Mutually orthogonal sets of ZCZ sequences

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In the construction of zero correlation zone (ZCZ) sequences, as the length of the interference free window (IFW) increases, fewer sequences are available and hence their utility in multiuser systems diminishes. To address this issue, it is shown that, for a given IFW, it is possible to design multiple sets of ZCZ sequences which are mutually orthogonal.

Introduction: In a synchronous CDMA system, orthogonal spreading sequences can be employed to eliminate multiple access interference. However, in a wireless channel, orthogonality among different users tends to diminish because of inter-path interference. To reduce the interference among users in a multipath environment or in an approximately synchronised CDMA [1] environment, the concept of generalised orthogonality was defined and a recursive construction of zero correlation zone (ZCZ) sequences was presented in [2] and [3]. In [4], analytical and simulation results show that the ZCZ sequences (also known as loosely synchronous sequences) are indeed more robust in multipath propagation channels, compared to traditional orthogonal sequences. Study of these sequences is further motivated by the recently proposed LAS-CDMA (large area synchronous CDMA) for 4G systems [5] of which ZCZ sequences are an integral part.

A recursive construction of ZCZ sequences starting from a Golay complementary pair [6] was proposed in [2]. The construction in [2] was extended in [1] by applying the same recursive method on a matrix the rows of which form a class of mutually orthogonal complementary sets. In a recent work [7], two new methods for constructing ZCZ sequences based on perfect sequences and unitary matrices have been proposed. In all the above constructions, the number of sequences in the set decreases with interference free window (IFW) length. Unavailability of a large number of ZCZ sequences prevents accommodating more users even though spectral efficiency of the system is improved by using these sequences [5]. It is known that, for a given IFW, the number of ZCZ sequences cannot be increased any further [7]. In this Letter we show that, for a given IFW, it is possible to have two mutually orthogonal sets such that each set has the maximum number of ZCZ sequences possible for that IFW. Towards this end, we introduce the notion of mutually orthogonal ZCZ sequence sets. We use these sets to increase the number of available sequences for a CDMA system. Throughout the Letter, we consider only binary sequences.

Orthogonal sets of ZCZ sequences: A set of sequences $\{\mathbf{b}^i\}_{i=1}^M$ each of length L, with IFW of T_{CZ} is denoted as ZCZ-(L, M, T_{CZ}), where $T_{CZ} = \min\{T_{ACZ}, T_{CCZ}\}$. T_{ACZ} and T_{CCZ} denote the zero periodic autocorrelation and zero periodic cross-correlation zones which are defined as [2]:

$$T_{ACZ} = \max\{T \mid \phi_{\mathbf{b}^i\mathbf{b}^i}(\tau) = 0, \forall i, |\tau| \le T, \tau \ne 0\}$$
$$T_{CCZ} = \max\{T \mid \phi_{\mathbf{b}^i\mathbf{b}^j}(\tau) = 0, \forall i \ne j, |\tau| \le T\}$$

In the above $\phi_{\mathbf{b}^i\mathbf{b}^j}(\tau)$ denotes the periodic cross-correlation between $\mathbf{b}^i$ and $\mathbf{b}^j$. Two distinct sets of ZCZ sequences $\{\mathbf{b}_1^i\}_{i=1}^M$ and $\{\mathbf{b}_2^i\}_{i=1}^M$ are said to be mutually orthogonal, if

$$\phi_{\mathbf{b}_1^i\mathbf{b}_2^j}(0) = 0 \quad \forall i \text{ and } j$$

Construction of mutually orthogonal ZCZ sequence sets: The construction presented in this Letter is based on mutually orthogonal complementary sets [1]. We begin with a collection of M_0 mutually orthogonal complementary sets each containing M_0 sequences of length L_0. Let these sets be arranged in matrix form such that the elements of the ith row are obtained from the ith complementary set. We denote this matrix as $\Delta_1^{(0)}$. Then,

$$\Delta_1^{(0)} = \begin{bmatrix} A_{11}^0 & A_{12}^0 & \cdots & A_{1M_0}^0 \\ A_{21}^0 & A_{22}^0 & \cdots & A_{2M_0}^0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{M_01}^0 & A_{M_02}^0 & \cdots & A_{M_0M_0}^0 \end{bmatrix}_{M_0 \times L_0M_0}$$

where each entry is a vector with L_0 components. The entry in the ith row and jth column denotes the ith sequence in the jth set. Let us introduce $\Delta_2^{(0)}$ which is identical to $\Delta_1^{(0)}$. For the nth iteration ($n \ge 1$), consider the following recursive relations for $\Delta_1^{(n)}$ and $\Delta_2^{(n)}$:

$$\Delta_1^{(n)} = \begin{bmatrix} \Delta_1^{(n-1)}\Delta_1^{(n-1)} & \Delta_1^{(n-1)}(-\Delta_1^{(n-1)}) \\ \Delta_1^{(n-1)}(-\Delta_1^{(n-1)}) & \Delta_1^{(n-1)}\Delta_1^{(n-1)} \end{bmatrix}$$

$$\Delta_2^{(n)} = \begin{bmatrix} \Delta_2^{(n-1)}\Delta_2^{(n-1)} & (-\Delta_2^{(n-1)})\Delta_2^{(n-1)} \\ \Delta_2^{(n-1)}(-\Delta_2^{(n-1)}) & (-\Delta_2^{(n-1)})(-\Delta_2^{(n-1)}) \end{bmatrix}$$

where $-\Delta$ denotes the matrix the ijth entry of which is the ijth negation of Δ, and $\Delta_1\Delta_2$ denotes the matrix the ijth entry of which is the concatenation of the ijth entry of Δ_1 and the ijth entry of Δ_2.

By straightforward extension of Theorem 13 in [8], it can be shown that the rows of $\Delta_1^{(n)}$ and $\Delta_2^{(n)}$ constitute mutually orthogonal complementary sets. It can also be shown that every row of $\Delta_1^{(n)}$ is orthogonal to every row of $\Delta_2^{(n)}$. Using the above mutually orthogonal complementary sets, we propose the following construction for orthogonal sets of ZCZ sequences.

Construction: Let M_n denote the number of rows in $\Delta_1^{(n)}$ and $\Delta_2^{(n)}$. The rows of $\Delta_1^{(n)}$ can be used to constitute $\{\mathbf{b}_1^k\}_{k=1}^{M_n}$. Similarly the rows of $\Delta_2^{(n)}$ can be used to constitute $\{\mathbf{b}_2^k\}_{k=1}^{M_n}$. It can be verified that the sets $\{\mathbf{b}_1^k\}_{k=1}^{M_n}$ and $\{\mathbf{b}_2^k\}_{k=1}^{M_n}$ form two different sets of ZCZ-($4^nL_0M_0$, 2^nM_0, $2^{n-1}L_0$) sequences. Also, $\{\mathbf{b}_1^k\}_{k=1}^{M_n}$ and $\{\mathbf{b}_2^k\}_{k=1}^{M_n}$ are mutually orthogonal sets.

Example 1: Let $\Delta_1^{(0)}$ and $\Delta_2^{(0)}$ be a 4×4 Hadamard matrix (special case of mutually orthogonal complementary sets for $L_0 = 1$). It can be seen that the rows of $\Delta_1^{(0)}$ constitute a ZCZ-(4, 4, 0) set. Using the above construction we obtain the following sequences for $n = 1$:

$$\begin{aligned}
\mathbf{b}_1^1 &= (+++++++++-+-+-+-)\\
\mathbf{b}_1^2 &= (++--++--+--++--+)\\
\mathbf{b}_1^3 &= (++++----+-+---+-+)\\
\mathbf{b}_1^4 &= (++----+++--+-++-)\\
\mathbf{b}_1^5 &= (+-+-+-+-++++++++)\\
\mathbf{b}_1^6 &= (+--++--+++--++--)\\
\mathbf{b}_1^7 &= (+-+--+-+++++----)\\
\mathbf{b}_1^8 &= (+--+-++-++-----++)
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{b}_2^1 &= (++++++++-+-+-+-+)\\
\mathbf{b}_2^2 &= (++--++----++--++-)\\
\mathbf{b}_2^3 &= (++++-----+-++-+-)\\
\mathbf{b}_2^4 &= (++----++-++-+--+)\\
\mathbf{b}_2^5 &= (+-+-+-+---------)\\
\mathbf{b}_2^6 &= (+--++--+---++--++)\\
\mathbf{b}_2^7 &= (+-+--+-+-----++++)\\
\mathbf{b}_2^8 &= (+--+-++----++++--)
\end{aligned}$$

Both $\{\mathbf{b}_1^k\}_{k=1}^8$ and $\{\mathbf{b}_2^k\}_{k=1}^8$ are ZCZ-(16, 8, 1) sets. Furthermore, the two sets are mutually orthogonal.

The above example produces double the number of sequences compared to the number of ZCZ sequences with IFW of 1 as given in [4]. A system utilising currently known ZCZ sequences will have only eight sequences of length 16 whereas with this construction we now have 16 sequences of the same length. However we pay a price, in the sense that a sequence from set 1 is not orthogonal to a sequence from set 2 for nonzero shifts. Next iteration ($n = 2$) will produce two mutually orthogonal sets, each forming a ZCZ-(64, 16, 2) set.

Example 2: For $L_0 = 2$ and $M_0 = 2$, using the mutually orthogonal complementary sets $\{\{(++), (+-)\}, \{(+-), (++)\}\}$,

$$\Delta_1^{(0)} = \Delta_2^{(0)} = \begin{pmatrix} (++) & (+-) \\ (+-) & (++) \end{pmatrix}$$

After the first iteration, we obtain

$$\mathbf{b}_1^1 = (++++++-+-++--+--+)$$

$$\mathbf{b}_1^1 = (+++++-+-++--+--+)$$
$$\mathbf{b}_1^2 = (+-+-+++++--+++--)$$
$$\mathbf{b}_1^3 = (++--+--++++++-+-)$$
$$\mathbf{b}_1^4 = (+--+++--+-+-++++)$$

and

$$\mathbf{b}_2^1 = (+++++-+---++-++-)$$
$$\mathbf{b}_2^2 = (+-+-++++-++---++)$$
$$\mathbf{b}_2^3 = (++--+--+-----+-+)$$
$$\mathbf{b}_2^4 = (+--+++---+-+----)$$

$\{\mathbf{b}_1^k\}_{k=1}^4$ and $\{\mathbf{b}_2^k\}_{k=1}^4$ are mutually orthogonal ZCZ-(16, 4, 2) sets. The second iteration will result in two mutually orthogonal sets each forming a ZCZ-(64, 8, 4) set.

Conclusion: The notion of mutually orthogonal ZCZ sequence sets has been introduced and a construction procedure for the same has been proposed. Use of these orthogonal sets increases the number of available sequences for CDMA systems.

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