CQC^{+++}	12.24	7.55	11.57	5.81	<u>25.37</u>	9.97
Example Building			*BD2*			
Proposed*	8.76	7.44	11.08	**9.63**	13.14	2.88
Kumari and Gupta^{+}	8.80	7.00	11.81	10.66	13.58	4.06
SRSS**	25.22	46.93	29.91	19.16	<u>83.98</u>	14.18
Quasi-SRSS***	27.01	49.76	31.99	18.84	<u>82.58</u>	15.68
Singh et al.$^{++}$	46.11	32.66	21.03	<u>50.06</u>	37.57	41.27
CQC^{+++}	10.19	6.81	10.70	12.80	11.57	4.47
Example Building			*BD3*			
Proposed*	7.91	**11.18**	3.28	1.23	3.87	1.05
Kumari and Gupta^{+}	4.34	10.47	5.38	<u>71.50</u>	7.43	3.21
SRSS**	10.45	6.23	14.35	1.21	18.41	10.02
Quasi-SRSS***	10.23	6.74	14.28	1.23	18.26	9.91
Singh et al.$^{++}$	4.99	5.74	3.28	41.16	7.64	6.98
CQC^{+++}	8.73	10.92	3.72	19.83	6.20	1.66
Note: 'BM' refers to Borrego Mountain motion, 'IV' to Imperial Valley motion, 'KC' to Kern County motion, 'MX' to Michoacan motion, 'PK' to Parkfield motion, and 'SF' to San Fernando motion						

*Equations (13) and (14); **Equation (15); ***Equations (9) and (14)
$^{+}$Equations (7) and (8); $^{++}$Equations (19)–(22); $^{+++}$Equations (16)–(18)

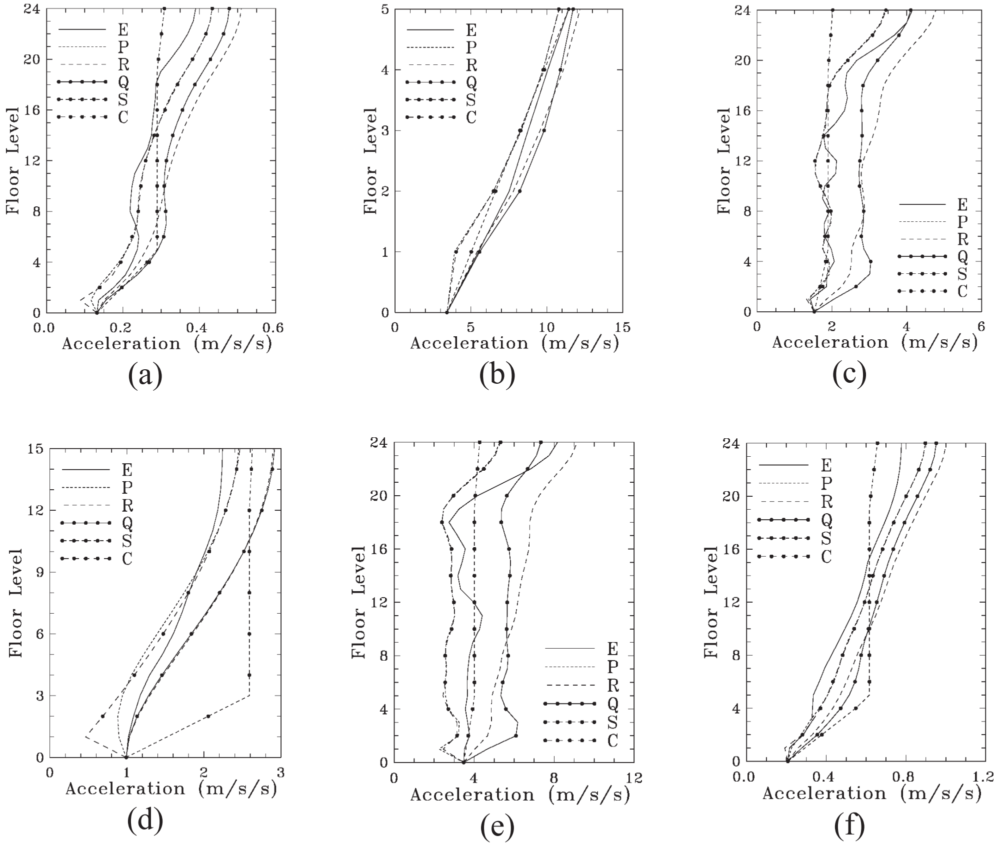

Figure 2 - Comparison of Floor Acceleration Envelopes for Exact (E), Proposed (P), SRSS (R), Quasi-SRSS (Q), Singh et al. [3] (S), and CQC (C) Estimates in the Cases of (a) BD1 and Borrego Mountain Motion, (b) BD3 and Imperial Valley Motion, (c) BD1 and Kern County Motion, (d) BD2 and Michoacan Motion, (e) BD1 and Parkfield Motion, and (f) BD1 and San Fernando Motion (see Equations (13) and (14) for Proposed Estimates, Equation (15) for SRSS Estimates, Equations (9) and (14) for Quasi-SRSS Estimates, Equations (19)–(22) for Singh et al. [3] Estimates, and Equations (16)–(18) for CQC Estimates).

It is seen from Figures 2(a)–2(f) that the results of the proposed rule follow the exact results fairly well despite those being the worst cases for each ground motion. The results of the SRSS and Quasi-SRSS variants also follow the exact results but on the conservative side. It will be useful to see also how the error in peak floor acceleration is distributed for different approximate methods. Hence, a cumulative probability density function for percentage error in peak floor acceleration is estimated for each method based on all the 264 results for the three example buildings and six example motions, and by finding the fractions of those results that have percentage errors below different levels varying from –60 to 150. Figure 3 shows the comparison of the cumulative probability density functions for the proposed rule, its SRSS variant, CQC rule, and the method by Singh et al. [3]. The cumulative probability density function for the Quasi-SRSS variant of the proposed rule is very close to that for the SRSS-variant and is not included in this figure. It is clear from the figure that the errors due to the proposed and CQC rules are distributed almost identically. The probability of a negative error, i.e. the chance of peak floor acceleration being underestimated, is about 60% for both methods. Further, dispersion in the errors is maximum for the method by Singh et al. [3] with errors ranging from –55% to 120%. The SRSS variant of the proposed rule is associated with relatively lesser dispersion and with stray cases of small negative error. Thus, the estimates from the SRSS variant are likely to be on the conservative side much more often compared to those from the method by Singh et al. [3].

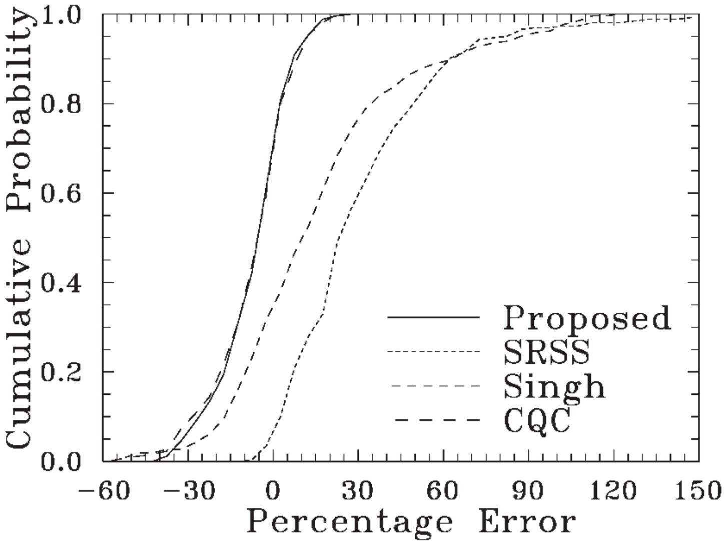

Figure 3 - Comparison of Cumulative Probability Density Functions for Percentage Error in Peak Floor Acceleration Estimate from the Proposed, SRSS-Variant and CQC Rules and from the Method by Singh et al. [3].

It follows from the above discussion that barring the case of structural system being stiff to the ground motion, the proposed and CQC rules perform at comparable levels, when all n modes are considered in estimating the peak floor accelerations. For most structural systems, however, first few modes dominate the total response, and modal combination rules are therefore applied by considering first p ($\ll n$) modes only. It will be therefore useful to compare the performances of the proposed and CQC rules, for the condition of $p < n$, in terms of the additional error introduced due to considering fewer modes than n. Peak floor accelerations are recomputed for different numbers of modes (p = 1, 2, …, n), and the percentage absolute error (averaged over all floors) with respect to the exact values for $p = n$ is computed for each of these cases for all 18 combinations of example buildings and excitations. Variation of the percentage absolute error with number of modes is compared for the proposed and CQC rules for each of these combinations. Figures 4(a)–4(f) show six such comparisons: Figures 4(b), 4(c) and 4(e) for BD1 in the cases of Imperial Valley, Kern County and Parkfield motions, respectively; Figures 4(a) and 4(f) for BD2 in the cases of Borrego Mountain and San

Fernando motions, respectively; and Figure 4(d) for BD3 in the case of Michoacan motion. For $p = n$, the errors indicated in these figures are same as those given in Table 4. It may be observed from these figures that, except for the first few modes, ignoring higher modes makes no difference to the errors associated with the proposed and CQC rules. The first few modes dominate the total response and, therefore, as these modes are ignored, the error due to the truncation of higher modes is usually increased. This increase is more in the case of CQC rule. For example, in the case of BD1 subjected to the Imperial Valley motion (see Figure 4(b)), the average absolute error grows from 7.9% to 43.1% for the proposed rule, while this error grows from 7.5% to 60% for the CQC rule, as the number of modes considered drops down to only the fundamental mode. This happens because the CQC rule considers the contribution of the base motion to the estimated peak floor acceleration in the form of separate modal contributions, and those contributions get ignored when all significant modes are not considered in the response calculations. Since the proposed rule considers the contribution of the base motion separately from the modal responses, this contribution is not affected by the truncation of higher modes.

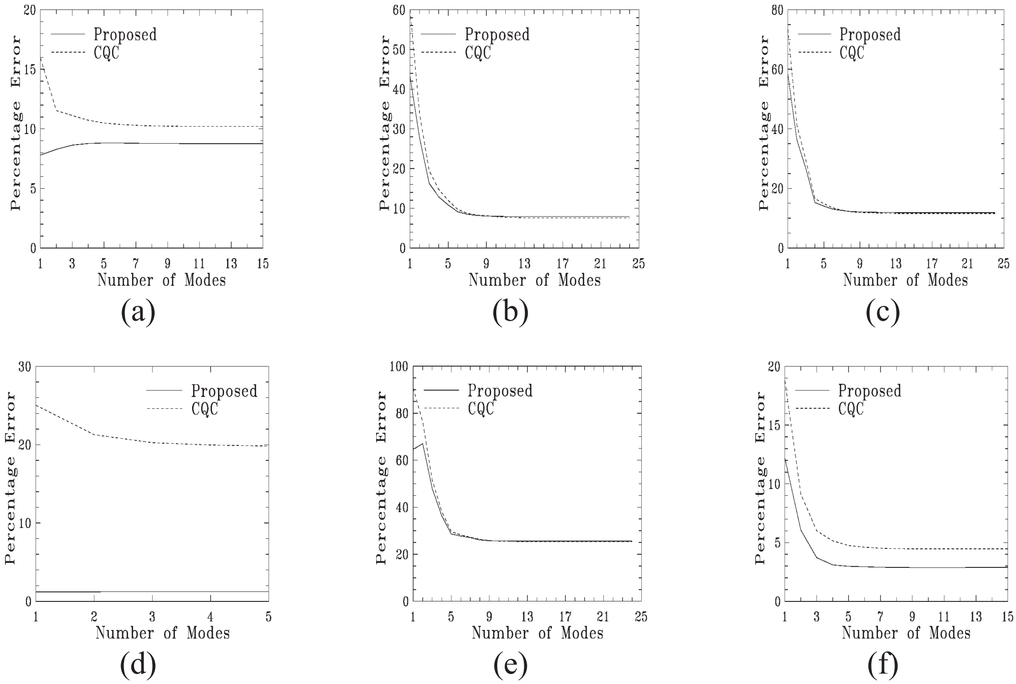

Figure 4 - Comparison of Averaged Percentage Absolute Errors in Peak Floor Estimates from the Proposed and CQC Rules for Different Numbers of Modes Considered in the Cases of (a) BD2 and Borrego Mountain Motion, (b) BD1 and Imperial Valley Motion, (c) BD1 and Kern County Motion, (d) BD3 and Michoacan Motion, (e) BD1 and Parkfield Motion, and (f) BD2 and San Fernando Motion.

5 CONCLUSIONS

In this study, several methods available to estimate the maximum values of absolute accelerations of floors in a multistoried shear building have been illustrated and compared. The building is assumed to be a linear, lumped mass, classically damped, fixed-base system, which is excited at its base by the ground motion of given PSA spectrum. The methods considered are: (i) the modal combination rule by Kumari and Gupta [7], and its SRSS and Quasi-SRSS variants, (ii) the CQC rule, and (iii) the method by Singh et al. [3]. The modal combination rule by Kumari and Gupta [7] has been modified to include a more accurate approximation of spectral velocity ordinates proposed by Gupta [22] for stiff modes in the structural system. The proposed rule also requires the knowledge of the dynamic properties of the building (mode shapes, modal frequencies and modal participation factors), the PSA ordinates, and the mean period of the ground motion.

A numerical study carried out with the help of three example buildings and six example ground motions with widely different characteristics has shown that the proposed rule improves the estimates of the modal combination rule by Kumari and Gupta [7] for the structural systems that are stiff to the ground motion. The absolute error averaged over various floors of the building has been found not to exceed 26%, and about 60% errors have been found to be less than 10%. It has been found that the extent of errors is significantly reduced in case the structural system is not flexible with respect to the ground motion. The probability of the estimates from the proposed rule being less than the exact values is, however, about 60%. The performance of the CQC rule has been found to be comparable to that of the proposed rule, when all modes are considered and the structural system is not stiff to the ground motion.

It has been found that the simpler variants of the proposed modal combination rule, i.e. SRSS and Quasi-SRSS rules, perform better than the proposed and CQC rules, and the simple method by Singh et al. [3], to the extent that those almost always give conservative estimates. However, the errors may be as large as 200%. The method by Singh et al. [3] has been found to give estimates with lesser errors, with the maximum error being about 120%. However, there is about 30% probability that the estimates by this method are nonconservative. The SRSS and Quasi-SRSS variants of the proposed rule may be used when the structural system is not more flexible compared to the ground motion, and that the method by Singh et al. [3] may be used when the structural system is not stiffer compared to the ground motion.

The above conclusions are based on a limited number of numerical examples and, therefore, are preliminary in nature. More exhaustive studies involving real-life multistoried buildings need to be carried out to confirm these conclusions and to further refine these methods. These further studies will require modeling of the existing buildings, where strong motion was recorded at different floors and where the recordings from multiple earthquakes are available, and testing of various methods for different amplitudes of excitation. An excellent candidate for such studies will be the Borik-2 building in Banja Luka, Republic of Srpska, where 20 small and intermediate earthquakes have been recorded in the basement and at the 7th and 13th floors [23–25]. Testing of the above-described methods, based on the data recorded in this building, will be the subject of one of our future investigations.

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