Supplement to "Effects of Air Loading on the Acoustics of an Indian Musical Drum"

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The purpose of this supplement is to provide additional details for certain results in the main paper. In Section 1 we provide a description of the modal synthesis procedure for the right hand tabla and obtain several results. This is followed by an analogous effort for the left hand tabla in Section 2. Finally, in Section 3, we provide some additional plots regarding the error behavior while studying optimum designs of tabla. *The notation is taken from the accompanying paper, which is called P for reference purposes in the following sections.* We also note that an earlier attempt at modal sound synthesis of right hand tabla, although using a different framework, is given in an unpublished thesis.[2]

1 Modal sound synthesis of right hand tabla

Modal sound synthesis is a physical modelling based sound synthesis based on the modal description of the vibrating system.[1] In order to synthesise sound of right hand tabla, we express the transverse displacement of the tabla membrane as a linear combination of nine modes (with coefficients a_i) given in Table II of P. Each of these modes is in turn a linear combination of five 'normal' modes having m number of nodal diameters. By 'normal', we refer to the basis functions of the right hand tabla membrane vibration without air loading (denoted as η^0_{mn}). The modal synthesis is performed subjecting the membrane to an initial velocity condition simulating a snap stroke at a point on the

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membrane. The initial velocity is given in terms of the raised cosine profile

$$v_{rc}(\rho,\phi) = \begin{cases} \frac{c_0}{2}\left[1+\cos\left(\frac{\pi}{\rho_{hw}}\sqrt{\rho^2+\rho_0^2-2\rho\rho_0\cos(\phi-\phi_0)}\right)\right], & \text{if } |\rho-\rho_0| \geq \rho_{hw} \\ 0, & \text{otherwise,} \end{cases}$$

which has amplitude c_0, half width ρ_{hw} and is centered at (ρ_0, ϕ_0). The initial velocity can also be written in terms of nine modes

$$\begin{aligned} v_{rc} = {} & a_1 i\omega_{01}\eta_{01} + a_2 i\omega_{02}\eta_{02} + a_3 i\omega_{03}\eta_{03} + a_4 i\omega_{11}\eta_{11} + a_5 i\omega_{12}\eta_{12} \\ & + a_6 i\omega_{21}\eta_{21} + a_7 i\omega_{22}\eta_{22} + a_8 i\omega_{31}\eta_{31} + a_9 i\omega_{41}\eta_{41}, \end{aligned} \tag{1}$$

where ω_{ms} are already determined by solving Equation (1) in P. In the above relation,

$$\eta_{ms} = \sum_{k=1}^{5} V_k^{ms}\eta_{mk}^0, \tag{2}$$

where V_{ms} are eigenvectors obtained by solving Equation (1) in P. Substituting the expansion of each η_{mn} in (1) and taking an inner product of (1) with each η_{mn} one by one, while using the orthogonality property of the normal modes η_{mn}^0

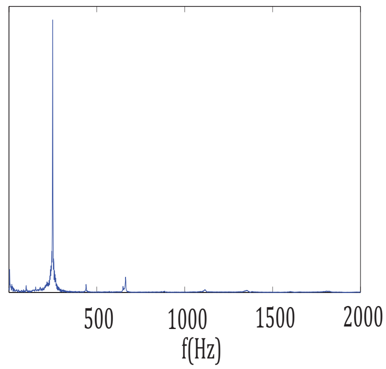
$$\begin{aligned} \int_0^{2\pi}\int_0^b \sigma\eta_{mn}^0\eta_{m'n'}^0\rho d\rho d\phi = \sigma_1\int_0^{2\pi}\int_0^a \eta_{mn}^0\eta_{m'n'}^0\rho d\rho d\phi + \sigma_2\int_0^{2\pi}\int_a^b \eta_{mn}^0\eta_{m'n'}^0\rho d\rho d\phi \\ = 0 \text{ if } m \neq m' \text{ or } n \neq n', \end{aligned} \tag{3}$$

we obtain a set of linear equations to solve for a_i in the form

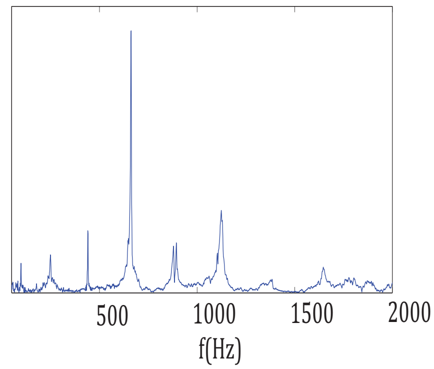
$$A_{9\times 9}a_{9\times 1} = C_{9\times 1}, \tag{4}$$

where the subscripts denote the size of the respective matrices. Once the vector **a** is solved from the previous equation, the evolution of the velocity profile for all points on the membrane with time is a straightforward computation. In order to verify our results we use the following scheme:

1. To begin with, we choose the parameters as given in the first row of Table I of P, except for tension, which is chosen so as to match the fundamental frequency with the obtained



(a) *tin.*



(b) *nā.*

Figure 1: Experimentally measured frequency spectra for two basic strokes of right hand tabla playing. The $x-$axis in each plot represents the frequency in Hz.

fundamental frequency from *tin* stroke.

2. Next, appropriate values of ρ_0, ϕ_0, ρ_{hw} and c_0 are taken depending on the stroke that we want to simulate.

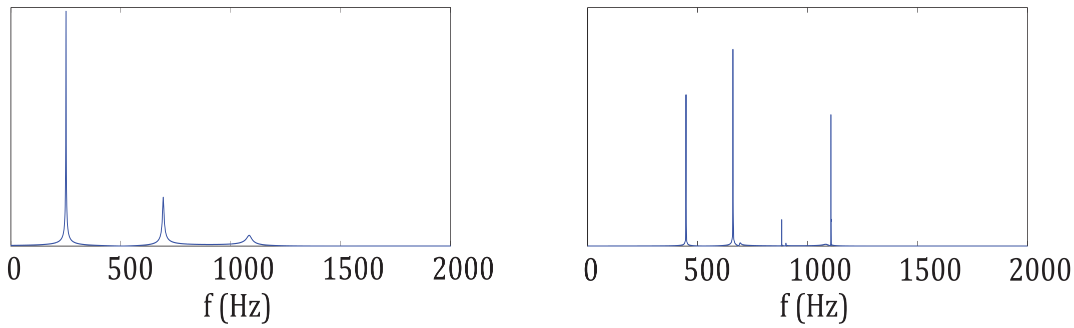
3. We select an arbitrary point on the membrane, find its velocity $v_p(t)$ with respect to time over the time interval $t \in (0, t_f)$, for a conveniently large value of t_f as well as a sufficiently large value of sampling frequency F_s.

4. Finally, we take the fast Fourier transform of $v_p(t)$ over the above mentioned time interval and compare the result with the spectrum obtained experimentally for the corresponding stroke.

1.1 Two playing strokes of right tabla playing

Tabla playing consists of a combination of different strokes each having their own playing style. We describe two such strokes here, along with the respective frequency spectrums (experimentally obtained). Both the strokes are associated with right hand tabla. Our nomenclature for the strokes is derived from the *Benaras Gharana.*[3] The playing style of each of the stroke is briefly mentioned here so as to better understand the corresponding frequency spectrum. In our experiments, the right hand tabla was tuned to the note A_4 such that the fundamental corresponds to 220 Hz.

1. ***tin***: This stroke is played using only the index finger, which is used to hit the black patch and



(a) *tin*.   (b) *nā*.

Figure 2: Fast Fourier transforms of simulations of *tin* and *nā*.

no damping of any mode is brought about by pressing the membrane lightly. This enables the fundamental mode to vibrate freely, see Figure 1(a). As we have discussed in the main paper, the fundamental has a major second note as the dominant one, and thus this stroke has a distinctly different pitch to it.

2. ***nā***: The *nā* stroke is played by resting the ring finger lightly on the edge of the black patch while striking the outer halo with the index finger. With a missing fundamental, several higher harmonics are seen to appear in the resulting spectrum. The lightly pressed ring finger keeps out the fundamental mode, see Figure 1(b).

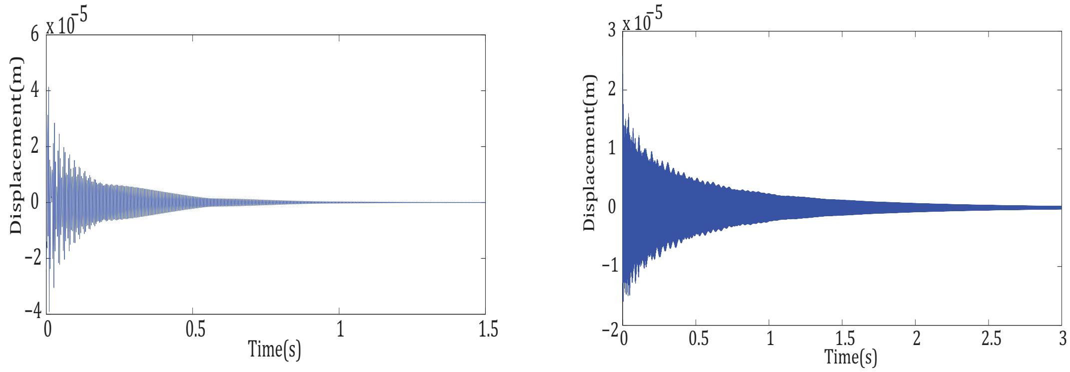
1.2 Results from the modal analysis

We present the modal sound synthesis results by simulating two commonly played strokes: *tin* and *nā*. As described above, *tin* is a thump at the center of the black patch whereas *nā* is a strike at the edge of the tabla membrane, using only the index finger, while keeping out the fundamental mode. To simulate these strokes as closely as possible,

1. We choose a higher value $(a/4)$ of ρ_{hw} for *tin*, as compared to *nā* $(a/8)$.

2. We choose the striking point (ρ_0, ϕ_0) for *tin* to be $(0, 0)$, and for *nā* to be $(0.04, 0)$. Note that 0.04 is slightly less than b.

3. We leave out the fundamental mode while performing the modal analysis of *nā* stroke. All



(a) *tin*.

(b) *nā*.

Figure 3: Transverse displacement at a point (0.02,0) for simulations of *tin* and *nā*.

the modes are retained in the modal analysis of *tin* stroke.

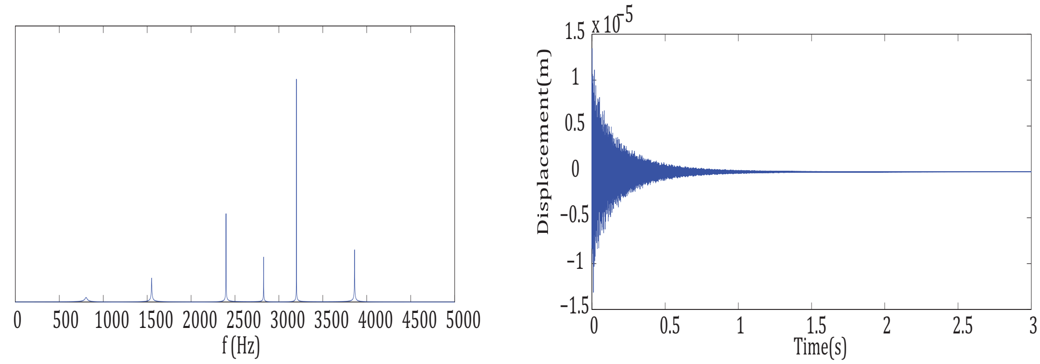
4. We choose $c_0 = 0.001$ for both the strokes.

Figures 2(a) and 2(b) show the fast Fourier transforms at a somewhat arbitrarily chosen point (0.02,0) for the two strokes. Both the figures confirm well with the corresponding experimental results (given in Figures 1(a) and 1(b)). We also plot the displacement evolution of this arbitrarily chosen point as obtained from modal analysis with respect to time for the two strokes. Note that the time axis goes from 0 to 1.5s in Figure 3(a) and upto 3s in Figure 3(b), indicating a low damping for the latter. This is expected since we have removed the fundamental mode, which otherwise dampens out quickly, in the simulation of *nā*.

2 Modal sound synthesis of left hand tabla

The numerical sound synthesis for the left hand tabla is done in the same manner as for the right hand tabla. The modal basis functions in terms of which a general displacement (or velocity) profile is expanded now consists of altogether twelve functions, as given in Table V of P. Note that six of these functions have an azimuthal dependence of $\cos\phi$, while the other six of $\sin\phi$. Choosing a 'raised cosine profile' for the initial velocity, we can write it in terms of the basis functions as

$$\begin{aligned} v_{rc} &= a_1 i\omega_{e01}\eta_{e01} + a_2 i\omega_{e02}\eta_{e02} + a_3 i\omega_{e11}\eta_{e11} + a_4 i\omega_{e12}\eta_{e12} + a_5 i\omega_{e21}\eta_{e21} + a_6 i\omega_{e31}\eta_{e31} \\ &+ a_7 i\omega_{o01}\eta_{o01} + a_8 i\omega_{o02}\eta_{o02} + a_9 i\omega_{o11}\eta_{o11} + a_{10} i\omega_{o12}\eta_{o12} + a_{11} i\omega_{o21}\eta_{o21} + a_{12} i\omega_{o31}\eta_{o31}, \end{aligned} \quad (5)$$



(a) (b)

Figure 4: (a) Fast Fourier transform and (b) Transverse displacement of an arbitrary point on the membrane of the left hand tabla.

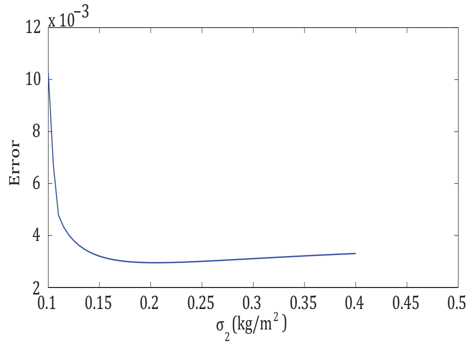
where an 'e' appearing in the subscripts represents the azimuthal dependence of $\cos\phi$ and 'o' of $\sin\phi$. Also,

$$\eta_{e/oms} = \sum_{k=1}^{5} V_k^{e/oms} \eta^0_{e/omk}, \tag{6}$$

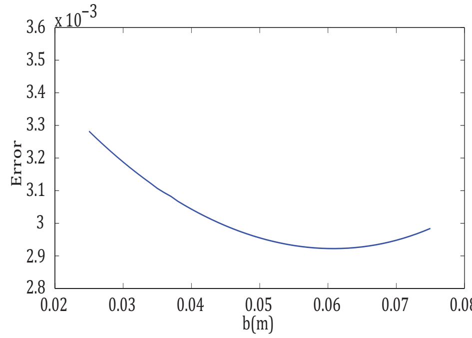
where $V^{e/oms}$ are eigenvectors already determined by solving Equation (21) in P. Again, the orthogonality property of the 'normal' modes is used to form a set of twelve linear equations to solve for a_i. For illustration purposes, we choose $(\rho_0, \phi_0) = (0, 0)$, $\rho_{hw} = a/8$, and $c_0 = 0.001$ in the raised cosine profile. The fast Fourier transform of the point $(\rho, \phi) = (a/2, 0.5)$ is shown in Figure 4(a). Note that a non-zero value of ϕ is deliberately chosen to bring out the effect of modes with azimuthal dependence of $\sin\phi$. The transverse displacement of this point with respect to time is also plotted. Unlike our examples for the right hand tabla, it is not possible, within the present framework, to simulate a playing stroke of a left hand tabla. This is because a typical left tabla stroke (such a *ghay*) is produced in conjunction with a varying pressure of the wrist palm junction on the edge of tabla membrane. It is presently not clear to us how to include such effects within our model.

3 Additional plots for the optimum design of tabla

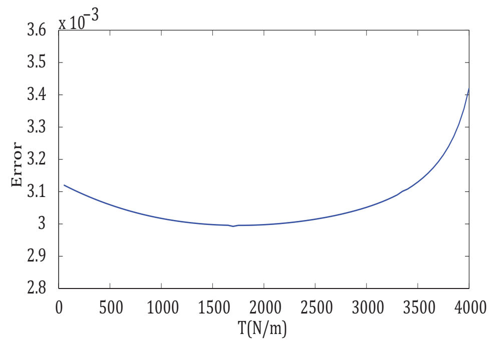
In this section, we collect some additional error plots indicating optimum parameter values for tabla design. In Figure 5, we show the error variation for the right hand tabla with respect to



(a) Error vs σ_2.



(b) Error vs b.



(c) Error vs T.

Figure 5: Error variations for the right hand tabla. The remaining parameters in each plot are fixed according to the values given in the first row of Table 1 in P.

density of the unloaded part of the membrane (σ_2), outer radius (b), and uniform tension in the membrane (T). The error plots with respect to varying density ratios, radii ratios, and cavity sizes are included in P (see Figure 3 therein). As noted in P, there is generally a flat valley around minima points making the error less sensitive to parameter variations. This flexibility in design parameters is also evident in the variety of tabla designs adapted by tabla makers.[3]

For the left hand tabla we have, in Figure 6, a surface plot of the error function E_l (defined in P) with respect to tension and density variations. It is an alternate representation of the contour plot given in P.

References

[1] S. Bilbao *Numerical Sound Synthesis* (John Wiley & Sons Ltd, 2009).

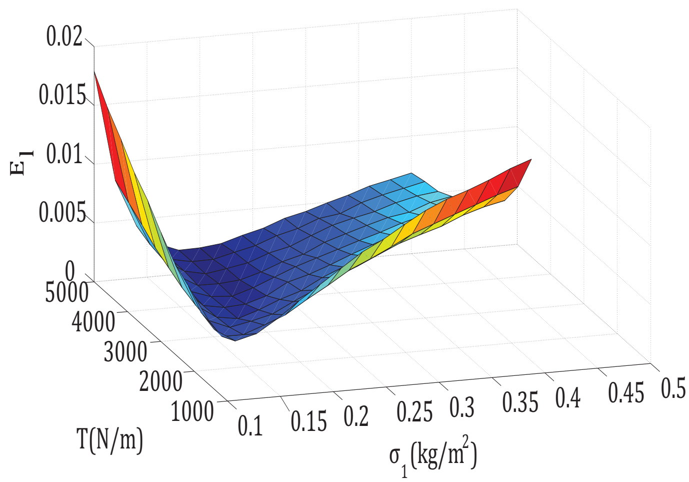


Figure 6: Surface plot of the error function E_l with respect to tension and density variations. The results are for the left hand tabla.

[2] G. Saraswat *Indian Musical Drum Eigenspectra and Sound Synthesis* (M.Tech. thesis, IIT Madras, 2011).

[3] P. A. Roda, "Resounding Objects: Musical Materialities and the Making of Banarasi Tablas", Ph. D. dissertation (New York University), 2013.