Performance analysis of predetection EGC receiver in Weibull fading channel

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The predetection equal gain combining (EGC) receiver is generally known to have a performance that is close to the maximal ratio combining (MRC) receiver while having relatively less implementation complexity. The bit error rate (BER) of an EGC receiver for binary, coherent and noncoherent modulations has been analysed for an independent Weibull fading channel. Numerical results have been compared with the available results for selection combining (SC) and MRC diversity receivers.

Introduction: The Weibull fading model exhibits an excellent fit to experimental fading channel measurements, for indoor [1] as well as outdoor environments [2]. In some recent work [3, 4] performance based on statistical parameters of the output signal-to-noise ratio (SNR) of switch-and-stay combining (SSC) and selection combining (SC) diversity receivers in Weibull fading channels has been obtained. In [5] the average symbol error probability for the SSC receiver for several binary and multilevel modulation schemes has been studied. The outage probability as well as BER performance of SC and MRC diversity receivers has been analysed in [6].

The predetection EGC receiver is generally known to have a performance close to a MRC receiver while at the same time being less complex from the implementation point of view. Hence, the performance analysis of this receiver is of interest.

In this Letter we analyse the performance of a predetection EGC receiver in independent Weibull fading channels for binary, coherent PSK and FSK, differential coherent PSK, and noncoherent FSK modulations.

BER performance analysis: Performance analysis of an EGC receiver usually needs the PDF of the output SNR, which is a function of the sum of the fading envelopes of the individual branches. However, a closed form expression for the PDF of this sum is not available for Weibull distribution. An alternative approach could be to use the BER expression derived in [7], provided the relevant expectations for the fading distribution can be determined. Hence, we begin by considering the BER expression [7],

$$P_e = \frac{2}{T}\sum_{\substack{n=1 \\ n\,\text{odd}}}^{\infty} A_n B_n \cos(\tau_n - \alpha_n) \tag{1}$$

where $A_n = \prod_{l=1}^{L}(\sqrt{(\mathrm{E}^2\{\cos(n\omega x_l)\} + \mathrm{E}^2\{\sin(n\omega x_l)\}}), \tau_n = \sum_{l=1}^{L}\tan^{-1}\{((\mathrm{E}[\sin(n\omega x_l)])/(\mathrm{E}[\cos(n\omega x_l)]))\}$, $\mathrm{E}(\cdot)$ is the expectation operator, $\omega = 2\pi/T$, and T is the period of the square wave used in deriving the infinite series expression for the sum of random variables in [8]. The minimum value of T required depends on the SNR as well as the desired accuracy of P_e. In the above, the random variable x_l denotes the fading envelope of the lth branch. We assume it to be Weibull distributed with a density function given by

$$f(x_l) = \frac{m}{\gamma} x_l^{m-1} e^{-x_l^m/\gamma}, \quad x_l \geq 0, \quad m > 0 \tag{2}$$

where $\mathrm{E}[X_l^2] = \gamma_l^{2/m}\Gamma(1 + 2/m)$. In (1) B_n and α_n are independent of the fading distribution and have been given in [7] (equation (27)) for coherent PSK and FSK modulations. For differential coherent PSK and noncoherent FSK modulations B_n has been denoted by D_n and α_n by d_n in [7] (equation (33)).

To evaluate (1) it is required to determine A_n and τ_n, which in turn require expressions for $\mathrm{E}[\cos(n\omega x_l)]$ and $[\sin(n\omega x_l)]$. One possible approach is to evaluate the integrals after multiplying the cos or sine term with the PDF of α_l. For Nakagami-m distribution these integrals can be readily found from standard integration and Fourier transform tables as a closed form expression containing hypergeometric functions. However, for Weibull distribution the integration is not straightforward and is also not available in standard tables. Another approach could be to evaluate the characteristic function and use its real and imaginary parts. But the characteristic function is also not available. Although [6] has derived the moment generating function, it is of limited use since the expression is in terms of Meijer's G function.

In this Letter we propose to evaluate these expectations by using the infinite series representation of cos and sine functions, as follows.

$$\cos(x) = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{2k!}$$
$$\sin(x) = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{(2k+1)!} \tag{3}$$

Interestingly, this is easily integrated as shown below, and the final expression is in the form of a convergent infinite series containing Gamma function.

$$\begin{aligned}
\mathrm{E}[\cos(n\omega x_l)] &= \int_0^{\infty}\sum_{k=0}^{\infty}(-1)^k \frac{(n\omega x_l)^{2k}}{2k!} \\
&\quad\times \frac{m}{\gamma} x_l^{m-1} e^{-x_l^m/\gamma} dx_l \\
&= \frac{m}{\gamma}\sum_{k=0}^{\infty}\frac{(-1)^k (n\omega)^{2k}}{2k!}\int_0^{\infty} x_l^{2k+m-1} e^{-x_l^m/\gamma} dx_l \\
&= \sum_{k=0}^{\infty}(-1)^k \frac{(n\omega\gamma^{1/m})^{2k}}{2k!} \\
&\quad\times \int_0^{\infty} t^{2k/m} e^{-t} dt, \text{ substituting } t = \frac{x^m}{\gamma} \\
&= \sum_{k=0}^{\infty}\frac{(-1)^k (n\omega\gamma^{1/m})^{2k}}{2k!}\Gamma\left(\frac{2k}{m} + 1\right)
\end{aligned} \tag{4}$$

$\mathrm{E}[\sin(n\omega x_l)]$ can also be derived similarly and the final expression obtained is

$$\mathrm{E}[\sin(n\omega x_l)] = \sum_{k=0}^{\infty}(-1)^k \frac{(n\omega\gamma^{1/m})^{2k+1}}{(2k+1)!}\Gamma\left(\frac{2k+1}{m} + 1\right) \tag{5}$$

Numerical results: The BER expression (1) has been evaluated numerically for different values of L and m and curves for average branch SNR against BER have been plotted. In numerical evaluation the value of T and the number of terms in the infinite series have been chosen to ensure an accuracy of $\pm 10^{-7}$ in BER. In Fig. 1 curves for coherent and differential coherent PSK modulations have been given for $L = 2$ and $L = 5$ each one for $m = 2$ and $m = 4$. It can be observed that the curve for coherent $L = 2$, $m = 2$ intersects the curve for differential coherent $L = 2$, $m = 4$ at an SNR of approximately 2.5 dB. This indicates that before 2.5 dB the coherent receiver for $m = 2$ gives better performance than the differential coherent receiver for $m = 4$ while after that it is inferior. Similar observations can be made from the results shown in [7] (Fig. 2). In Fig. 2 curves have been shown for coherent and noncoherent binary FSK modulations. A comparison of the results for coherent FSK with that of the SC receiver [6] shows that at a BER of 10^{-3} there is a gain of approximately 1.8 dB in SNR for $L = 2$, $m = 4$. For the same values of L and m a comparison with the MRC results in [6] shows that to achieve a BER of 10^{-3} the EGC receiver requires only 0.2 dB more SNR than the MRC receiver.

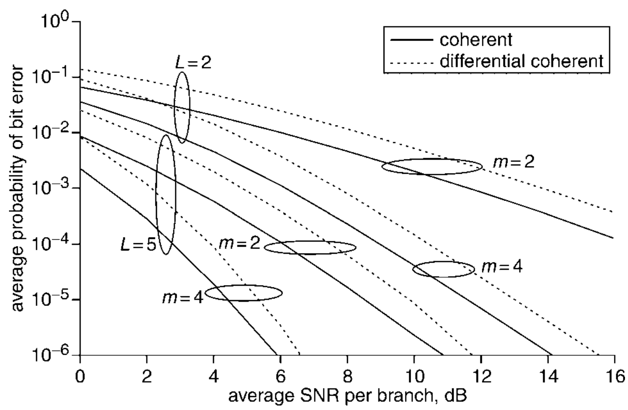

Fig. 1 *BER for coherent and differential coherent binary PSK*

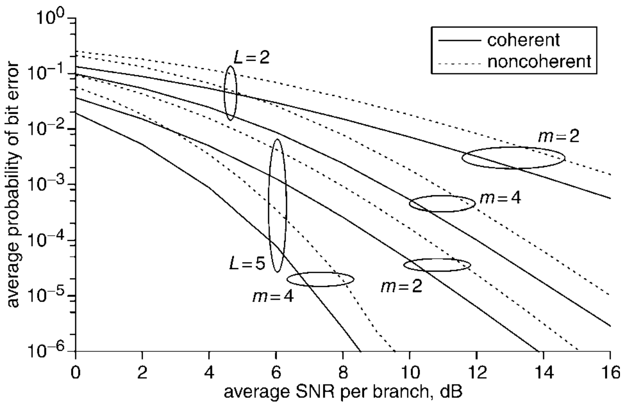

Fig. 2 *BER for coherent and noncoherent binary FSK*

Conclusions: We have analysed the performance of a predetection EGC receiver in an independent Weibull fading channel. BER performance for coherent PSK and FSK, differential coherent PSK and noncoherent FSK modulations has been given for a varying number of branches and fading parameters. The results have been compared with the available results for SC and MRC receivers.

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