Information Theory: A Lighthouse for Understanding Modern Communication Systems

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References

- Fundamentals of Digital Communication by Upamanyu Madhow, Cambridge, 2008.
- Fundamentals of Wireless Communication by David Tse and Pramod Viswanath, Cambridge, 2006.

Outline

- Introduction
- Information Theory for beginners - challenges
- Insights for Physical Layer Design - examples
- Insights for Wireless Communication Systems - examples

Introduction

- Information Theory is seemingly neither inspired by, nor a result of, observing or analyzing any natural phenomena.
- Yet, it has a huge influence on the life engineered around us.
- “The underlying philosophy of using simple models to understand the essence of an engineering problem has pervaded the development of the communication field ever since.” – David Tse.
- Should Information Theory not be given greater importance in the undergraduate EE curriculum ?

Pedagogical Issues

- Ideally, a study of an engineering subject should start with a statement of the goal, followed by a study of the building blocks, and their design, to achieve that goal.
- Thus, a study of communications should start by providing a way to quantify or measure information that needs to be communicated, followed by a design of the resources required to communicate it over a given channel.
- The situation is further not helped by the fact that analog communication is taught before digital communications.
- Interestingly, this order is reversed when information theory is taught. Thus differential entropy can be taught only after entropy has been taught.

A Beginner's Perspective

- Is there a result in analog communications which can be related to Information Theory ?
- Yes, frequency modulation can trade-off bandwidth with signal to noise ratio.
- How does Shannon's AWGN capacity result:
  - Reconcile with the Nyquist rate for avoiding inter symbol interference in a given finite bandwidth ?
  - Compare with the bit rate on the channel of any communication system ?

Understanding Modern Communication Systems

Let

- W denote the bandwidth of the signal
- N_o denote the spectral density of the AWGN
- C denote the capacity of the channel
- P denote the power of the signal

$$C = W \log_2 \left(1 + \frac{P}{N_0 W}\right) \quad \text{bit/s.}$$

- Let R denote the information rate, then R < C
- If E_b denotes the energy per information bit, then $P=E_bR$

Understanding Modern Communication Systems

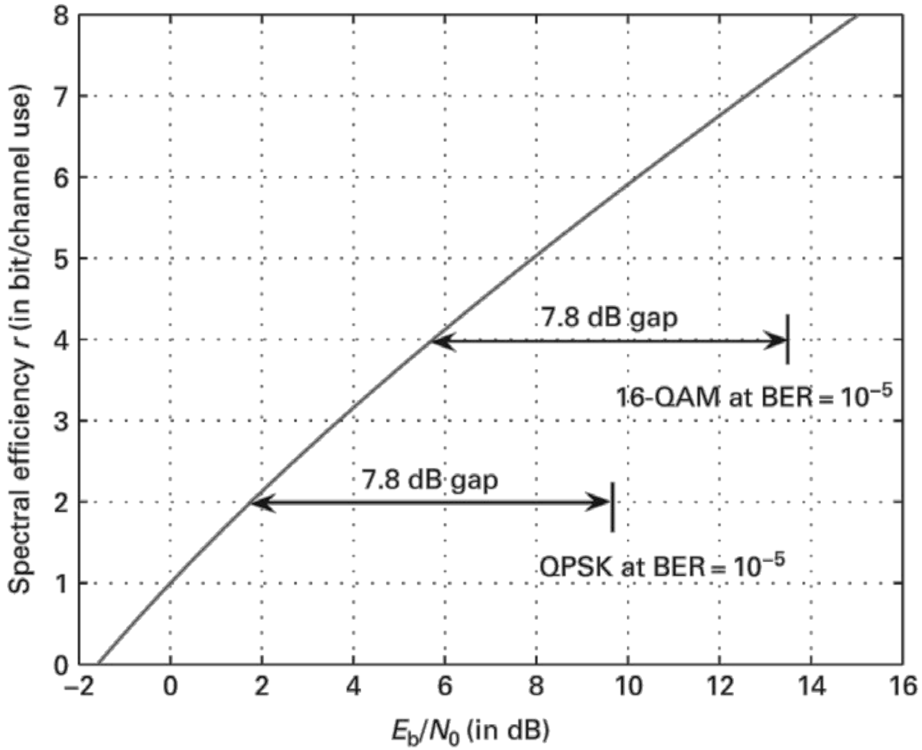
- Define r=R/W as the spectral efficiency
- Using the fact P=E_bR and C > R, and the relation

$$C = W \log_2 \left(1 + \frac{P}{N_0 W}\right) \quad \text{bit/s.}$$

- We obtain the following condition for reliable communication

$$\frac{E_b}{N_0} > \frac{2^r - 1}{r}.$$

Insights for Physical Layer Design

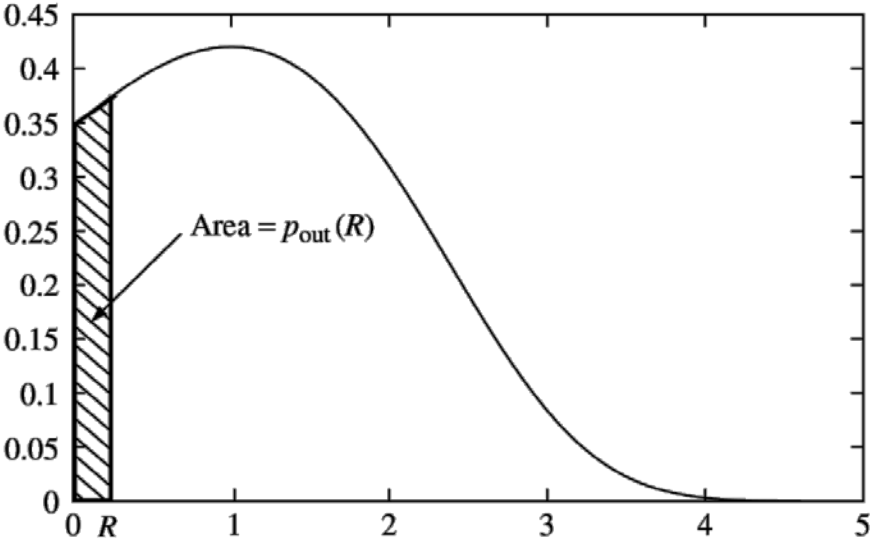


- As we let spectral efficiency r → 0, we enter a power-limited regime

$$\frac{E_b}{N_0} > \ln 2 \quad (-1.6\ \text{dB})$$

Insights into Wireless Communications

- Consider the following SISO fading channels:
    - Slow fading channel
    - Fast fading channel
    - CSI at the transmitter
- Density of $\log(1+|h|^2\mathrm{SNR})$, for Rayleigh fading. For any target rate R, there is a non-zero outage probability.



Slow Fading Channel

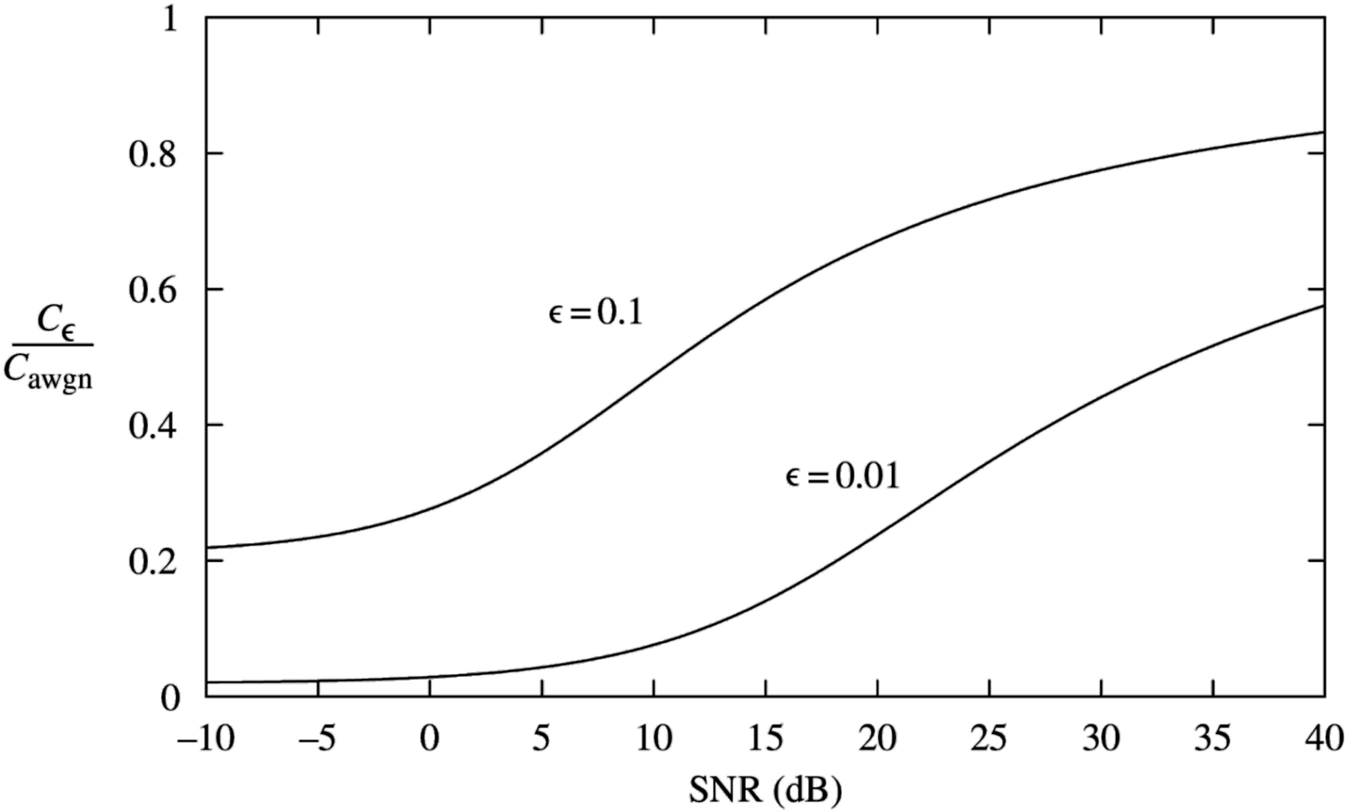
- If the channel realization h is such that $\log(1+|h|^2\mathrm{SNR}) < R$, then whatever the code used by the transmitter, the decoding error probability cannot be made arbitrarily small.
- The system is said to be *in outage, and the outage* probability is

$$p_{\mathrm{out}}(R) := \mathbb{P}\{\log(1 + |h|^2\mathsf{SNR}) < R\}$$

- Thus, the capacity of the slow fading channel in the strict sense is zero. An alternative performance measure is the ϵ-*outage capacity* C_ϵ.

Slow Fading Channel

- This is the largest rate of transmission R such that the outage probability $p_{out}(R)$ is less than ϵ.



Fast Fading Channel

- Suppose coding is done over L coherence periods, each of T_c symbols. If T_c >>1, we can model this as L parallel sub-channels that fade independently. The outage probability is

$$p_{\text{out}}(R) = \mathbb{P}\left\{ \frac{1}{L} \sum_{\ell=1}^{L} \log(1 + |h_\ell|^2 \mathsf{SNR}) < R \right\}$$

- For finite L, the quantity

$$\frac{1}{L} \sum_{\ell=1}^{L} \log(1 + |h_\ell|^2 \mathsf{SNR})$$

  is random and there is a non-zero probability that it will drop below any target rate R. Hence we have to again resort to the notion of outage.

Fast Fading Channel

- However as L → ∞, the law of large numbers says that

$$\frac{1}{L}\sum_{\ell=1}^{L}\log(1+|h_\ell|^2\mathsf{SNR}) \to \mathbb{E}[\log(1+|h|^2\mathsf{SNR})]$$

- Now we can average over many independent fades of the channel by coding over a large number of coherence time intervals and a reliable rate of communication can indeed be achieved.
- In this situation it is now meaningful to assign a positive capacity to the fast fading channel

$$C = \mathbb{E}[\log(1+|h|^2\mathsf{SNR})]\ \text{bits/s/Hz}$$

CSI at the Transmitter

- With channel knowledge, we can control the transmit power such that R can be delivered no matter what the fading state is.
- This is known as the channel inversion strategy: the received SNR is kept constant irrespective of the channel state.
- This means that huge amount of paper is required when the channel is bad.
- Practical systems are peak-power constrained and this will not be possible beyond a threshold.

CSI at the Transmitter

- The capacity of the fast fading channel with CSI at the transmitter is given by

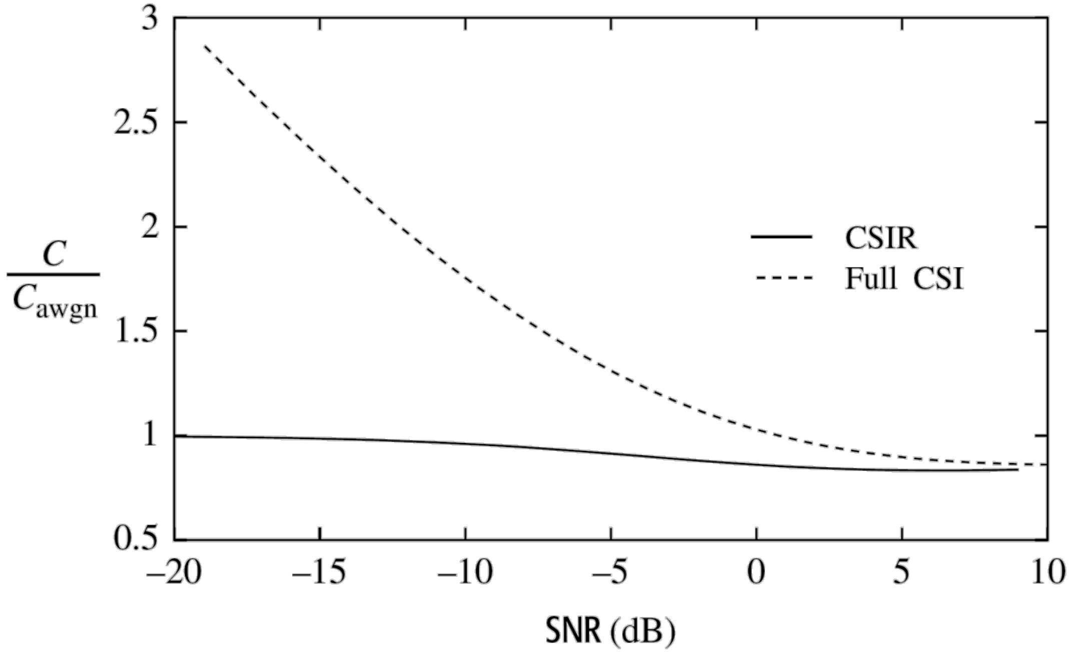
$$C = \mathbb{E}\left[\log\left(1 + \frac{P^*(h)|h|^2}{N_0}\right)\right] \text{ bits/s/Hz}$$

where

$$P^*(h) = \left(\frac{1}{\lambda} - \frac{N_0}{|h|^2}\right)^+$$

- λ depends only on the channel statistics but not on the specific realization of the fading process.
- In general, the transmitter allocates more power when the channel is good and less or even no power when the channel is poor.
- This is opposite of the channel inversion strategy.

Performance as a Fraction of AWGN Capacity



- At low SNR, the capacity with full CSI is significantly larger than the CSIR capacity.
- This means that the capacity of the fading channel can be much larger than when there is no fading.

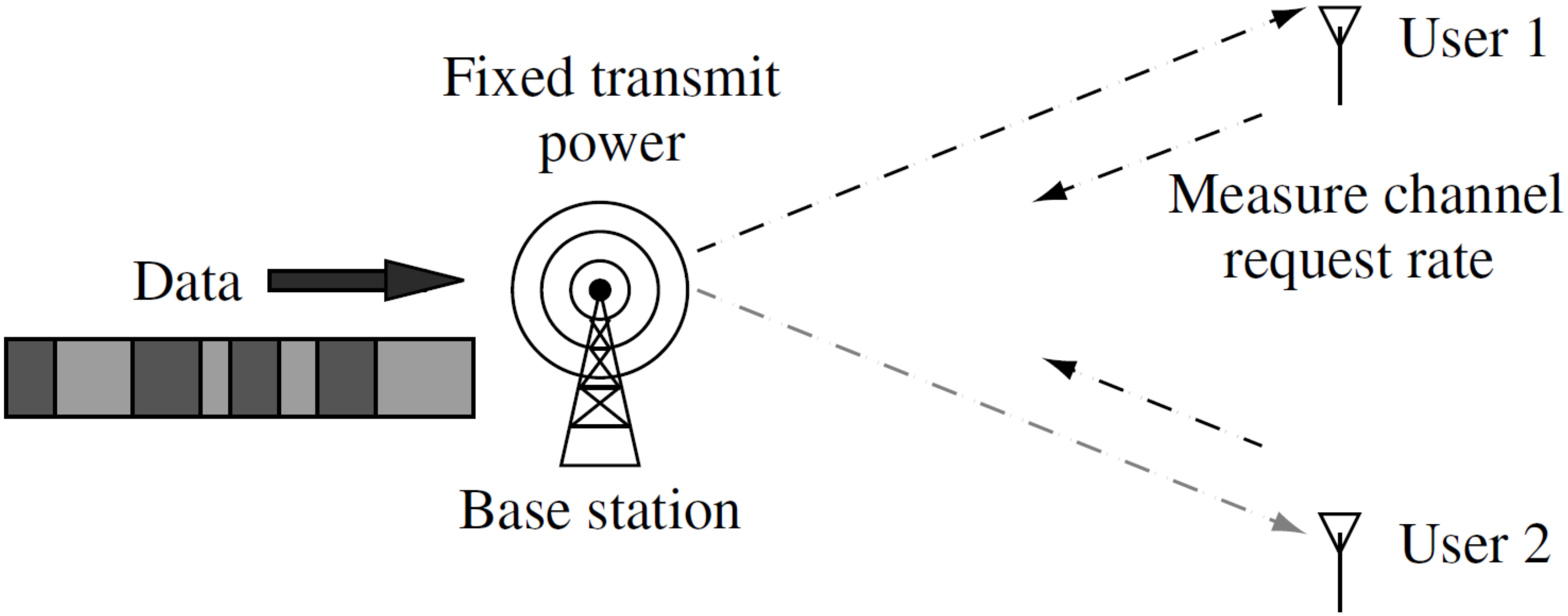
Discussion

- In a fading channel when SNR is low, with CSI the transmitter opportunistically transmits only when the channel is near it peak.
- In contrast, in a non-fading AWGN channel the channel stays constant and there are no peaks to take advantage of.
- Overall the performance gain from full CSI is not that large compared to CSIR, unless the SNR is very low.
- Channel inversion is power inefficient as compared to waterfilling, but it offers a constant rate of flow of information and so the associated delay is independent of channel variations.

2G (IS-95) and 3G (IS-856)

- The contrast between power control in IS-95 and rate control in IS-856 is roughly analogous to that between channel inversion and waterfilling.
- In IS-95 power is allocated dynamically to a user to maintain a constant target rate at all times.
- In IS-856 rate is adapted to transmit more information when the channel is strong. This is suitable for data since it does not have a stringent delay requirement.
- However, unlike waterfilling there is no dynamic power adaptation in IS-856, only rate adaptation.

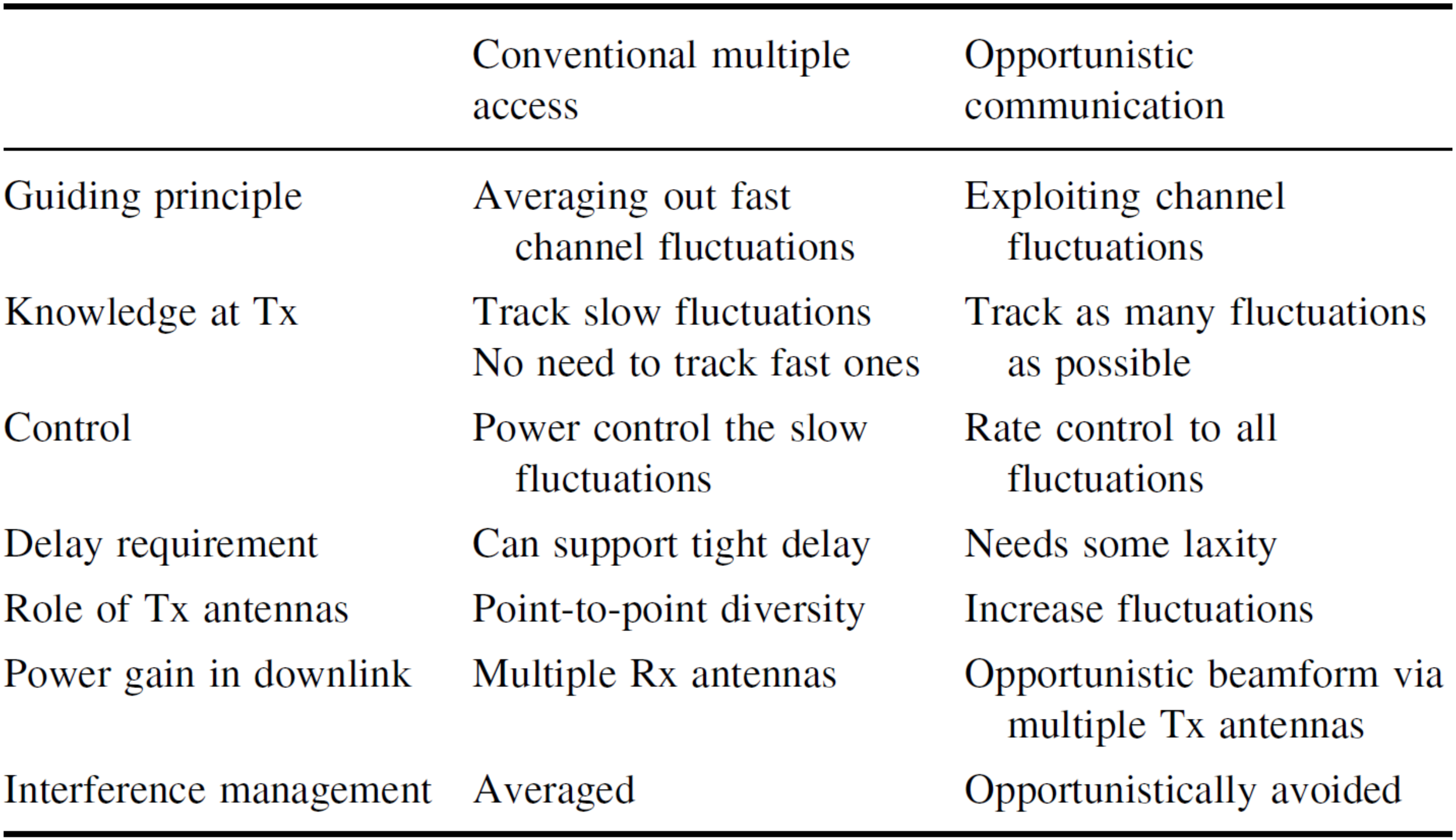
Rate Adaptation in IS-856



Rate Adaptation in IS-856

Requested rate (kbits/s)	SINR threshold (dB)	Modulation	Number of slots
38.4	−11.5	QPSK	16
76.8	−9.2	QPSK	8
153.6	−6.5	QPSK	4
307.2	−3.5	QPSK	2 or 4
614.4	−0.5	QPSK	1 or 2
921.6	2.2	8-PSK	2
1228.8	3.9	QPSK or 16-QAM	1 or 2
1843.2	8.0	8-PSK	1
2457.6	10.3	16-QAM	1

Conventional versus Modern Viewpoint



	Conventional multiple access	Opportunistic communication
Guiding principle	Averaging out fast channel fluctuations	Exploiting channel fluctuations
Knowledge at Tx	Track slow fluctuations No need to track fast ones	Track as many fluctuations as possible
Control	Power control the slow fluctuations	Rate control to all fluctuations
Delay requirement	Can support tight delay	Needs some laxity
Role of Tx antennas	Point-to-point diversity	Increase fluctuations
Power gain in downlink	Multiple Rx antennas	Opportunistic beamform via multiple Tx antennas
Interference management	Averaged	Opportunistically avoided