SIAM Review Vol. 58, Issue 4 (December 2016)

Book Reviews

to the well-known monograph of Ambrosio, Gigli, and Savaré.

All in all, this long textbook is very complete and pleasant to read, with a progressive level of difficulty and complexity and many nice examples which illustrate the theoretical results. It contains deep and precise information on many important tools in variational analysis (functional analysis, convex analysis) and many advanced methods, together with a general overview of most of the modern techniques. It should be useful for both students and researchers, whether they need to learn or review some advanced techniques in analysis or are looking for an introduction to more recent theories.

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Plasticity: Mathematical Theory and Numerical Analysis. Second Edition. *By Weimin Han and B. Daya Reddy.* Springer, New York, 2013. $149. xvi+424 pp., hardcover. ISBN 978-1-4614-5939-2.

The discipline of plasticity is concerned with the study of irreversible deformations in solids. The governing equations for plastic evolution can take widely different forms depending on the phenomenological model and loading conditions considered. This renders plasticity a challenging pursuit for both the solid mechanician and the mathematician. Moreover, due to the rapid progress made in plasticity research, the mechanician has been left rather unfamiliar with the developments in the mathematical theory and likewise the mathematician with the physical background of the recent plasticity models. The book under review is an ambitious effort to fill this gap by making recent mathematical research in plasticity accessible to the nonmathematician without losing sight of both the rigor as well as the physical basis for plasticity theories. The book is unique in its broad consideration of analytical and numerical aspects of plasticity. In this second edition, important material relevant to strain gradient and single crystal plasticity theories has been added.

The book is divided into three parts. The first part, consisting of four chapters, provides a quick summary of the relevant notions from continuum mechanics and introduces several formulations of rate-independent elastoplasticity. The latter is presented first in a classical framework and then in a convex-analytic setting. After fixing the notation in Chapter 1, concepts from continuum mechanics, including kinematics, balance laws, dissipation, and linearized elasticity, are collected in Chapter 2. Next, in Chapter 3, this is followed by a survey of rate-independent elastoplastic theories within a classical framework, with an emphasis on isotropic strain gradient models of plasticity (particularly the Gurtin–Anand model) and small deformation single crystal plasticity. Chapters 2 and 3 provide only a minimalistic, although well-written, exposure of the physical aspects of plasticity to a nonspecialist, who will have to look elsewhere for more comprehensive treatments. The elastoplastic problems are restated again in Chapter 4, now within a convex-analytic framework. In this form, the problems can be formulated in more generality, such as allowing for nonsmooth yield loci, in addition to being amenable for further mathematical analysis.

The second part of the book seeks to resolve the well-posedness of the initial-boundary value problems of elastoplasticity introduced in Chapters 3 and 4. To this end, the problems are posited in terms of variational inequalities which are subsequently used to demonstrate existence and uniqueness of the solutions. The prerequisite notions from functional analysis and variational inequalities are collected in Chapters 5 and 6, respectively. An interested reader, looking for details, would have to look at several excellent texts available on these subjects. Chapters 7 and 8, combined with their numerical counterparts in Chapters 12 and 13, form the core of the book. They contain a detailed mathematical analysis of the primal and the dual variational problems of elastoplasticity, respectively. Whereas the former has displacement, plastic strain, and hardening parameters as unknowns, the latter solves for generalized stress as the unknown variable. In both the cases, the emphasis is on proving existence and uniqueness of solution to the weak formulation of classical and strain-gradient plasticity problems for polycrystalline and single-crystalline materials. Drawing heavily from their research papers, the authors have successfully managed to present a developed picture of the mathematical theory. This should be extremely valuable for both the mechanician and the mathematician.

The final part of the book is concerned with the numerical analysis of computational algorithms for solving elastoplasticity problems. The temporal and spatial variations are approximated using the finite difference and finite element methods, respectively. After succinctly recalling pertinent aspects of finite element analysis in Chapter 9, approximation of variational equations and inequalities using the finite element method is discussed in Chapter 10. The majority of the discussion is on obtaining reasonable error estimates for the finite element solution of the variational problems. Chapter 11 takes another step toward that goal by introducing semidiscrete and fully discrete approximations and establishing the relevant error estimates. Convergence under minimal regularity assumptions is also established. Finally, in Chapters 12 and 13, we come back to elastoplasticity. Chapter 12 focuses on the implementation of numerical schemes for the primal elastoplastic problem. The error estimates and solution algorithms are derived for various plasticity models and convergence of the algorithms is rigorously established. In Chapter 13, numerical analysis of the dual variational problem is undertaken. Again, several numerical schemes are introduced and analyzed. The focus of this chapter is on implementing time-discrete schemes for classical problems in plasticity.

The book is well written and carefully presented overall. It presents a wealth of useful material otherwise absent from other plasticity books. It exposes both the interested mechanician and the interested mathematician to mathematical problems in plasticity in a unified manner, albeit to be pursued in their own way. My only disappointment with the book is the absence of bibliographic notes at the end of each chapter (aside from one in Chapter 3). Their inclusion would have not only provided further reading directions and open problems, but also given an overall perspective on an active research discipline within which the contents of the book are placed.

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Spline Functions: Computational Methods. *By Larry L. Schumaker.* SIAM, Philadelphia, PA, 2015. $83.00. xii+412 pp., hardcover. ISBN 978-1-611973-89-1.

The book is a long-awaited sequel to two previous books on splines, [2] and [1], by the same author. The new book is a product of over fifty years of research, teaching, and collaboration with numerous scientists within and outside the field of splines. The complete bibliography comprised of over 100 pages was too long for the hard copy and has been put online; see [3]. A very valuable part of the book is a MATLAB package, *SplinePak*, which is freely available both on [3] and on Larry L. Schumaker's website [4]. For the impatient reader, I would recommend downloading the package and going straight to the examples in the book, which, admittedly, is exactly what I did. Within minutes, I had a beautiful picture of a minimal energy quadratic smooth spherical spline interpolating $f(x, y, z) = x^4 + 1.1y^4 + 1.3z^4$ at 42 vertices of a triangulation of the unit sphere; see Figure 3. Each numerical example in the text describes a problem and has a reference to the code that provides a solution. The only drawback is that the MATLAB functions in *SplinePak* are currently p-files, and thus cannot be modified. The script files are the usual m-files. A complete list of the scripts and the functions in the package is included at the end of the book.

For the more patient reader, I recommend downloading the package and getting hold of both of Larry Schumaker's previous books on splines, *Spline Functions: Basic Theory* [2] and *Spline Functions on Triangulations* [1] (coauthored with M. J. Lai). This takes us to the next truly valuable feature of the book: it is equally useful to the reader looking for algorithms to solve