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Acoustics of bifacial Indian musical drums with composite membranes

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Abstract

We are interested in a certain class of bifacial Indian drums which consist of composite circular membranes stretched over an enclosed air cavity on both sides of an axisymmetric wooden shell. There is a large variety of such drums in Indian music which differ from each other in shapes and sizes of the shell and in the nature of the composite membranes. These drums produce sounds with a definite pitch. Whereas the effect of the composite nature of the membrane is well studied in the context of monofacial Indian drum tabla, the acoustical implications of the coupling between two composite membranes through an air cavity remains largely unexplored. The purpose of this work is to present some initial results from our study of this acoustical problem using a finite element method based numerical methodology. We use the developed framework, first to verify some existing results on Japanese wa-daiko, followed by an acoustical study of dholak, an Indian drum with composite membrane on one side, and finally to note the effect of curvature of the shell on modal frequencies.

Keywords: Bifacial drums, Indian drums, Composite membranes

1 INTRODUCTION

The importance of incorporating an enclosed air cavity below the vibrating membrane has been unambiguously demonstrated for monofacial drums such as kettledrum and tabla [1, 2]. The air cavity should arguably play a greater role in the acoustics of bifacial drums where the two membranes are coupled to each other via the enclosed air cavity and the surrounding shell. The most significant examples of such bifacial drums are the snare drums [3], the taiko family of percussion instruments from Japan [4], and the drums such as pakhawaj, mrdangam, dholak, dhol, iddakka, etc. from India [5, 6, 7]. The Indian drums usually have composite membranes (as in tabla) and distinguish themselves in generating sound with a definite pitch. In the following, we will begin by posing a general boundary-value-problem under some simplifying assumptions, followed by a variational principle which will form the basis of a finite element procedure. The developed framework will be verified by recovering some results on Japanese wa-daiko as presented by Suzuki and Hwang [4]. We will then proceed towards an acoustical study of dholak where membrane on one side of the barrel is composite and the two drum heads are unequal in size. We will also discuss the effect of curvature of the barrel shape on modal frequencies.

2 PROBLEM FORMULATION

The vibro-acoustic problem of bifacial Indian musical drums can be described by a system of coupled partial differential equations. These include equations which govern the displacement of the membranes and an acoustic wave equation which governs the internal pressure in the cavity. These equations are supplemented with appropriate set of initial and boundary conditions. In order to simplify the present discussion, we assume the membranes to be two-dimensional elastic continuum which do not resist or transmit bending moment and shear force. The restoring forces arise from the pre-stretching in the plane of the membrane. We neglect the acoustic and structural damping and assume the side walls of the cavity to be perfectly rigid. Moreover, we consider bifacial musical drums with axisymmetric cavity. The cavity is closed by stretched circular composite or homogeneous membranes on both the ends. The cavity is closed in such a manner that the air inside the cavity is confined and the motion of the membranes changes the volume of the air in the cavity. This changes the pressure of the air confined in the cavity. The pressure of the confined air generates a force on the membranes.

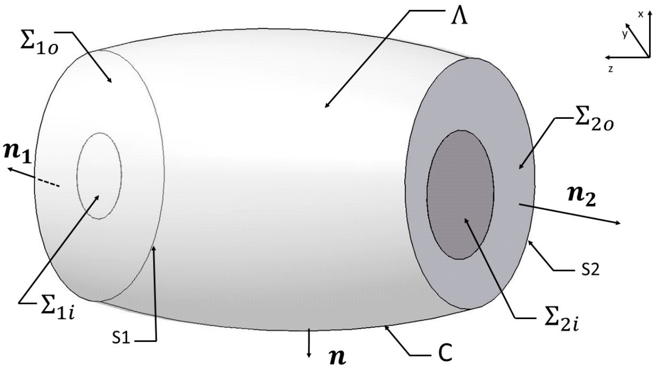


Figure 1. Schematic diagram of the problem domain.

In the following, we will consider a general model, of bifacial drums with composite membranes, as shown in Figure 1. The cavity domain Λ is bounded by surface C of a rigid shell with composite membranes on the two sides. The left side of the cavity has a composite membrane Σ_1 (radius b_1) with a centrally loaded patch (i.e. a patch of added material) Σ_{1i} (radius a_1) and the right side has a composite membrane Σ_2 (radius b_2) with an eccentrically loaded patch Σ_{2i} (radius a_2 and eccentricity d). The composite membrane with a centric loaded patch has a fixed edge S1 whereas the composite membrane with an eccentrically loaded patch has a fixed edge S2. The left side composite membrane is subjected to uniform tension T_1 per unit length such that its transverse motion $\bar{u}_1(x,y,t)$ is governed by

$$\sigma_1(r)\frac{\partial^2 \bar{u}_1}{\partial t^2} - T_1\Delta\bar{u}_1 = \bar{p}, \tag{1}$$

where $\sigma_1(r)$ is the density per unit area which is piecewise continuous with a constant value σ_{1a} for radius $0 \le r \le a_1$ and σ_{1b} for radius $a_1 < r \le b_1$; the operator Δ represents the Laplacian. The acoustic pressure field $\bar{p}(x,y,z,t)$ is also an unknown variable. At radius $r = a_1$, both the transverse motion (for compatibility) and the normal force (for equilibrium) should be continuous. At $r = b_1$, $\bar{u}_1 = 0$. The right side composite membrane is subjected to a uniform tension T_2 per unit length such that its transverse motion $\bar{u}_2(x,y,t)$ is governed by

$$\sigma_2(r)\frac{\partial^2 \bar{u}_2}{\partial t^2} - T_2\Delta\bar{u}_2 = \bar{p}, \tag{2}$$

where $\sigma_2(r)$ is the density per unit area which is piecewise continuous with a constant value σ_{2a} for the loaded patch and σ_{2b} for the remaining part. At the boundary of the loaded patch, both the transverse motion and the normal force should be continuous. At $r = b_2$, $\bar{u}_2 = 0$. The acoustic air cavity domain Λ is assumed to be filled with an inviscid fluid (air) with pressure field $\bar{p}(x,y,z,t)$, which is governed by the acoustic wave equation

$$\frac{\partial^2 \bar{p}}{\partial t^2} - c_p^2\Delta\bar{p} = 0, \tag{3}$$

where c_p is the speed of sound in the medium (air). The boundary condition at the rigid wall surface C is given by $\partial\bar{p}/\partial\mathbf{n} = 0$; at the left side membrane by $\partial\bar{p}/\partial\mathbf{n_1} = -\rho_a\ddot{\bar{u}}_1$, ρ_a is the density of air (1.21 kg/m^3); and at the right side membrane by $\partial\bar{p}/\partial\mathbf{n_2} = -\rho_a\ddot{\bar{u}}_2$.
Substituting the modal solutions

$$\bar{u_1} = u_1(x,y)e^{-i\omega t}, \ \bar{u_2} = u_2(x,y)e^{-i\omega t}, \ \text{and} \ \bar{p} = p(x,y,z)e^{-i\omega t} \tag{4}$$

into Equations (1), (2), and (3), where ω is the modal frequency, we can simplify them as

$$\omega^2\sigma_1(r)u_1 + T_1\Delta u_1 + p = 0 \tag{5}$$

for the composite membrane Σ_1, such that $u_1 = 0$ at edge S1,

$$\omega^2\sigma_2(r)u_2 + T_2\Delta u_2 + p = 0 \tag{6}$$

for the composite membrane Σ_2, such that $u_2 = 0$ at edge S2, and

$$\omega^2 p + c_p^2\Delta p = 0 \tag{7}$$

for the internal pressure field in the cavity Λ, such that $\partial p/\partial \mathbf{n} = 0$ on C, $\partial p/\partial \mathbf{n_1} = \omega^2\rho_a u_1$ on Σ_1 and $\partial p/\partial \mathbf{n_2} = \omega^2\rho_a u_2$ on Σ_2.

The preceding boundary-value-problem can be recast in terms of a variational principle. The solution of the problem, given in terms of smooth functions $u_1(x,y)$, $u_2(x,y)$, and $p(x,y,z)$, extremizes the variational functional $I(u_1,u_2,p) = I_1 + I_2 + I_3$, where

$$I_1 = \int_{\Sigma_1} \frac{1}{2}\Big(T_1\nabla u_1 \cdot \nabla u_1\Big)dA - \omega^2 \int_{\Sigma_1} \frac{1}{2}\Big(\sigma_1 u_1^2\Big)dA - \int_{\Sigma_1}\Big(pu_1\Big)dA, \tag{8}$$

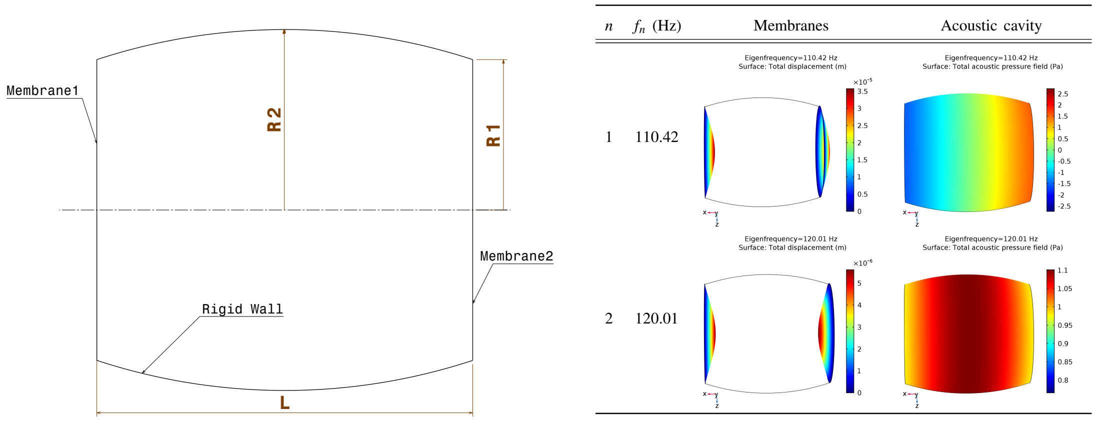
$$I_2 = \int_{\Sigma_2} \frac{1}{2}\Big(T_2\nabla u_2 \cdot \nabla u_2\Big)dA - \omega^2 \int_{\Sigma_2} \frac{1}{2}\Big(\sigma_2 u_2^2\Big)dA - \int_{\Sigma_2}\Big(pu_2\Big)dA, \text{ and} \tag{9}$$

$$I_3 = \int_{\Lambda} \frac{1}{2\omega^2\rho_a}\Big(\nabla p \cdot \nabla p\Big)dV - \int_{\Lambda} \frac{1}{2\rho_a c_p^2}\Big(p^2\Big)dV, \tag{10}$$

subjected to $\delta u_1 = 0$ on $S1$ and $\delta u_2 = 0$ on $S2$, the three variations δu_1, δu_2, and δp otherwise allowed to vary independently but smoothly over their respective domains; the operator ∇ represents the gradient. This variational principle forms the basis for our finite element procedure for determination of modal frequencies and modeshapes. We choose four-nod quadrilateral elements for discretizing the membranes and the rigid boundary C and eight-nod hexahedral elements for discretizing the acoustic domain, ensuring that the membrane elements match well with acoustics domain elements at the nodes. The basis functions used for the former are $\{1,x,y,xy\}$, whereas the basis functions used for the latter are $\{1,x,y,z,xy,xz,yz,xyz\}$. The integration over domains is evaluated using the standard Gauss quadrature rule for polynomials. The efficacy of our code is tested by using it to verify the existing results for kettledrum and tabla as reported in earlier literature [1, 8, 2], in particular considering non-cylindrical kettle shapes in the former case, see [9] for further details. Besides verifying our framework for monofacial drums, we use it to revisit Japanese bifacial drums, which have homogeneous membranes, as discussed next.

3 WA-DAIKO

We consider the bifacial drum wa-daiko whose geometry is shown in Figure 2(a). It is rotationally symmetric with respect to its central axis. The total length L of the barrel is 0.5 m. The radius R1 of both the membranes is 0.2 m. The maximum radius of the barrel R2 is 0.24 m. The curvature of the drum can be obtained by the three-point circle which passes through the maximum radius point and the two edge points. The other parameters, taken from Suzuki and Hwang [4], are volume density (2000 kg/m^3) and thickness (2 mm) of the membrane. With membrane tensions $T_1 = T_2 = 14$ kN/m, the first two modal frequencies, using our formalism, are 110.42 and 120.01 Hz. These are close to those obtained by Suzuki and Hwang [4] (as 110.5 and 120.0 Hz). The membranes move in phase at 110.42 Hz and in the opposite directions at 120.01 Hz. The corresponding mode shapes are shown in Figure 2(b). Furthermore, keeping tension $T_1 = 14$ kN/m in membrane 1, the modal frequencies for different values of tension in membrane 2 are evaluated and collected in Table 1; these are in agreement with Suzuki and Hwang [4]. Any further increase in tension T_2 beyond 20 kN/m does not affect the value of f_1. At such high tension values, the displacement of membrane 2 becomes very small. The membrane then behaves like a rigid body and stops interacting with membrane 1.



(a) (b)

Figure 2. (a) Geometry of wa-daiko; (b) Mode shapes.

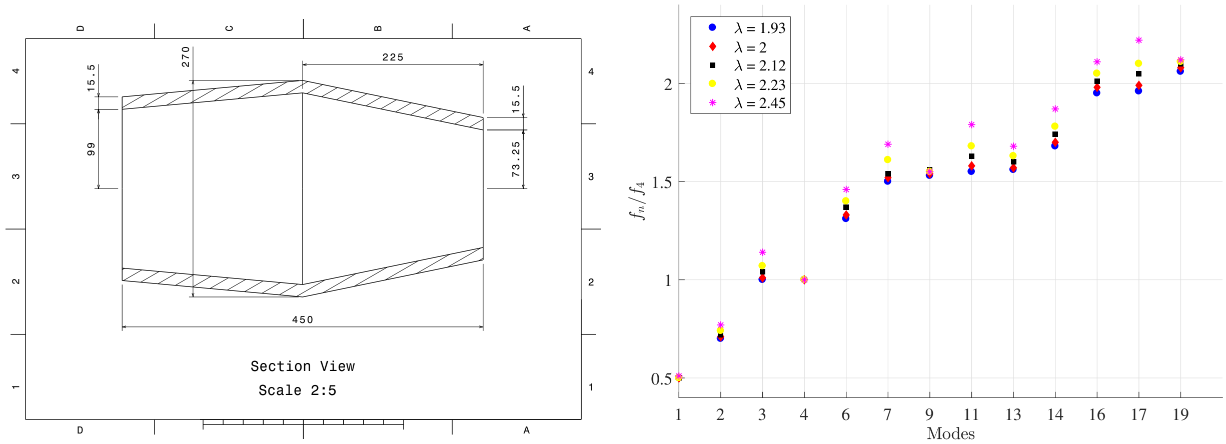
Table 1. Modal frequencies for the first two modes of wa-daiko for different tension values.

T_2(kN/m)	14	15	16	17	18	20
f_1(Hz)	110.42	111.99	112.94	113.51	113.87	114.23
f_2(Hz)	120.01	122.2	124.91	127.88	130.97	137.17

4 DHOLAK

Dholak is a barrel-shaped bifacial drum made of a single piece of wood; it is one of the most widely used drums in north India. The two membrane heads of dholak are different in size, where the smaller head is covered with a homogeneous membrane while the larger head is covered with a composite membrane. The latter has a patch, much like that in tabla, made of a mixture of sand, tar, and clay. However, unlike, tabla, the patch is on the inner side of the membrane and hence not visible on the drum surface. The length of dholak is around 41 cm. The outer diameter of the larger and the smaller drum heads are about 23 and 18 cms, respectively. The thickness of the hollow barrel is around 2 cm. A cross-sectional view of a typical dholak is shown in Figure 3(a). The diameter of dholak at the waist, at mid-length, is around 27.5 cm. The waist forms the base of the two truncated conical shapes which end at the drum heads. The membranes can be tuned separately to different tension values. In the simulations, we have used different tensions in both the membranes. The tensions in the larger and the smaller membranes are taken as $T_1 = 3.5$ kN/m and $T_2 = 3$ kN/m, respectively. The radius of the axisymmetric patch on the larger membrane is taken as 3.8 cm. The density of the homogeneous membrane (and the outer part of the composite membrane) is taken to be 0.18 kg/m^2. The ratio of the central patch density to outer part density is denoted by λ^2. We have fixed all the parameters except the value of λ, which we will vary to obtain different results. In Figure 3(b), we compare frequency ratios of several modes for various values of λ; the values are tabulated in Table 2. The mode shapes for the first twelve modes, corresponding to $\lambda = 1.93$, are given in Table 3 along with the frequency ratios f_n/f_4.

The frequencies f_1 and f_2 correspond to in phase and out of phase membrane motion, respectively, with larger membrane having a larger deflection. The frequencies f_3 and f_6 correspond to in phase and out of phase membrane motion, respectively, with smaller membrane having a larger deflection. All of these modes show a strong coupling between the membrane heads. The frequencies $f_4 = f_5$ correspond to a mode with one nodal diameter on the larger membrane and negligible activity in the other membrane and the air cavity. The frequencies $f_7 = f_8$ correspond to the case when both the membranes have one nodal diameter. The two membranes inter-



(a) (b)

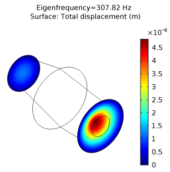
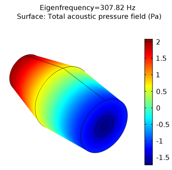
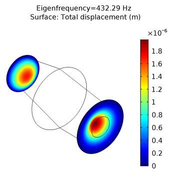
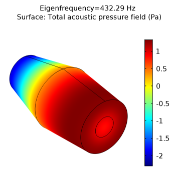
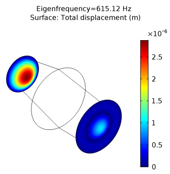
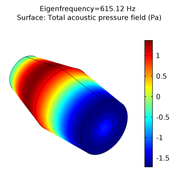
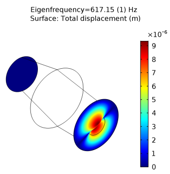
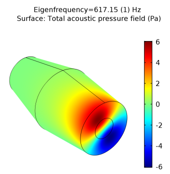
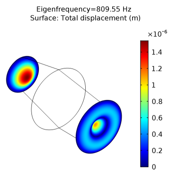
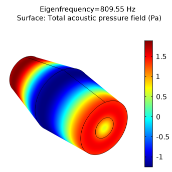
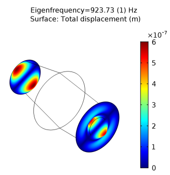
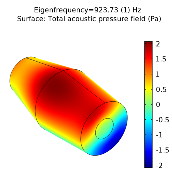
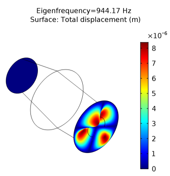
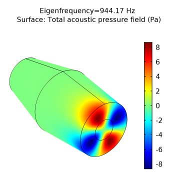
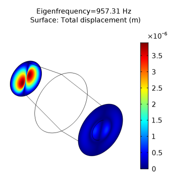
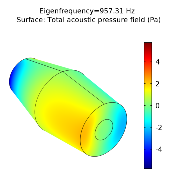
Figure 3. (a) A sectional view of dholak (all dimensions in mm); (b) Frequency ratios with an increasing value of λ. The missing modes (on the x-axis) are degenerate with respect the preceding mode number.

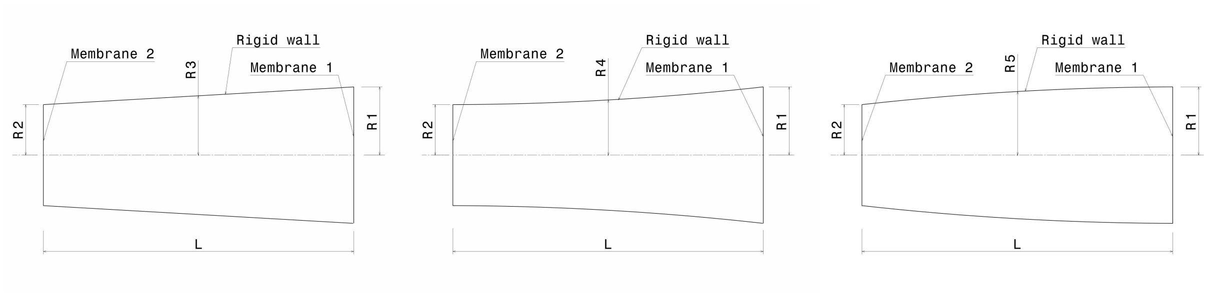
act strongly even in this case. The frequencies $f_9 = f_{10}$ represent a pair of degenerate modes with two nodal diameters on the larger membrane and negligible vibrations in the other membrane as well in the air cavity. The frequencies $f_{11} = f_{12}$ correspond to the case when there is one nodal diameter in the smaller membrane in addition to small vibrations both in the other membrane and the cavity. Clearly, there are modes in which membranes interact with each other and those in which they do not, the former mostly corresponding to the ones having no nodal diameters on the membranes. If the two drum sizes are equal then there is an increased degeneracy in the spectrum. A change in the value of λ will effect only those modes in which the larger membrane participates. The frequency ratios over a range of λ are presented in Table 2 (graphically in In Figure 3(b)). For $\lambda = 1.93$, we obtain many ratios close to being multiples of 0.25, which is indicative of sound with a definite pitch.

Table 2. Frequency ratios with respect to f_4 for dholak with an increasing value of λ.

n	f_n/f_4 (λ = 1.93)	f_n/f_4 (λ = 2.00)	f_n/f_4 (λ = 2.12)	f_n/f_4 (λ = 2.23)	f_n/f_4 (λ = 2.45)
1	0.50	0.50	0.50	0.50	0.51
2	0.70	0.71	0.72	0.74	0.77
3	1.00	1.01	1.04	1.07	1.14
4 and 5	1.00	1.00	1.00	1.00	1.00
6	1.31	1.33	1.37	1.40	1.46
7 and 8	1.50	1.52	1.54	1.61	1.69
9 and 10	1.53	1.54	1.56	1.55	1.55
11 and 12	1.55	1.58	1.63	1.68	1.79
13	1.56	1.57	1.60	1.63	1.68
14 and 15	1.68	1.70	1.74	1.78	1.87
16	1.95	1.98	2.01	2.05	2.11
17 and 18	1.96	1.99	2.05	2.10	2.22
19 and 20	2.06	2.08	2.10	2.11	2.12

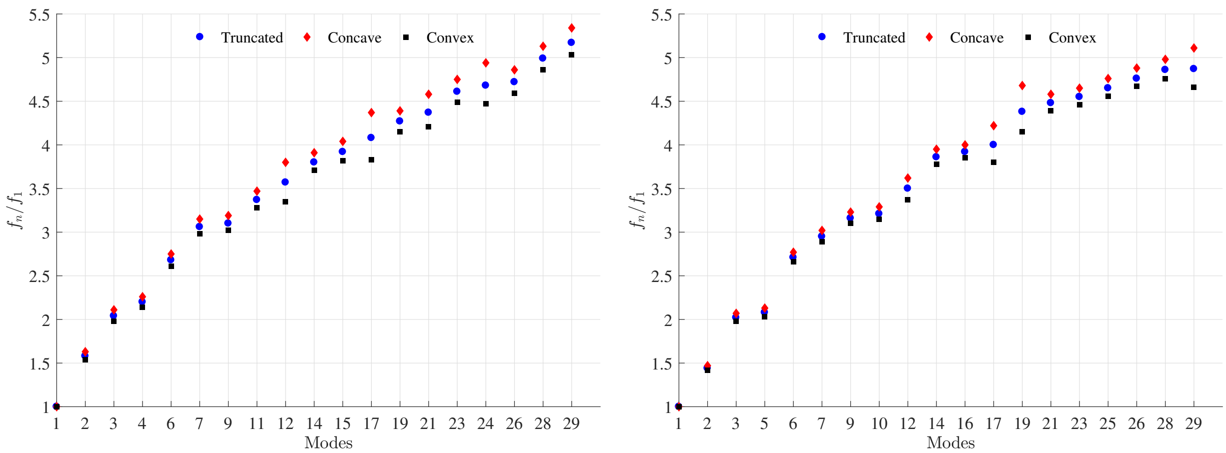
Table 3. Mode shapes and frequency ratios f_n/f_4 for dholak (λ = 1.93, T_1 =3.5 kN/m and T_2 =3 kN/m).

n	f_n/f_4	Membrane	Acoustic cavity
1	0.50		
2	0.70		
3	1.00		
4 and 5	1.00		
6	1.31		
7 and 8	1.50		
9 and 10	1.53		
11 and 12	1.55		



(a) (b) (c)

Figure 4. Section of a bifacial drum with varying barrel curvature: (a) truncated conical, (b) concave, and (c) convex shape.



(a) (b)

Figure 5. f_n/f_1 ratios for bifacial drums of three different barrel shapes with equal tension ($T_1 = T_2 = 4$ kN/m). (a) Both the membranes are homogeneous; (b) Smaller membrane is homogeneous while the larger one is composite. In both the cases, the missing modes (on the x-axis) are degenerate with respect the preceding mode number.

5 EFFECT OF CURVATURE OF THE BARREL

In this section we will discuss the effect of curvature of the axisymmetric shell which encloses the air cavity. We consider a drum geometry with membrane heads of unequal sizes (as is the case with most of the Indian drums) and a barrel with either conical, concave, or convex shape. Figure 4 shows a section of the three geometries considered in the following. With reference to the figure, R2 = 73.25 mm, R1 = 99 mm, L = 450 mm, R4 = R3 - 6.125 mm, and R5 = R3 + 6.125 mm, where R3 is the radius of the truncated cone at its mid point. The tensions in the larger and the smaller membranes are T_1 and T_2, respectively. We will consider two cases, one where both the membranes are homogeneous and the other where only one membrane is composite.

First, let the two membranes be homogeneous with a density of 0.2451 kg/m^2 and tensions $T_1 = T_2 = 4$ kN/m. The modal frequencies for several modes are compared for the three cases in Figure 5(a). Most of the modes have modeshapes qualitatively similar to those described for dholak, however there are some variations, for instance modes 7 and 8 which here have isolated deformation of the smaller membrane with one nodal diameter; for more details see [9]. Of course, only those frequencies which correspond to the modes where air cavity participates actively are affected by the change in shape of the barrel. The trends are captured in Figure 5(a). Second, we consider the case where the smaller membrane is homogeneous with a density of 0.2451 kg/m^2 but the other membrane is a composite membrane having a centric loaded patch of density twice as that of the outer portion. The outer density of the composite membrane is same as that of smaller membrane. The tension values in both the membranes are identical, $T_1 = T_2 =$ 4 kN/m. The modal frequencies for the three drum shapes are compared in Figure 5(b). The composite nature of the membrane lowers the frequencies corresponding to the modes in which the larger membrane participates actively. The degeneracy in the spectrum shifts by a mode, for several modes, when compared to the preceding scenario of homogeneous membranes. Overall, we note that the concave shaped bifacial drums have higher frequency ratios while the convex shaped ones have the lower frequency ratios with respect to the truncated cone shaped drums.

6 CONCLUSIONS

A variational formulation was proposed, and used for a finite element implementation, to study the acoustics of bifacial drums with axisymmetrically curved barrels and composite drumheads. Such drums are found commonly in different cultures across India. The developed method is applied to study a specific Indian drum, dholak, which has a convex shaped barrel and one composite membrane. Modal frequencies were investigated for a range of parametric values and the most optimum solution was identified in the considered range. Further experimental and simulation work, required to discuss a more complete picture of the emergent acoustical phenomena in such bifacial drums, will be discussed in a future study.

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