Supplement to "Acoustics of Idakkā: An Indian Snare Drum with Definite Pitch"

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I. Idakkā design

We provide some additional figures to elucidate the design of the instrument. First, in Figure 1, we show a full profile view of an idakkā clearly showing the connection of the barrel with the drumheads via a cotton rope. The pegs and the tassels are also visible. Secondly, in Figure 2, we show the side view of an assembled idakkā accompanied by a schematic with detailed dimensions.

Figure 1: A Profile view of a fully assembled idakkā.

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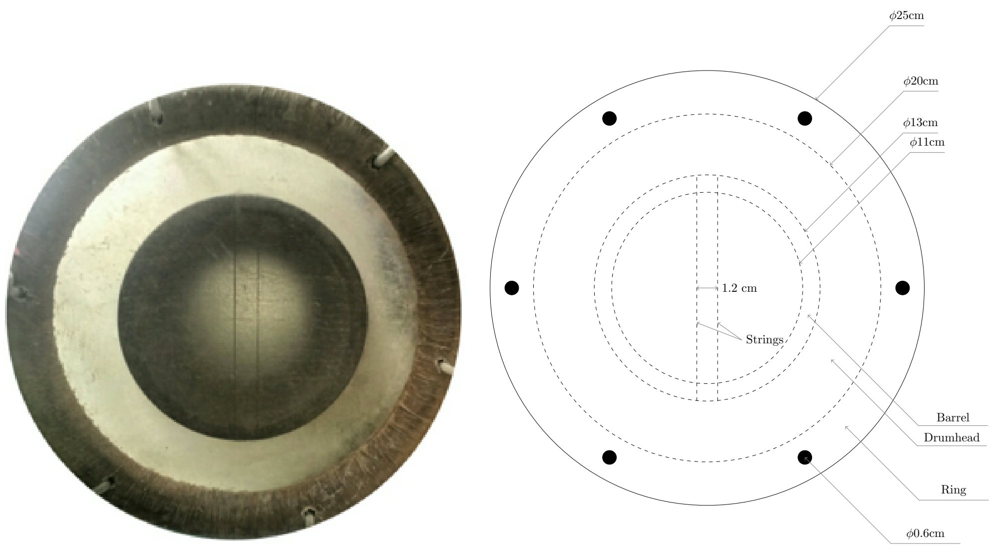


(a) (b)

Figure 2: A side view of a fully assembled idakkā with a schematic. The central region appears dark in (a) due to a converging barrel in the background.

II. Additional information regarding experiments

A. Estimation of material properties

The palmyrah strings are fixed to the barrel by first making them taut and then rolled around the copper nails on the edge. The snares are fixed without any scope for fine tuning. This would, presumably, lead to a variation in the snare tension during the installation. We need to estimate the range of tension values that a palmyrah fibre can sustain without breaking. From our interaction with an expert (Mr. P. Nanda Kumar),[1] we found out that a successful installation of the snares may come after many unsuccessful attempts due to breaking of the fibres in the process. This suggests that the tensions are close to the breaking force. We estimated this force using a Universal Testing Machine. The results are collected in Figure 3. We observe that, for a force more than 5 N, the force-displacement behavior becomes nonlinear and the final breakage happens around 8-10 N. Given the uncertainty involved in the process, in addition to the differences that may arise in individual samples, we shall use a nominal value of 6.5 N for all our analyses.

The area density of the drum was estimated by weighing a 4 cm $\times$ 4 cm sample of the drumhead material. The linear density of the palmyrah fibres was estimated by weighing two samples of around 20 cm length.

B. PSD evolution

In Figures 4(a) and 4(b) we plot the evolution of power for certain harmonic frequency segments

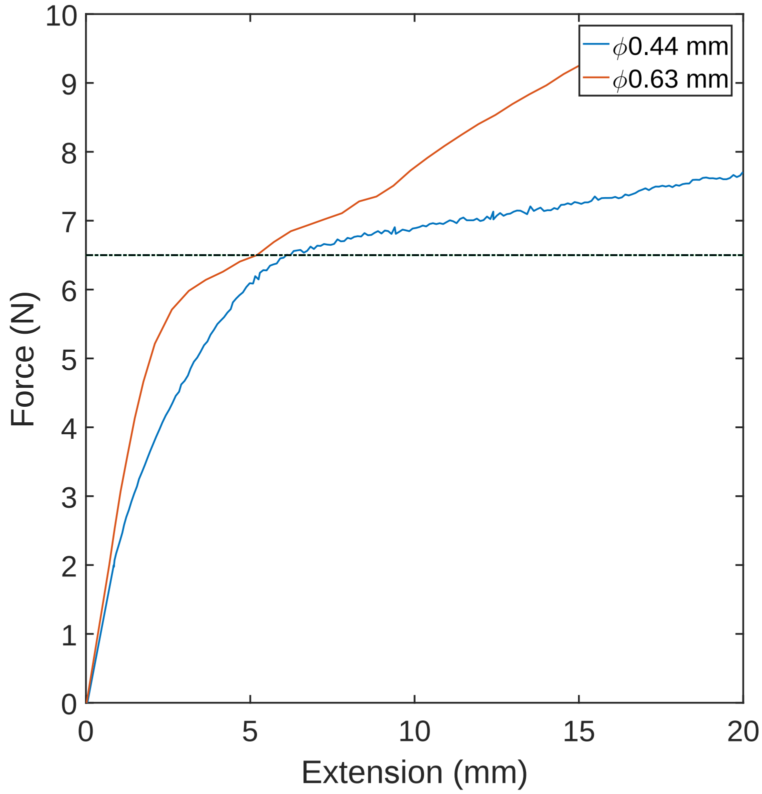
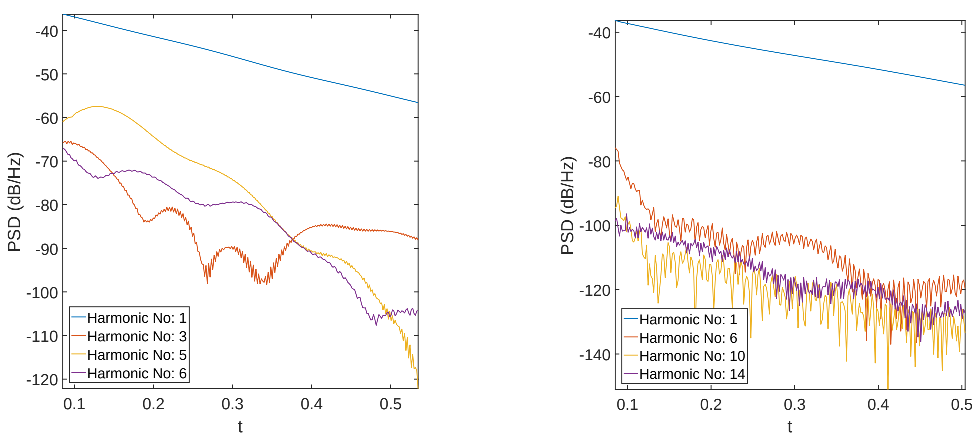


Figure 3: Force vs. extension curve for fibres of two different average diameters.



(a) (b)

Figure 4: Evolution of power/frequency of certain harmonic frequency segments corresponding to spectrograms in Figure 4 of the main paper.

corresponding to spectrograms in Figures 4 (left) and 4 (right), respectively, of the main paper. The nature of these plots is oscillatory, explaining the slight buzzing tone in the idakkā's sound.

C. Error in harmonicity

In Table 1 we have collected the percentage error, with respect to nearest integer, for the first

Table 1: Percentage error, with respect to nearest integer, for the first 8 near harmonic peaks for the three cases in the top row of Figure 5 of the main paper.

f_0 (Hz)	Peak 1	Peak 2	Peak 3	Peak 4	Peak 5	Peak 6	Peak 7	Peak 8
115	-0.2911	-0.7336	0.1601	-0.0125	0.2293	0.3105	0.4978	0.0794
158	-1.1574	-0.4893	-0.4167	-1.2626	-0.8207	-1.3731	-1.0101	-0.1326
192	0.3652	-0.4825	-0.5008	-0.2534	0.3246	-0.9285	-0.6023	-0.5869

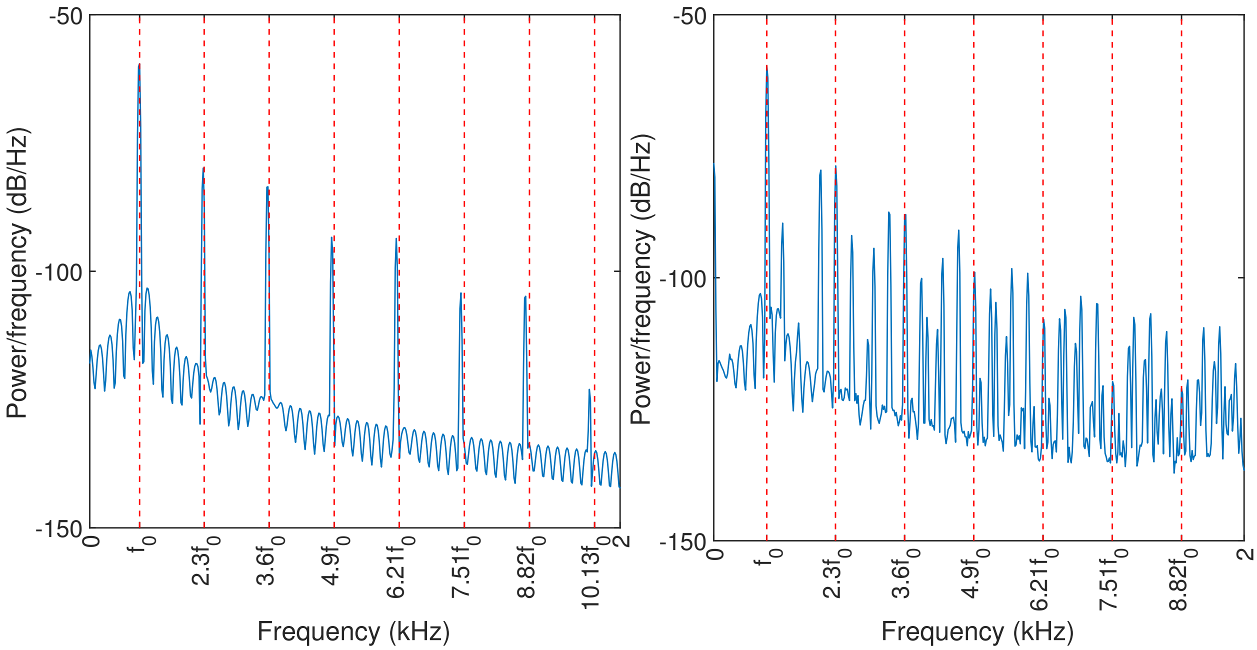
8 near harmonic peaks for the three cases in the top row of Figure 5 of the main paper. Most of the deviations are below 1% and the highest magnitude of deviation is less than 1.4%.

III. Nonlinear Normal Modes

A Nonlinear Normal Mode (NNM) of an undamped system is defined as a synchronous periodic oscillation where all material points of the system reach their extreme values or pass through zero simultaneously.[3;4] We will work with the undamped form of the governing equation (the notation is consistent with the main paper):

$$\ddot{\tilde{\eta}}_{mn} = -B_{mn}^2 \tilde{\eta}_{mn} + \chi \sum_{i=1}^{N_S} \left(\int_{-\sqrt{1-(\psi^i)^2}}^{\sqrt{1-(\psi^i)^2}} \tilde{h}^i_{\tilde{\xi}^i \tilde{\xi}^i} \phi_{mn}(\tilde{\xi}^i) d\tilde{\xi}^i \right) \tag{1}$$

Let $z(t, z_0) = \begin{bmatrix} \tilde{\eta}(t) \\ \dot{\tilde{\eta}}(t) \end{bmatrix}$, where $z_0 = \begin{bmatrix} \tilde{\eta}_0 \\ \dot{\tilde{\eta}}_0 \end{bmatrix}$. We define the shooting function $H(t, z_0) = z(t, z_0) - z_0$. If an initial condition z_{p0} and a time period T corresponds to a NNM then $z_p(t, z_{p0}) = z_p(t + T, z_{p0})$. As a result, $H(T, z_{p0}) = 0$. The problem of finding the NNMs is therefore equivalent to finding the solution of the nonlinear system of equations $H(t, z_0) = 0$.[5] These equations form a system of $2n$ equations for $2n + 1$ unknowns. In order to make the problem solvable we can impose an additional constraint, $|\tilde{\eta}_{p0}| = 1$. This excludes NNMs in which $\tilde{\eta}_0 = 0$. The problem can be further simplified by considering specific initial conditions such as $\dot{\tilde{\eta}}_0 = 0$.[5] The equations to be solved are now reduced to $\begin{bmatrix} \dot{\tilde{\eta}}(t,z_0) \\ |\tilde{\eta}_0| - 1 \end{bmatrix} = 0$. This is a system of $n + 1$ equations for the same number of unknowns. We can simplify the system of equations further by noting that if $z_{0*} = \begin{bmatrix} \tilde{\eta}(T, \eta_{0*}) \\ 0 \end{bmatrix}$ satisfies $\dot{\tilde{\eta}}(T, z_{0*}) = 0$ then $\dot{\tilde{\eta}}(T, cz_{0*}) = 0$, as long as c is positive. Therefore we can choose the displacement corresponding to one the modes and fix it to a constant value, say 1. The solution will scale accordingly. As a result, we can do away with the $|\tilde{\eta}_0| = 1$ constraint, since fixing one of the displacements to a non-zero value will automatically exclude the trivial solution while still retaining the possibility of convergence to a non-trivial solution. Hence, $\dot{\tilde{\eta}}(t, z_{0*}) = 0$ is to be solved which is a system of n equations for n unknowns. This formulation excludes the NNMs for which the displacement corresponding to the fixed drum mode is actually zero. This can be mitigated by



(a) (b)

Figure 5: PSD plots corresponding to the membrane center displacement (a) without and (b) with a finite rim. The dotted lines indicate the frequencies corresponding to axisymmetric modes of vibration of a uniform membrane without a finite rim. The non axisymmetric modes do not appear due to the geometry and the initial conditions.

running the simulation again after fixing the displacement corresponding to a different drum mode.

IV. The curved rim

In this section, we reformulate the membrane vibration problem as a unilateral constraint boundary value problem by considering a finite rim at the edge of the membrane. The membrane, around its edge, will therefore wrap and unwrap during its motion similar to the string motion in Indian string instruments such as sitār, tānpurā, etc.[6;7] We ignore the effect of string-membrane interaction from our present considerations. The effect of a finite rim was also noted in our analysis of audio recordings of the idakkā sound without strings in the bottom row of Figure 5 of the main paper. Considering the barrel rim to have an internal radius R_i and outer radius R, we take the cross section of the rim shape to be an inverted parabola centered at $r = (R_i + R)/2$ and of height p above the center point on the horizontal plane. The parabolic profile of the rim is hence of the form

$$S_{rim}(r) = p - \frac{4p}{(R - R_i)^2}\left(r - \frac{R + R_i}{2}\right)^2. \tag{2}$$

The membrane displacements, considered to be axisymmetric, can be solved using the equation

$$\mu W_{tt} = T_M \left(\frac{W_r}{r} + W_{rr} \right) + K[S_{rim} - W]^{\alpha}, \tag{3}$$

where subscript r indicates a partial derivative with respect to the radial variable; the constants K and α are associated with the contact interaction of the membrane with the rim. The membrane is clamped at $r = R$. The equation is solved by discretizing the spatial derivatives using a central difference scheme and then using `ode113` solver in MATLAB to solve for the time evolution. For simulation purposes, we have taken $\mu = 0.1$ kg-m^{-2}, $T_M = 100$ N-m^{-1}, $R_i = 5.5$ cm, $R = 6.5$ cm, $p = 0.2$ mm, $K = 10^{15}$, and $\alpha = 1.3$. An initial displacement of 5 mm at the center of membrane was considered. The PSD plots corresponding to membrane displacement at the center are given in Figure 5 both without and with a finite rim. The former case shows frequency peaks corresponding to an ideal circular membrane clamped at the edge. The effect of a finite rim, in the latter plot, is clearly to add a rich set of overtones to the spectrum in addition to increasing the overall intensity of various overtones. We end by noting that the above treatment is at best preliminary and would need to be discussed with rigorous details in a future work.

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