Acoustics of Mizhāvu

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ABSTRACT

The vibro-acoustical nature of mizhāvu, a large pitcher-shaped monofacial membranophone with an indefinite pitch, is studied. A coupled structure-acoustics, Finite Element Method (FEM) based, methodology is developed and used for conducting the modal analysis of the drum consisting of a clamped membrane backed by an acoustic air cavity. The results of the FEM simulation are interpreted keeping in mind the recorded sound of the drum and the strokes that are used for playing the drum. The distinctive acoustical signature of mizhāvu is identified through a coupling of axisymmetric membrane modes with longitudinal pressure modes resulting in a rather rich spectrum of overtones. The effect of varying parametric values on the frequency spectrum of the drum is discussed and the acoustics of mizhāvu is compared with two large drums, the Indian nagāḍā and the western timpani.

1. INTRODUCTION

Mizhāvu[a] is a big-bellied pitcher-shaped monofacial membranophone.[1,2] It has a short narrow cylindrical neck over which a parchment is stretched and tied tightly using a rope. The pitcher is made of thin copper sheet which thickens towards the neck. The parchment is developed out of the outer calf skin and is noticeably thicker than the ones used in tablā, for instance. A tiny hole, also called the ear of the drum, is drilled through one side of the vessel at around halfway height from the bottom, purposefully so as to improve the resonance.[1] A full mizhāvu, as well as a closeup of the neck region, is shown in Fig. 1. The drum is placed in front of the player within a cage (piñjara) of wooden slats such that its bottom remains away from the ground. Mizhāvu is played with hands and yields a loud metallic tone often described as thunderous. Unlike some other Indian drums, such as tablā, pakhāwaja, mradangam, and idakkā,[3-6] mizhāvu sound has no definite pitch.

Traditionally, mizhāvu has been used exclusively in the ritualistic Sanskrit theatre forms of Kerala such as kuṭiyaṭṭam, cākyar kuttu, and nāṅgyār kuttu.[7,8] Kuṭiyaṭṭam is the most prominent survivor of the ancient Indian theatrical culture with several links to nātyaśastra. The theatre is performed within a highly formalized rectangular structure called kuṭṭambalam which has a square-shaped

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[a] Mizhāvu is phonetically written as [mɪɻaːʋ]. The retroflex approximant [ɻ] is a trademark of Malayalam language. The *virama* at the end cancels the inherent vowel after the consonant ‘v’ as per the schwa deletion rule.

Figure 1: Mizhāvu (notice the hole) with a closeup of the neck portion.

stage in addition to an auditorium. The stage is connected to the dressing room (nepathya), located behind it, with two narrow doors (one for entrance and the other for exit). Two mizhāvus are placed between these two doors in the backside of the stage, see Fig. 2. Besides mizhāvu, idakkā (hour-glass shaped drum), kuzhittāla (cymbal), śankha (conch), and two wind instruments (kompa and kuchal) are also present on stage (altogether constituting a paṅcavādya ensemble).

Mizhāvu is played with an open palm using two basic strokes: thā, by hitting the middle of the membrane, and thom (or thu),[b] by hitting near the edge of the membrane.[1] These strokes are played with varying intensities to produce a structure of rhythmic beats or tālas. There are seven tālas: chempaṭa, tripuṭa, jhampa, dhruva, aṭanta, eka, and lakṣmī; these are used according to the situation, character,

[b] The phonemic representation of thā and thom strokes are /t̪aː/ and /t̪oːm/, respectively.

Figure 2: Mizhāvu in performance during the kuṭiyaṭṭam festival at the Natanakairali institute in Irinjalakuda, Kerala in the first week of January 2020.

etc. (for instance, lakṣmī tāla is used to enact the jatāyu dance piece from Rāmāyana).

The purpose of this article is to study and characterize the vibro-acoustical nature of mizhāvu. In Section 2, we report a brief analysis of the recording of mizhāvu sound. The recordings were conducted by one of the authors (SP) on the sidelines of the kuṭiyaṭṭam festival at the Natanakairali Institute in Irinjalakuda, Kerala, in early January 2020. These field studies provide us with the motivation for pursuing the simulation work in the following sections. In Section 3, we present an idealized mathematical model, and the associated variational formulation, for the finite element implementation of the structure-acoustic coupled problem. The results of the developed numerical methodology are presented and discussed in Section 4. The results are first justified in the light of frequency spectrums collected from the sound recordings and then studied under a variation in the parametric values of mouth diameter and membrane tension values. Subsequently, a simplified analytical model of the drum is discussed in order to understand the coupling between an axisymmetric membrane mode with the longitudinal acoustical modes. Towards the end, we briefly discuss the acoustics of nagāḍā and timpani and contrast them to that of mizhāvu. This is to emphasize the distinctiveness of mizhāvu in comparison to other big-bellied drums which also have a much larger mouth diameter than that of mizhāvu. Finally, we present some results which justify our assumption of ignoring the neck of the drum. The article is concluded in Section 5.

2. THE AUDIO RECORDINGS

The recordings were done using a mizhāvu which was 76 cm high, had a mouth with inner and outer diameter of 14 cm and 16 cm, respectively. The rim of the neck, which provided the boundary for the vibrating membrane, was therefore 1 cm thick. The membrane was clamped at the outer edge of the rim. The maximum diameter of the pitcher was 52 cm. The hole on the side had a diameter of 9 mm and was located 48 cm above the bottom of the drum. The drum overall had an axisymmetric shape (modulo the hole on the side). The audio recordings were conducted using the Audio-Technica AT2020 USB cardioid condenser microphone. Several recordings were made (different drummers playing with different intensity) for each of the following cases: (i) basic strokes thā and thom with hole open, (ii) basic strokes thā and thom with hole closed, (iii) lakṣmī tāla on one drum, and (iv)

thā (with hole)	293.9, 354.7, 571.3, 741, 834.7, 936.1, 957.6, 1116, 1173
thom (with hole)	305.6, 355.2, 572.3, 738.6, 935.7, 959.3, 1115, 1173
thā (hole closed)	296.2, 355, 571.6, 741.3, 844.4, 935.7, 959.2, 1116, 1173
thom (hole closed)	306.3, 361.3, 571.8, 936.7, 959.4, 1116
lakṣmī tāla	306.5, 358.5, 571.6, 743.8, 862.7, 938.5, 960.6, 1118, 1175
two mizhāvus	305.5, 359.7, 575.2, 617.8, 746.7, 853.1, 940, 961.6, 1118, 1175

Table 1: Dominant natural frequencies (in Hz) for various playing styles.

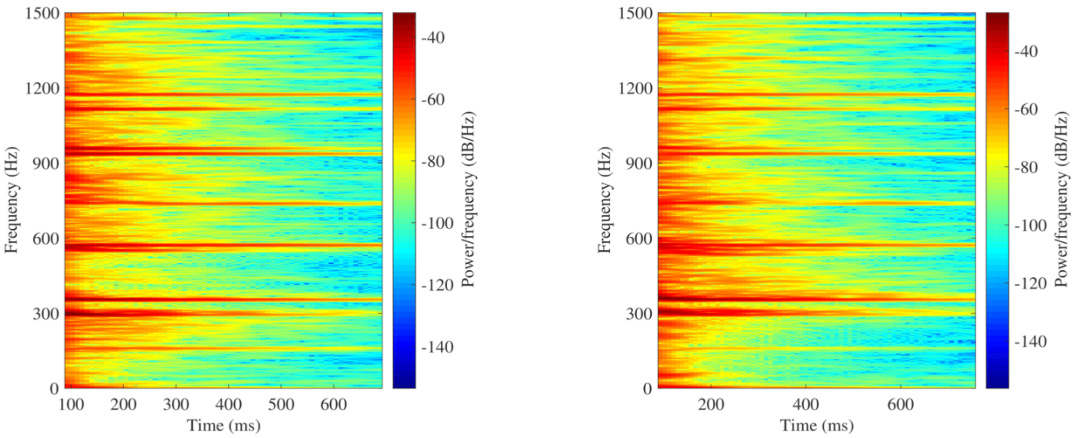


Figure 3: Spectrograms for thā and thom strokes with hole open.

freestyle playing using two mizhāvus. The recorded data is processed to generate spectrograms and frequency spectrum plots. The modal frequencies are identified from the dominant peaks appearing in the latter. Out of the several recordings for each of the case, mentioned above, we present results for one representative sample after noting little variation among all the available candidates. The dominant modal frequencies are summarized in Table 1. The spectrograms corresponding to thā and thom strokes, with hole open, are given in Fig. 3. The associated frequency spectrum plots are given in Fig. 4. The spectrum plots for the remaining four cases are collected in Figs. 5 and 6. The spectrograms for thā and thom strokes showed no perceptible difference when the hole was closed.

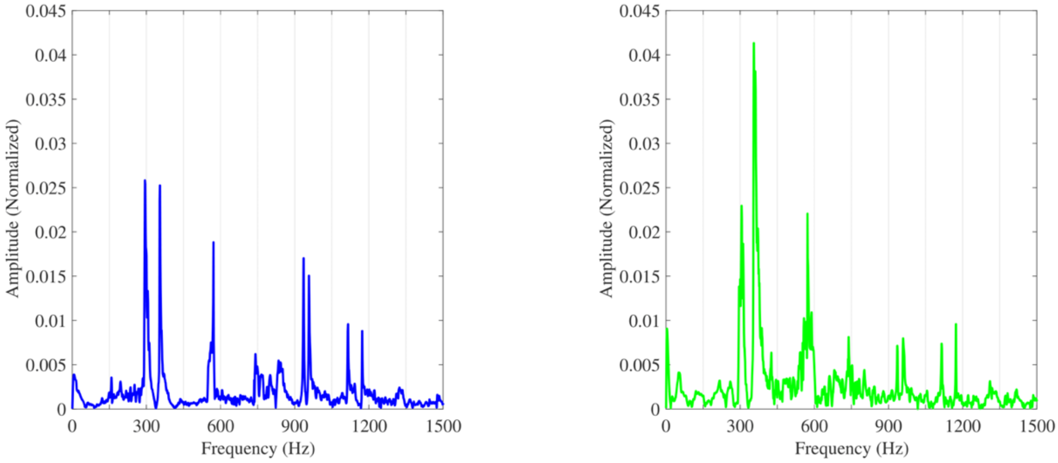


Figure 4: Frequency spectrum plots for thā and thom strokes with hole open.

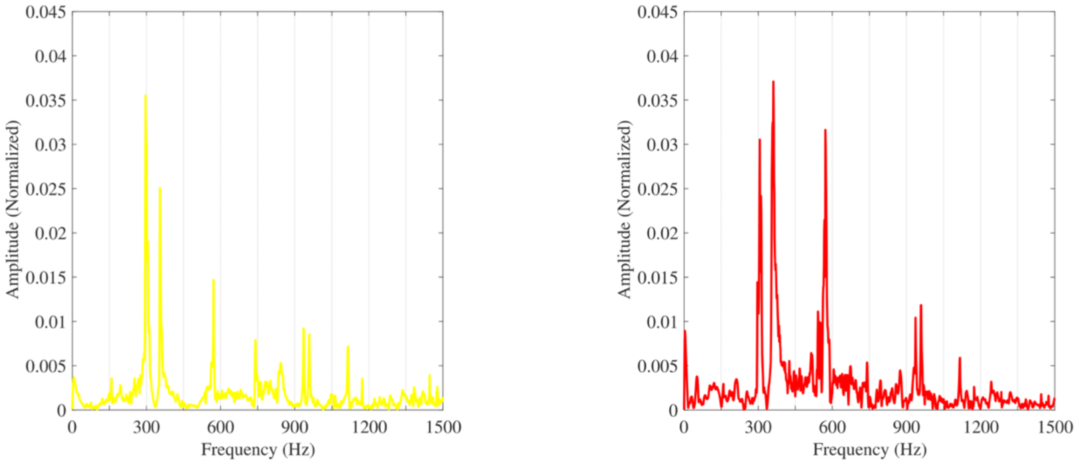


Figure 5: Frequency spectrum plots for thā and thom strokes with hole closed.

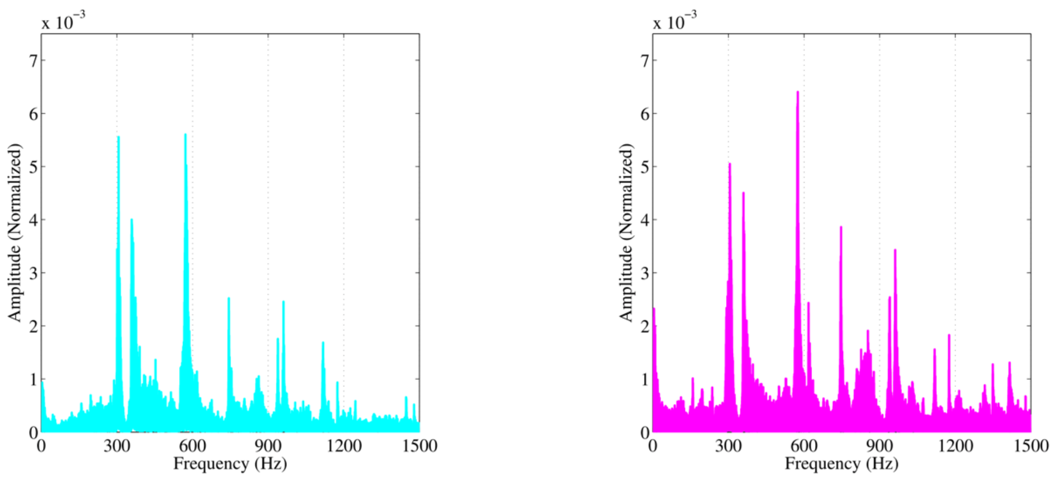


Figure 6: Frequency spectrum plots for lakṣmī tāla and a pair of mizhāvus.

Several observations are in order concerning these plots. First, recall that with the thā and thom strokes, the membrane is struck at the centre and close to the edge, respectively (both with an open palm). Therefore, we expect the spectrum of thā stroke to be dominated by the membrane modes with antinodes at the centre; all the *0m* modes satisfy this. Here, and elsewhere, the mode shapes of circular membranes are denoted through the convention *nm*, where *n* indicates the number of nodal diameters and *m* the number of nodal circles.[9] The spectrum for the thom stroke will also include the other modes (the *nm* modes). This is evident from the spectrograms in Fig. 3, although the modes which sustain are identical in both the cases. Given that the strikes are made with an open palm (i.e., a finite area over the small membrane), it is expected that the vibration modes are dominated by modes with *01* and *02* membrane modes (assuming that the higher membrane modes will be more damped). Even then, certain modes can dominate overall due to the resonance of specific membrane modes with the air cavity modes. Second, the hole in the drum shell appears to be inconsequential as far as spectrograms and frequency spectrums are concerned. Its acoustical importance therefore remains inconclusive. Finally, there is a consistent pattern in peaks which can be noticed from all the spectrum plots. There are clear peaks close to 300 Hz, 600 Hz, 750 Hz, 900 Hz, and 1200 Hz, indicating semblance of a definite pitch with a fundamental of 150 Hz (the fundamental is discernible in the spectrograms). The peaks at 300 Hz, 900 Hz, and 1200 Hz always appear in pair. The difference in the frequencies of the pair are around 50 Hz, 20 Hz, and 50 Hz, respectively. The presence of these nearby peaks indicate a beat-like phenomena in mizhāvu sound. The splitting of frequencies can be a consequence of asymmetric tuning of the membrane, or due to asymmetry in the membrane density distribution, or asymmetry in the construction of the instrument; we do not explore any of these possibilities in this work. To summarize, the presence of air cavity in mizhāvu yields several dominant natural frequencies in close harmonic relationships in addition to appearance of beats due to pairs of nearby frequency values.

3. THE VIBRO-ACOUSTIC MODEL

The vibro-acoustic problem of monofacial drums can be described in terms of a system of coupled partial differential equations. These include the membrane vibration equation and the acoustic wave equation, the latter governing the internal pressure field of the cavity. The differential equations are supplemented with an appropriate set of initial and boundary conditions. We neglect both acoustic and structural damping and assume the walls of the cavity to be perfectly rigid. We also neglect the acoustic environment external to the drum. The cavity of the drum is closed in such a manner that the air inside the cavity is confined and the motion of the membrane changes the volume of the air in the cavity. This changes the pressure of the air confined in the cavity. The pressure of the confined air in turn generates a force on the membrane.

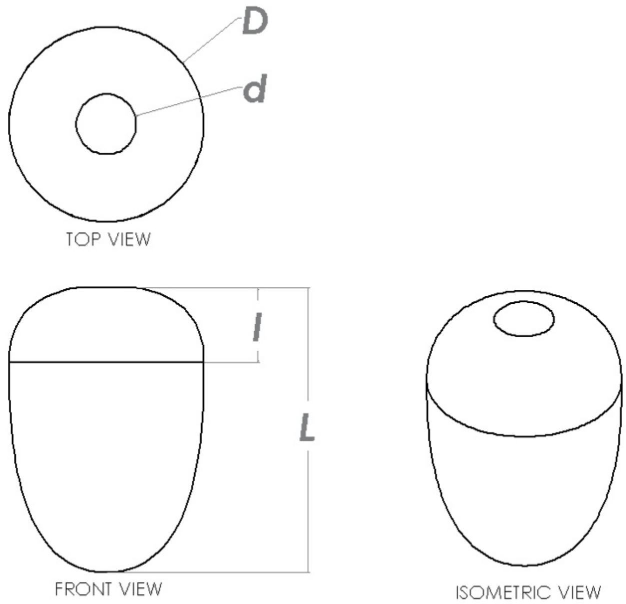


Figure 7: An idealized model of mizhāvu shell used for FEM simulations; d represents diameter of the mouth, D the maximum diameter of the shell, L the height of the drum, and l the position (from top) of the maximum diameter.

We consider an idealized model of mizhāvu, whose geometry is illustrated in Fig. 7. We assume the neck height (and width) of mizhāvu to be vanishingly small and the hole to be absent. The validity of the former assumption is discussed in Section 4.5 whereas the latter is justified in the preceding section on the basis of audio recordings. The membrane is clamped at the edge of the open face of the curved shell. The latter is assumed to be elastically rigid. In general, due to thickness of the neck region, the membrane will wrap and unwrap over the finite rim, somewhat analogous to the behaviour of a vibrating string over the bridge in several Indian string instruments such as tānpurā and sitār.[10] The feature of a finite rim is also present in a large variety of Indian drums (e.g., tablā, pakhāwaja, and idakkā) and it will be important to study its role in the vibro-acoustical behavior of these drums. However, in our idealized model, the rim is assumed to be sharp without any finite width. The density of the membrane is assumed to be uniform.

In formal terms, the cavity domain Λ is bounded by a rigid axisymmetric shell surface C and a circular membrane Σ of diameter d. The membrane, with a fixed edge S, is subjected to uniform tension T per unit length such that its transverse motion $\bar{u}(x, y, t)$ is governed by the differential equation

$$\sigma \frac{\partial^2 \bar{u}}{\partial t^2} - T\Delta\bar{u} = \bar{p}, \tag{1}$$

where σ is the uniform density (per unit area) of the membrane and the operator Δ represents the two-dimensional Laplacian. The acoustic pressure field $\bar{p}(x, y, z, t)$ is also an unknown variable. At radius $r = d/2$, $\bar{u} = 0$. The acoustic air cavity domain Λ is assumed to be filled with an inviscid fluid (air) whose pressure field is governed by the acoustic wave equation

$$\frac{\partial^2 \bar{p}}{\partial t^2} - c_p^2 \widetilde{\Delta}\bar{p} = 0, \tag{2}$$

where c_p is the speed of sound in the medium (air) and $\widetilde{\Delta}$ is the three-dimensional Laplacian. The boundary conditions at the rigid wall surface C and at the membrane are given by $\partial\bar{p}/\partial\boldsymbol{n} = 0$ and $\partial\bar{p}/\partial z = -\rho_a \ddot{\bar{u}}$, respectively, where ρ_a is the density of air and $\boldsymbol{n}$ is the outward normal to the surface. The modal solutions

$$\bar{u} = u(x, y)e^{-i\omega t} \; and \; \bar{p} = p(x, y, z)e^{-i\omega t}, \tag{3}$$

where ω is the frequency, when substituted into Eqs. (1) and (2), respectively, yield

$$\omega^2\sigma u + T\Delta u + p = 0 \tag{4}$$

for the membrane Σ, such that $u = 0$ at edge S, and

$$\omega^2 p + c_p^2 \widetilde{\Delta} p = 0 \tag{5}$$

for the internal pressure field in the cavity Λ, such that $\partial p/\partial \boldsymbol{n} = 0$ on C and $\partial p/\partial z = \omega^2 \rho_a u$ on Σ.

The preceding boundary-value-problem can be recast in terms of a variational principle. The solution of the problem, given in terms of smooth functions $u(x, y)$ and $p(x, y, z)$, extremizes the variational functional $I(u, p) = I_1 + I_2$, where

$$I_1 = \int_\Sigma \frac{1}{2} T(\nabla u \cdot \nabla u)\, dA - \omega^2 \int_\Sigma \frac{1}{2} \sigma u^2\, dA - \int_\Sigma pu\, dA \text{ and} \tag{6}$$

$$I_2 = \frac{1}{2\omega^2 \rho_a} \int_\Lambda \tilde{\nabla} p \cdot \tilde{\nabla} p\, dV - \frac{1}{2\rho_a c_p^2} \int_\Lambda p^2\, dV \tag{7}$$

subjected to $\delta u = 0$ on S; the operators ∇ and $\tilde{\nabla}$ represent the two-dimensional and the three-dimensional gradient; and $\cdot$ denotes the dot product. This variational principle forms the basis for our finite element procedure (implemented using Matlab) for the determination of modal frequencies and modal shapes. We choose four-node quadrilateral finite elements for discretizing the membrane and the rigid boundary C and eight-node hexahedral finite elements for discretizing the acoustic domain while ensuring that the membrane elements match well with acoustics domain elements at the nodes. The basis functions used for the former are $\{1, x, y, xy\}$ whereas the basis functions used for the latter are $\{1, x, y, z, xy, xz, yz, xyz\}$. The integration over domains is evaluated using the standard Gauss quadrature rule for polynomials. The efficacy of our code is tested by using it to verify the existing results for timpani and tablā as reported in the earlier literature.[4,11,12] The details, including those related to convergence and mesh refinement, are available elsewhere.[13]

4. RESULTS AND DISCUSSION

The idealized shape of mizhāvu for simulation purposes is considered as given in Fig. 7. The curve generating the axisymmetric shape of the drum is drawn using a three-point spline interpolation. This is done in two parts, one from the mouth to the maximum diameter and the other from the maximum diameter to the bottom. The curves are chosen so as to mimic the shape of the actual drum. We fix the height of the drum as L=76 cm, the maximum diameter of the shell as D=52 cm, and the distance of the maximum diameter circle from top as l=28 cm, all in accordance with the mizhāvu used for the audio recordings. The diameter d of the mouth however will be allowed to vary (between 14 cm and 16 cm). Besides d, we will also allow the membrane tension T to take different values. The material parameters σ=0.5445 kg/m^2 (areal density of the membrane), ρ_a=1.21 kg/m^3 (volume density of air), and c_p=344 m/s (speed of sound in air) will be fixed throughout. The areal density of the mizhāvu membrane is calculated from the samples collected during the field work. It should be noted that the density value is almost twice as much as that of tablā and timpani membranes.

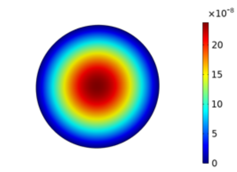
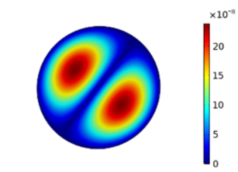
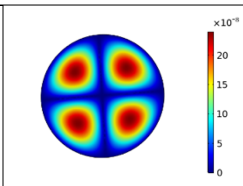
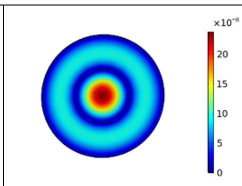
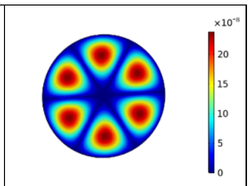
				
365.32 Hz	582.12 Hz	780.4 Hz	838.96 Hz	970.04 Hz

Table 2: Mode shapes and natural frequencies of an ideal membrane (without air cavity) with T=3500 N/m, d=16 cm; the membrane displacements shown are in m.

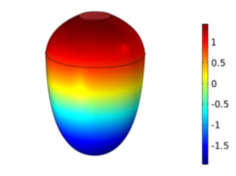
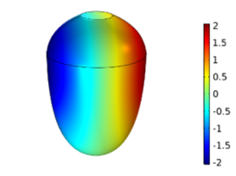
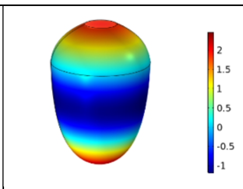
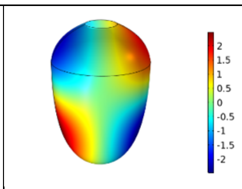
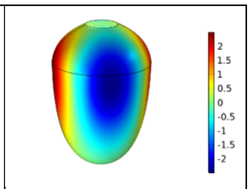
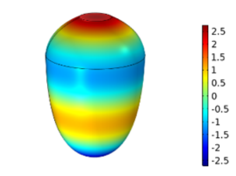
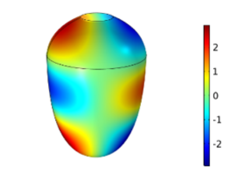
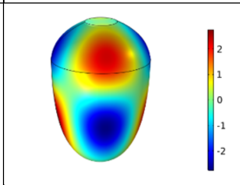
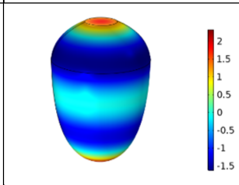
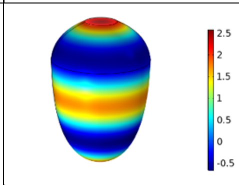
				
285.22 Hz	429.27 Hz	515.05 Hz	578.35 Hz	691.82 Hz
				
712.47 Hz	757.52 Hz	832.61 Hz	861.68 Hz	920.12 Hz

Table 3: Mode shapes and natural frequencies of the air cavity without a membrane closing the facing with d=16 cm; the pressure values shown are in Pa.

4.1. The eigen-spectrum and mode shapes of the drum

In order to understand the frequency spectrum, as obtained from the audio recordings, we begin by fixing d=16 cm and T=3500 N/m. The effect of varying d and T values on the frequency spectrum will be discussed in Section 4.2. To set the background, we look at the natural frequencies and mode shapes of the two building blocks of our drum taken separately, i.e., an ideal membrane clamped at its edge, on one hand, and an air cavity in the shape of the drum but with a face not covered by the membrane, on the other. In the latter case, the air cavity is assumed to be surrounded by an acoustically hard boundary (i.e., $\partial p/\partial \boldsymbol{n} = 0$ everywhere on the boundary). The pertinent results are collected in Tables 2 and 3, respectively. We recall from Section 2 that the strokes of mizhāvu playing will predominantly activate the *01* and *02* membrane modes (the axisymmetric modes) and consequently the longitudinal pressure modes (again axisymmetric) in the air cavity. With the considered parameters, these modes appear, respectively, at 365.32 Hz and 838.96 Hz for an isolated membrane, and at 285.22 Hz, 515.05 Hz, 712.47 Hz, 861.68 Hz, and 920.12 Hz for an isolated air cavity. The frequency values are close to the peaks observed in the spectrums from audio recordings.

A more complete picture is obtained when the frequency spectrum of the full drum (membrane and cavity coupled) is obtained. The results are given in Table 4. The relevant frequencies are those corresponding to the axisymmetric membrane modes. These are 279.46 Hz, 350,6 Hz, 520.42 Hz, 716 Hz, corresponding to the *01* membrane mode, and 829.39 Hz, 865.09 Hz, 926.23 Hz, corresponding to the *02* membrane mode. The acoustic mode shapes which accompany these frequencies are all longitudinal and axisymmetric, see Table 4. These frequency values are in reasonable agreement with the frequency spectrum obtained from the audio recordings of the thā stroke where the first seven frequency peaks were observed at 293.9 Hz, 354.7 Hz, 571.3 Hz, 741 Hz, 834.7 Hz, 936.1 Hz, 957.6 Hz, see Table 1, first row. While comparing these values we should remember that we have ignored the effect of the acoustic environment external to the drum and have neglected the finiteness of the rim, among other idealizations in terms of geometry and material properties.

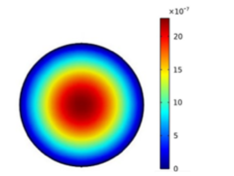
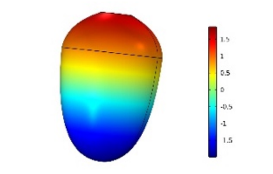
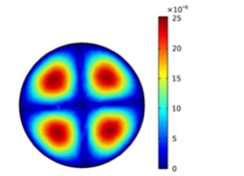
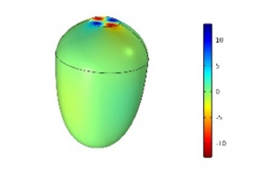
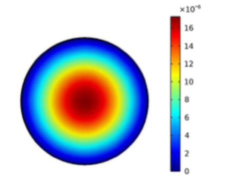
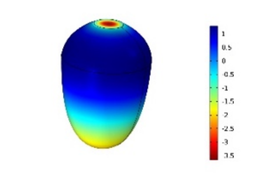
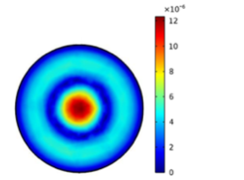
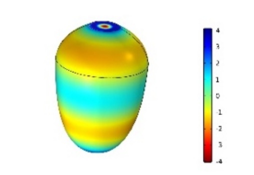
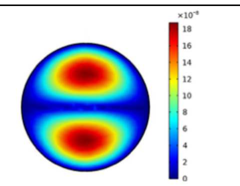
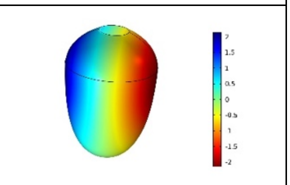
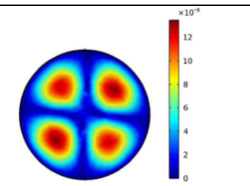
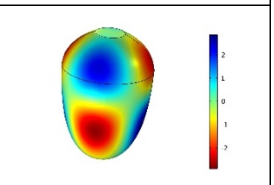
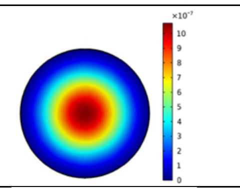
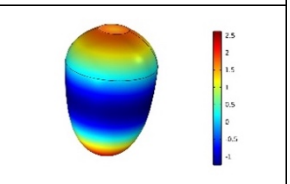
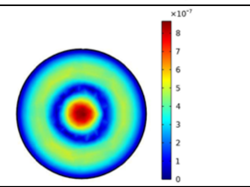
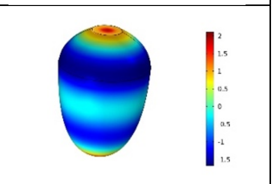
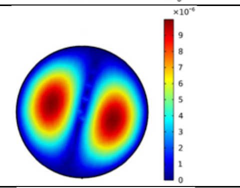
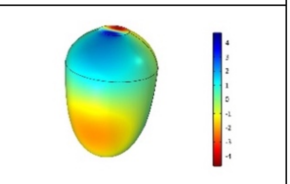
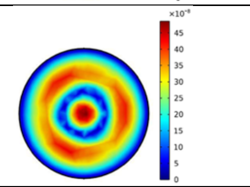
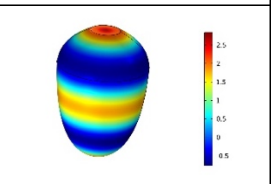
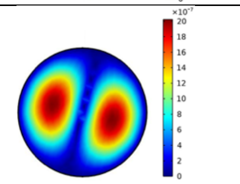
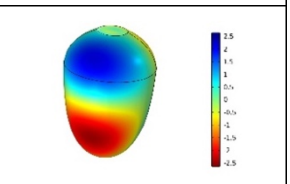
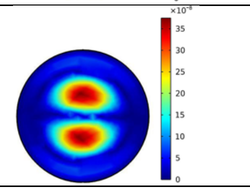
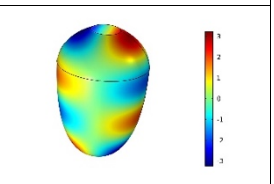
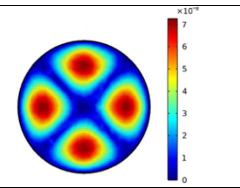
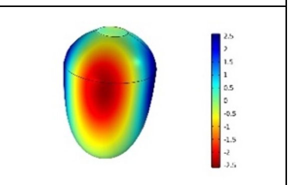
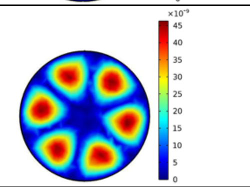
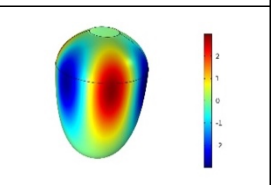
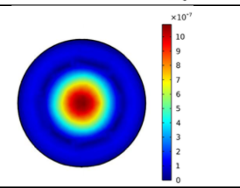
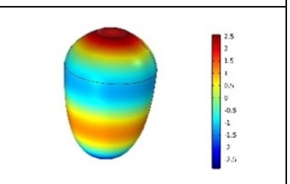
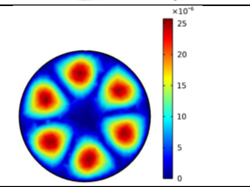
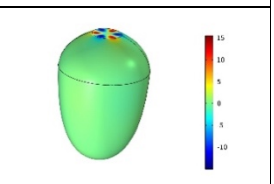
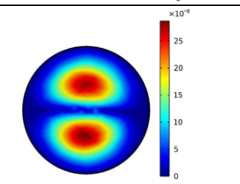
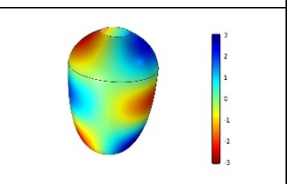
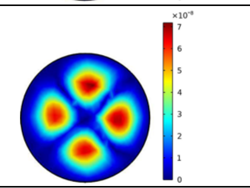
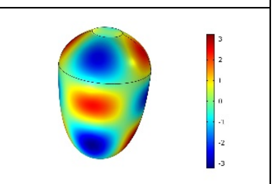
Natural Frequency (Hz)	Membrane Mode	Air Cavity Mode		Natural Frequency (Hz)	Membrane Mode	Air Cavity Mode
279.46				768.38		
350.6				829.39		
429.3				832.72		
520.42				865.09		
561.8				926.23		
581.46				935.01		
691.78				937.39		
716.23				967.88		
758.21				997.69		

Table 4: Mode shapes and natural frequencies (in Hz) of an idealized mizhāvu with T=3500 N/m and d=16 cm. The membrane and air cavity modes shapes are shown separately for clarity. The membrane displacements are in m and pressure in Pa.

The acoustical nature of mizhāvu is distinct from that of both tablā and timpani. The tablā acoustics is dominated by the vibrations of the non-uniform membrane (which yields a definite pitch) with the air cavity playing a leading role in dampening out the unwanted modes. In timpani acoustics, the acoustic cavity exerts a sufficient pressure load on the membrane so as to bring slight (but important) changes in the membrane frequency values. The mizhāvu acoustics is dominated by the longitudinal (axisymmetric) pressure modes multitude of which are coupled with the axisymmetric membrane modes. In fact, in all but a few cases, the pressure modes generate the vibration pattern in the membrane (as if it is a forced vibration of the membrane due to acoustic pressure). Unlike both tablā and timpani, several distinct modes appear each with *01* and *02* membrane modes (see also Section 4.4). Consequently, even the sound generated by exciting only the first one or two axisymmetric membrane modes (by striking the drum membrane at the center, as in the thā stroke) yields rich overtones with a somewhat distinctive pitch.

4.2. Effect of varying membrane diameter and tension

We now investigate how the spectrum changes when we change the mouth diameter and tension in the membrane, while keeping other parameters fixed (as mentioned above). The natural frequency values, appended with the corresponding membrane mode shape identifiers, are collected in Table 5. The acoustic modes are not mentioned for the sake of brevity and because they are of the kind plotted in Table 4. The membrane modes can be identified with the numbers written in the *nm* format in a smaller font next to the frequency values. For instance, 277$_{01}$ indicates that the frequency 277 Hz (rounded off to the nearest integer) is associated with a mode shape having *01* membrane mode (in addition to some pressure mode). Wherever the mode shape was unclear, it is indicated with a U next to the frequency value. Some columns are shorter than others because the membrane mode shapes are no longer discernible. We first note the trend in the frequency spectrum change as we modify the diameter of the mouth from d=14 cm to 15 cm and then 16 cm, all for a fixed tension value. The frequencies are, in general, seen to decrease with increasing d, sometime staying more or less constant but sometimes changing drastically. Frequently, particularly for higher frequencies, the order of mode shapes is modified and, in some cases, new membrane modes replace existing ones. These observations remain invariant for all the seven membrane tension values between 2000 N/m and 5000 N/m considered in Table 5. We also note the change in the spectrum for a fixed diameter but varying tension in the membrane. The frequency values generally increase with an increasing tension, but there are multiple instances when they remain invariant. The latter will clearly occur whenever the mode is dominated by the acoustic cavity and the membrane vibration has little overall influence. The axisymmetric modal frequencies, agreeable with mizhāvu, are obtained at higher tension values for lower mouth diameters and vice versa.

T= 2000 N/m			T= 2500 N/m			T= 3000 N/m			T= 3500 N/m			T= 4000 N/m			T= 4500 N/m			T= 5000 N/m		
d=14	*d=15*	*d=16*	*d=14*	*d=15*	*d=16*	*d=14*	*d=15*	*d=16*	*d=14*	*d=15*	*d=16*	*d=14*	*d=15*	*d=16*	*d=14*	*d=15*	*d=16*	*d=14*	*d=15*	*d=16*
277_{01}	269_{01}	251_{01}	281_{01}	277_{01}	269_{01}	282_{01}	278_{01}	276_{01}	282_{01}	279_{01}	279_{01}	283_{01}	280_{01}	281_{01}	283_{01}	280_{01}	281_{01}	283_{01}	281_{01}	282_{01}
323_{01}	307_{01}	298_{01}	356_{01}	335_{01}	310_{01}	387_{01}	362_{01}	329_{01}	415_{01}	389_{01}	350_{01}	419_{11}	413_{01}	371_{01}	419_{11}	417_{11}	391_{01}	419_{11}	417_{11}	410_{01}
419_{11}	418_{11}	423_{11}	419_{11}	418_{11}	429_{11}	419_{11}	417_{11}	429_{11}	419_{11}	417_{11}	429_{11}	441_{01}	417_{11}	429_{11}	464_{01}	436_{01}	429_{11}	482_{01}	456_{01}	429_{11}
515_{11}	480_{11}	433_{11}	519_{01}	516_{01}	478_{11}	519_{01}	517_{01}	519_{01}	521_{01}	517_{01}	520_{01}	523_{01}	519_{01}	521_{01}	526_{01}	520_{01}	522_{01}	532_{01}	523_{01}	523_{01}
518_{01}	516_{01}	518_{01}	559_{11}	535_{11}	519_{01}	560_{11}	555_{11}	522_{11}	560_{11}	555_{11}	561_{11}	560_{11}	556_{11}	576_{11}	560_{11}	556_{11}	577_{11}	560_{11}	556_{11}	577_{11}
561_{11}	557_{11}	579_{11}	578_{11}	558_{11}	579_{11}	630_{11}	588_{11}	576_{11}	677_{21}	634_{11}	581_{11}	677_{21}	675_{21}	605_{11}	677_{21}	675_{21}	640_{11}	677_{21}	675_{21}	673_{11}
677_{21}	653_{21}	581_{21}	678_{21}	676_{21}	650_{21}	677_{21}	675_{21}	691_{21}	680_{11}	675_{11}	691_{21}	720_{01}	677_{11}	691_{21}	720_{01}	719_{01}	691_{21}	721_{01}	719_{01}	691_{21}
699_{21}	676_{21}	626_{02}	718_{01}	715_{02}	692_{02}	680_{11}	717_{U}	711_{21}	719_{01}	717_{01}	716_{01}	724_{11}	718_{01}	717_{01}	739_{11}	738_{11}	718_{01}	740_{11}	732_{11}	718_{01}
714_{02}	694_{02}	692_{21}	741_{11}	729_{21}	728_{02}	741_{11}	735_{11}	758_{11}	742_{11}	735_{11}	758_{11}	744_{11}	736_{11}	758_{11}	774_{11}	801_{21}	758_{11}	806_{21}	760_{11}	759_{11}
741_{11}	728_{02}	720_{U}	782_{21}	735_{11}	757_{11}	806_{21}	798_{21}	773_{02}	806_{21}	801_{21}	768_{21}	806_{21}	801_{21}	820_{21}	806_{21}	845_{01}	832_{21}	814_{11}	801_{21}	832_{21}
760_{02}	735_{11}	732_{31}	806_{21}	788_{02}	818_{31}	848_{02}	802_{21}	832_{21}	848_{01}	845_{01}	829_{02}	849_{01}	845_{01}	833_{21}	849_{01}	908_{01}	862_{01}	849_{01}	845_{01}	862_{01}
806_{21}	801_{21}	757_{12}	839_{02}	801_{21}	832_{21}	856_{21}	843_{U}	863_{U}	909_{02}	862_{21}	832_{21}	911_{01}	907_{02}	860_{02}	911_{01}	917_{31}	871_{21}	912_{01}	909_{01}	917_{21}
849_{02}	822_{31}	808_{12}	852_{02}	846_{02}	863_{U}	903_{02}	862_{02}	896_{31}	919_{31}	902_{02}	865_{02}	919_{31}	917_{31}	886_{02}	919_{31}	919_{11}	915_{02}	919_{31}	917_{31}	920_{U}
880_{31}	845_{U}	832_{21}	915_{U}	911_{U}	900_{12}	919_{U}	914_{02}	925_{U}	925_{31}	917_{31}	926_{02}	925_{11}	919_{11}	929_{02}	925_{11}	960_{21}	935_{11}	926_{11}	920_{11}	935_{11}
913_{02}	903_{12}		919_{31}	917_{31}		925_{11}	917_{U}	934_{12}	968_{21}	919_{11}	935_{11}	968_{21}	922_{21}	935_{11}	968_{21}	978_{21}	937_{31}	968_{21}	961_{21}	937_{31}
920_{31}	910_{U}		925_{11}	919_{12}		935_{02}	919_{11}	937_{31}	1000_{02}	940_{02}	937_{31}	989_{21}	961_{21}	937_{31}		995_{02}	951_{02}	1002_{02}	995_{02}	995_{02}
924_{12}	917_{31}		968_{21}	961_{21}		968_{21}	961_{21}	989_{12}	1003_{02}	961_{21}	967_{31}	1002_{02}	995_{02}	997_{21}						
						1002_{02}	995_{U}	1030_{01}	1042_{31}	995_{02}	1030_{01}		998_{02}	1030_{01}						

Table 5: Natural frequencies (in Hz, rounded to closest integers) for various mouth diameters (d in cm) and tension values. Each frequency value is appended (in a smaller font) with the associated membrane mode designation. For instance, 277_{01} indicates that the frequency 277 Hz is associated with a mode shape having 01 membrane mode (in addition to some pressure mode). The notation U appearing next to some values indicate that the corresponding mode shape was unclear.

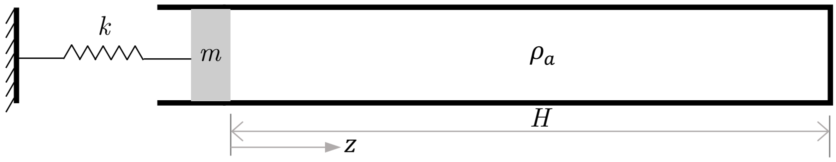


Figure 8: A simple model to illustrate coupling of one membrane mode with axisymmetric cavity modes.

4.3. A simple analytical model

To see how one (axisymmetric) membrane mode can couple with several longitudinal acoustic cavity modes, we consider a simple model of the membrane acoustic interaction. We assume the membrane to vibrate only in its *01* mode and allow only one-dimensional longitudinal pressure variations in the cavity. As a result, we have a simple harmonic oscillator (mass m and stiffness k) coupled with a one-dimensional acoustic tube filled with air (piston-like arrangement), see Fig. 8. The length of the air column is taken as H, and we retain the values of the density of air ρ_a and the speed of sound in air c_p as before. The value of k is obtain from the natural frequency of the *01* mode of an ideal (no cavity) circular membrane clamped at its edge. The mass m is taken to be the mass of the membrane. The area of cross-section of the tube, A, is taken to be the area of the membrane (i.e., $\pi d^2/4$). The displacement of mass m is denoted as $w(t)$ and the pressure field in the tube as $q(z, t)$.

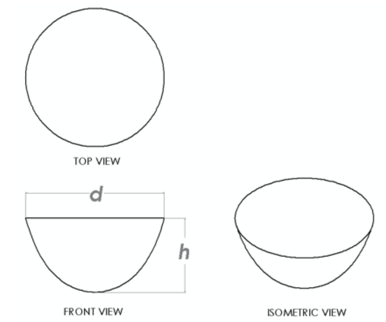
The governing equations include $m\ddot{w} + kw = q(0, t)A$ for the motion of mass m, $\partial^2 q/\partial t^2 = {c_p}^2 \partial^2 q/\partial z^2$ for the cavity acoustics, and the boundary conditions

$\partial q/\partial z = -\rho_a \ddot{w}$ at $z = 0$ and $\partial q/\partial z = 0$ at $z = H$. As usual, the solutions are assumed to be of the form $w(t) = w_0 e^{-i\omega t}$ and $q(z,t) = f(z)e^{-i\omega t}$. The simplified equations are solved by $f(z) = B_1 \sin(\omega z/c_p) + B_2 \cos(\omega z/c_p)$, where the constant coefficients B_1 and B_2 are such that $B_1/B_2 = \tan(\omega H/c_p) = A\rho_a c_p \omega/(k - m\omega^2)$. The second equality in the last expression can be used to calculate the natural frequencies and the first for determining the corresponding mode shapes.

To obtain quantitative results, we fix d=0.16 m, hence A=0.02 m^2, H=0.76 m, and T=3500 N/m. We take $m = \sigma A$ and $k = m{\omega_0}^2$, where $\omega_0 = 2.405\sqrt{T/\sigma}/(d/2)$. The natural frequencies, of values less than 1000 Hz, are calculated as 251.5 Hz, 659.4 Hz, 893.3 Hz. These correspond to longitudinal pressure modes in the tube with one, three, and four nodes, respectively. Although it is not wise to draw a direct comparison with the numerical results of Section 4.1 (where we had considered a drum of same diameter, same height, and same tension), the present calculations lead to values which are comparable with a subset of those obtained for mizhāvu. The simplified model otherwise is limited in its scope and should be used with care.

4.4. Nagāḍā and timpani

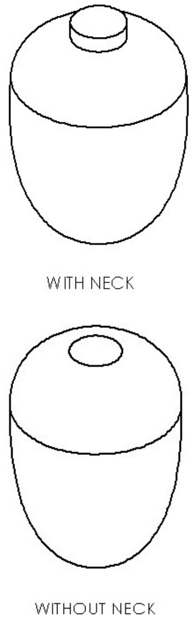
It is relevant to compare mizhāvu acoustics with some other big-bellied drums such as the Indian nagāḍā and the western timpani. Both have a membrane stretched over the mouth of a kettle, see Table 6 for a representative shape and dimensions. The former is typically twice as big as the latter. Both of these however



nagāḍā (d=1.4 m, h=0.75 m)				timpani (d=0.656 m, h=0.414 m)	
T=3000 N/m		T=3500 N/m		T=3500 N/m	
σ=0.2650	σ=0.5445	σ=0.2650	σ=0.5445	σ=0.2650	σ=0.5445
55_{11}	47_{11}	60_{11}	51_{11}	158_{11}	127_{11}
87_{01}	69_{01}	92_{01}	74_{01}	184_{01}	140_{01}
88_{21}	71_{21}	94_{21}	77_{21}	237_{21}	181_{21}
118_{31}	93_{31}	127_{31}	100_{31}	272_{02}	207_{02}
128_{02}	101_{12}	132_{02}	106_{02}	309_{31}	231_{31}
129_{12}	103_{02}	138_{12}	109_{12}	330_{12}	251_{12}
147_{41}	114_{41}	158_{41}	123_{41}	378_{41}	279_{41}
164_{22}	126_{22}	177_{22}	136_{22}		

Table 6: Nagāḍā and timpani. The natural frequency (in Hz, rounded to closest integer) spectrum for various tension (T) and membrane density (σ, in kg/m^2) values. The associated membrane modes are indicated in a smaller font. For instance, 55_{01} indicates that the frequency 55 Hz corresponds to a mode shape with *01* membrane mode (in addition to some pressure mode).

have a significantly larger mouth diameters when compared to mizhāvu. The acoustics of timpani has been well studied.[9,11,12] In Table 6, we report natural frequency values for these drums for two typical tension values. The density value σ=0.5445 kg/m^2 corresponds to that of mizhāvu and σ=0.2650 kg/m^2 to that of timpani membrane. The low frequency values of nagāḍā renders the drum useful to be heard over large distances, as has been historically the purpose of such drums. In any case, we note that unlike mizhāvu, there are no repeated membrane modes in either of these drums. Therefore, if one is to activate only the axisymmetric membrane modes (by striking at the center of the drum), then only one frequency will be heard corresponding to each membrane mode. In this way, mizhāvu distinguishes itself from some other big-bellied drums. Consequently, we can conjecture that the uniqueness of mizhāvu sound is due to its large pot-like belly



T=3000 N/m		T=3500 N/m	
With Neck	*Without Neck*	*With Neck*	*Without Neck*
280_{01}	276_{01}	286_{01}	279_{01}
326_{01}	329_{01}	345_{01}	350_{01}
414_{11}	429_{11}	414_{11}	429_{11}
519_{11}	519_{01}	537_{01}	520_{01}
536_{01}	522_{11}	557_{11}	561_{11}
562_{11}	579_{11}	565_{11}	581_{11}
669_{21}	691_{21}	669_{21}	691_{21}
709_{21}	711_{21}	719_{01}	716_{01}
711_{U}	713_{U}	755_{11}	758_{11}
755_{11}	758_{11}	766_{21}	768_{21}
774_{02}	773_{02}	800_{21}	829_{02}
800_{21}	832_{21}	820_{02}	832_{21}
840_{02}	863_{02}	843_{02}	865_{02}
894_{31}	896_{31}	904_{02}	926_{02}
900_{U}	925_{U}	909_{31}	935_{11}
909_{31}	933_{12}		

Table 7: Effect of mizhāvu neck. The natural frequency (in Hz, rounded to closest integer) spectrum for two tension (T) values with, and without, the neck. The associated membrane modes are indicated in a smaller font (U represents an unclear mode shape). For instance, 280_{01} indicates that the frequency 280 Hz corresponds to a mode shape with *01* membrane mode (in addition to some pressure mode). The mouth diameter is d=16 cm.

covered with a membrane over a small mouth. The authors were not able to locate any pitcher-type drums of this sort in other musical cultures.

4.5. The relevance of neck height

In our idealization of mizhāvu, in Section 3, we had ignored height of the neck altogether. We now justify our assumption by reporting natural frequency values for a mizhāvu, with and without neck, for two tension values, see Table 7. The shape of the neck is illustrated therewith. The height of the neck is taken as 5 cm. It is clear that inclusion of the neck has a limited, possibly negligible, influence on the frequency values. The neck, however, is essential for tying the membrane around the mouth using ropes, see Figure 1.

5. CONCLUSION

We have discussed the vibro-acoustical character of mizhāvu, which is a big-bellied, but small mouthed, drum used extensively as a primary accompaniment in the Sanskrit theatre forms of Kerala. The drum is played with a limited number of distinct strokes, particularly those which excite the axisymmetric membrane modes *01* and *02*. Each of these membrane modes appear with a large number of longitudinal pressure acoustic modes. Hence, even by striking the membrane at the center one can hear an overtone rich sound which has a near harmonic character. Such a drum is indeed unique and has no equal among other known drums in the world culture. We have argued our viewpoint by providing a brief comparison with two kettledrums but further comparisons should be taken up with respect to big-bellied monofacial drums from Africa and Japan. On the other hand, the simulation methodology is being presently extended to include external acoustic environment so that more realistic results can be obtained. Such a framework will also help us understand the reception of mizhāvu sound by the performers and the audience present in the formalized theatre environment within which it is usually performed.

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