The Deborah Number

By M. Reiner

The following lines are from an after-dinner talk presented at the Fourth International Congress on Rheology, which took place last August in Providence, R. I. Marcus Reiner, research professor at the Israel Institute of Technology, is currently in the United States as a visiting professor at the Polytechnic Institute of Brooklyn.

In 1928 I came from Palestine to Easton, Pa., to assist Eugene Cook Bingham at the birth of Rheology. I felt strangely at home. There was Bethlehem quite near, there was a river Jordan and a village called little Egypt. The situation was, however, also slightly confusing. To go from Bethlehem to Egypt, one had to cross the river Jordan, a topological feature which did not conform to the original. Then there were, here, places such as Allentown to which there was no analogy. And this could lead to strange situations, such as when a girl at school was asked where Christ was born and replied, "In Allentown". When corrected by "No, in Bethlehem," she remarked, "I knew it was somewhere around here."

In Palestine I was working as a civil engineer doing science as a hobby. In 1926 a chemist had asked my help in the problem of the flow of a plastic material through a tube. I solved the problem and derived what is now known as the Buckingham-Reiner equation, Buckingham at the US National Bureau of Standards having derived the equation before. When Bingham learned of my work, he invited me to Lafayette College.

When I arrived, Bingham said to me, "Here you, a civil engineer, and I, a chemist, are working together at joint problems. With the development of colloid chemistry, such a situation will be more and more common. We therefore must establish a branch of physics where such problems will be dealt with."

I said, "This branch of physics already exists; it is called mechanics of continuous media, or mechanics of continua."

"No, this will not do," Bingham replied. "Such a designation will frighten away the chemists."

So he consulted the professor of classical languages and arrived at the designation of rheology, taking as the motto of the subject Heraclitus' πavτα ρει or "everything flows."

Rheology has become a well-known branch of physics, but most typists think it is a misprint for theology. I constantly receive mail addressed to the Theological Laboratory of the Israel Institute of Technology and, on the occasion of the Second International Congress at Oxford ten years ago, there was a special coach in the train at Paddington Station reserved for the members of the Theological Congress. This seems ridiculous, but there is some relation between rheology and theology, and on this I want to say a few words.

Heraclitus' "everything flows" was not entirely satisfactory. Were we to disregard the solid and deal with fluids only? There *are* solids in rheology, even if they may show relaxation of stress and consequently creep.*

The way out of this difficulty had been shown by the Prophetess Deborah even before Heraclitus. In her famous song after the victory over the Philistines, she sang, "The mountains flowed before the Lord." When, over 300 years ago, the Bible was translated into English, the translators, who had never heard of Heraclitus, translated the passage as "The mountains *melted* before the Lord"—and so it stands in the authorized version. But Deborah knew two things. First, that the mountains flow, as everything flows. But, secondly, that they flowed before the Lord, and not before man, for the simple reason that man in his short lifetime cannot see them flowing, while the time of observation of God is *infinite*. We may therefore well define as a nondimensional number the Deborah number

$$D = \text{time of relaxation/time of observation.}$$

The difference between solids and fluids is then defined by the magnitude of D. If your time of observation is very large, or, conversely, if the time of relaxation of the material under observation is very small, you see the material flowing. On the other hand, if the time of relaxation of the material is larger than your time of observation, the material, for all practical purposes, is a solid. In problems of industrial design, you may introduce the *time of service* for the time of observation. When designing a concrete bridge you make up your mind to decide how long you expect it to serve, and then compare this time-interval with the time of relaxation of concrete.

It therefore appears that the Deborah number is destined to become the fundamental number of rheology, bringing solids and fluids under a common concept, and leaving Heraclitus' πavτα ρει as a special case for infinite time of observation, or infinitely small time of relaxation. The greater the Deborah number, the more solid the material: the smaller the Deborah number, the more fluid it is.

There is a story they tell about two students of theology. They were praising the Almighty God. Said one: "For God, one thousand years are like a minute. And as He is the Creator of all, a thousand dollars are for Him like a cent." Said the other: "Wonderful; next time I pray to God, I shall pray, 'God, give me a cent'." Said the first: "What will it help you? He will say 'Wait a minute'."

This man did not take care of the difference between God's and his own time scale. And this is the connection between rheology and theology. In every problem of rheology make sure that you use the right Deborah number.

* and at this Congress a large number of papers deal with solids.