Performance Analysis of Predetection EGC in Exponentially Correlated Nakagami-m Fading Channel

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***Abstract*—Using a Gil-Palaez lemma-based approach, we derive a general expression for the bit-error rate of a predetection equal gain combining receiver for coherent binary phase-shift keying modulation in exponentially correlated, Nakagami-m fading channel for an arbitrary number of branches. The obtained expressions are different for even and odd numbers of branches. Numerical results have been corroborated with simulations.**

***Index Terms*—Binary phase-shift keying (BPSK), diversity, exponential correlation, Nakagami-m fading channel, predetection equal gain combining (EGC).**

I. Introduction

PERFORMANCE of predetection equal gain combining (EGC) receivers in Nakagami-m fading channels is known for an arbitrary number of independent branches [1]. However, for correlated branches, it is known only for dual diversity, i.e., the two-antennas case for Rayleigh [2] as well as Nakagami-m [3] fading channels. In a recent publication [4], Karagiannidis *et al.* have derived the statistics of the output singal-to-noise ratio (SNR) for predetection EGC in correlated, Nakagami-m fading channels for any number of branches. However, the bit-error rate (BER) performance still remains an open problem.

The difficulty in the analysis of the predetection EGC receiver for an arbitrary number of branches is that the closed-form expression for the joint probability density function (pdf) of the multivariate, correlated Nakagami-m fading statistics is not available for more than two variables. In [5] Karagiannidis *et al.* have derived a closed form expression for the multivariate, Nakagami-m pdf for the special case of exponential correlation, which can be extended to the case of an arbitrary correlation using Green's matrix-approximation approach [6].

The Gil-Palaez lemma [7] can be used to find the probability of bit error of a detector when the characteristic function of the detector output decision variable is known [8]. Using this approach, we derive the BER performance of a predetection EGC receiver for coherent binary phase-shift keying (BPSK) modulation in exponentially correlated, Nakagami-m fading channels for an arbitrary number of branches.

In Section II, we describe the channel and receiver, and in Section III, we derive the BER expression. Numerical and simulation results have been given in Section IV, followed by Section V, wherein we conclude the letter.

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II. Channel and Receiver

The channel has been assumed to be slow and frequency nonselective, with Nakagami-m fading statistics. The complex low-pass equivalent of the signal received at the lth input branch over one bit duration T_b can be expressed as

$$r_l(t) = \alpha_l e^{j\phi_l} s(t) + n_l(t), \quad 0 \le t \le T_b, \quad l = 1, 2, \ldots, L \tag{1}$$

where $s(t)$ is the transmitted signal with bit energy E_s, and $n_l(t)$ is zero-mean complex Gaussian noise with two-sided power spectral density $2N_{ol}$. Random variable ϕ_l is uniformly distributed over $[0, 2\pi]$, while α_l is Nakagami-m distributed, with the density function given by

$$f(\alpha_l) = \frac{2\alpha_l^{2m-1}}{\Gamma(m)\Omega_l^m} \exp\left(-\frac{\alpha_l^2}{\Omega_l}\right), \quad \alpha_l \ge 0 \tag{2}$$

where $\Omega_l = (1/m)\mathrm{E}[\alpha_l^2]$, E is the expectation operator, and m is the fading parameter whose value lies in the range $1/2 \le m < \infty$. We assume α_l is independent of ϕ_l.

We assume that the signal envelopes are exponentially correlated, i.e., the correlation matrix $\Sigma_{i,j} \equiv \rho^{|i-j|}$, $1 \le i, j \le L$, where $\rho^{|i-j|}$ (ρ is the power correlation coefficient) is given by

$$\rho^{|i-j|} = \frac{\mathrm{cov}\left(\alpha_i^2, \alpha_j^2\right)}{\sqrt{\mathrm{var}\left(\alpha_i^2\right)\mathrm{var}\left(\alpha_j^2\right)}}, \quad 0 \le \rho < 1. \tag{3}$$

The closed-form expression for the joint pdf of L exponentially correlated, Nakagami-m fading envelopes is given by [5]

$$\begin{aligned} f(\alpha_1, \alpha_2, \ldots, \alpha_L) = {} & \frac{\alpha_1^{m-1}\alpha_L^m e^{\left(-\frac{\alpha_1^2+\alpha_L^2}{2(1-\rho^2)} - g_1\right)}}{2^{m-1}\Gamma(m)(1-\rho^2)^{m(L-1)}} \\ & \times \prod_{k=1}^{L-1} \left(\frac{\rho}{1-\rho^2}\right)^{-(m-1)} \alpha_k \\ & \times I_{m-1}\left[\frac{\rho\alpha_k\alpha_{k+1}}{1-\rho^2}\right] \end{aligned} \tag{4}$$

where $I_\nu(\cdot)$ is the νth-order modified Bessel function of the first kind, and

$$g_1 = \begin{cases} 0, & \text{for } L = 2 \\ \frac{1+\rho^2}{2(1-\rho^2)} \sum_{k=2}^{n-1} \alpha_k^2, & \text{for } L > 2. \end{cases}$$

The EGC combiner cophases the received signals, and adds them algebraically. This is followed by a matched-filter detector. For coherent BPSK modulation, assuming a "1" was transmitted, the receiver output decision variable is given by using the analysis in [2]

$$D_1 = \sum_{l=1}^{L} (\alpha_l + w_l) \tag{5}$$

where w_l's are independent zero-mean Gaussian random variables at the detector output, with variances $(N_{0l}/2E_s)$.

III. BER Analysis

Using the Gil-Palaez lemma, the BER can be obtained from [8]

$$P_e = \Pr(D_1 < 0) = \frac{1}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\Im\{\Phi_{D_1}(j\omega)\}}{\omega} d\omega \qquad (6)$$

where $\Phi_{D_1}(j\omega)$ is the characteristic function of the detector output decision variable D_1, and $\Im\{\cdot\}$ denotes its imaginary part. The characteristic function of D_1 can be expressed as

$$\begin{aligned}\Phi_{D_1}(j\omega) &= \Phi_{\alpha_1+\alpha_2+\ldots+\alpha_L}(j\omega)\Phi_{w_1+w_2+\ldots+w_L}(j\omega)\\ &= \Phi_{\alpha_1,\alpha_2,\ldots,\alpha_L}(j\omega, j\omega,\ldots,j\omega)\prod_{l=1}^{L}\Phi_{w_l}(j\omega) \qquad (7)\end{aligned}$$

where $\Phi_{w_l}(j\omega) = \exp(-N_{0l}\omega^2/4E_s)$ is the characteristic function of w_l. The characteristic function of exponentially correlated, Nakagami-m random variables $\alpha_1, \alpha_2, \ldots, \alpha_L$ for $L \geq 3$ can be derived as shown in the Appendix. Following the same approach, it can be found for $L = 2$ also. Combining and rearranging the expressions for $L = 2$ and $L \geq 3$, the characteristic function for L-variate, exponentially correlated, Nakagami-m distributed random variables can be given as

$$\begin{aligned}&\Phi_{\alpha_1,\alpha_2,\ldots,\alpha_L}(j\omega_1, j\omega_2,\ldots,j\omega_L)\\ &= \frac{2^{L(1-2m)}\pi^{\frac{L}{2}}(1-\rho^2)^m}{\Gamma(m)(1+\rho^2)^{m(L-2)}}\sum_{i_1,i_2,\ldots,i_{L-1}=0}^{\infty}\left(\frac{\rho}{4}\right)^{2\sum_{j=1}^{L-1} i_j}\\ &\times\frac{(1+\rho^2)^{-\left(i_1+2\sum_{j=2}^{L-2} i_j + i_{L-1}\right)\beta}}{\prod_{j=1}^{L-1} i_j!\Gamma(m+i_j)}\\ &\times\prod_{k=1}^{L}\Gamma(2\xi_k)\left[A_k\, {}_1F_1\left(\xi_k;\frac{1}{2};-c_k\omega_k^2\right) + j\omega_k B_k \right.\\ &\qquad\left.\times {}_1F_1\left(\frac{1}{2}+\xi_k;\frac{3}{2};-c_k\omega_k^2\right)\right] \qquad (8)\end{aligned}$$

where $\beta = 0$ for $L = 2$ and $\beta = 1$ for $L \geq 3$, $\xi_k = i_{k-1} + i_k + m$, and

$$A_k = \left[\Gamma\left(\frac{1}{2}+\xi_k\right)\right]^{-1}, \quad k = 1, 2, \ldots, L$$

$$B_k = \begin{cases}\sqrt{2(1-\rho^2)}\,[\Gamma(\xi_k)]^{-1}, & k = 1, L\\ \sqrt{\frac{2(1-\rho^2)}{1+\rho^2}}\,[\Gamma(\xi_k)]^{-1}, & k = 2, 3, \ldots, L-1\end{cases}$$

$$c_k = \begin{cases}\frac{1-\rho^2}{2}, & k = 1, L\\ \frac{1-\rho^2}{2(1+\rho^2)}, & k = 2, 3, \ldots, L-1\end{cases}$$

where $i_0 = i_L = 0$. As expected, for $L = 2$ and $m = 1$, (8) reduces to [2, eq. (3)] (the characteristic function for bivariate Rayleigh random variables, with $\Omega_1 = \Omega_2 = 2$).

We need $(\Im\{\Phi_{D_1}(j\omega)\}/\omega)$ to evaluate (6). Consider the product of L complex numbers of the form $a_i + jb_i$ for $i = 1, 2, \ldots, L$. It can be shown that

$$\begin{aligned}&\Im\left\{\prod_{i=1}^{L}(a_i + jb_i)\right\}\\ &= \begin{cases}\sum_{i=1}^{\frac{L}{2}}(-1)^{i-1}\sum_{p=1}^{{}^LC_{2i-1}}\prod_{\substack{r=1\\ r\neq\theta_{(2i-1)}^{(L)}(p,1:2i-1)}}^{L} a_r\prod_{n=1}^{2i-1} b_{\theta_{(2i-1)}^{(L)}(p,n)}, & L \text{ even}\\ \sum_{i=0}^{\frac{L-1}{2}}(-1)^{\frac{L-2i-1}{2}}\sum_{p=1}^{{}^LC_{2i}}\prod_{r=1}^{2i} a_{\theta_{(2i)}^{(L)}(p,r)}\prod_{\substack{n=1\\ n\neq\theta_{(2i)}^{(L)}(p,1:2i)}}^{L} b_n, & L \text{ odd}\end{cases}\end{aligned} \qquad (9)$$

where we define $\theta_{(M)}^{(L)}(i,j)$ as follows. We consider all possible combinations of M integers out of L consecutive integers 1 to L, where $M \leq L$. There will be ${}^LC_M$ possible combinations. We number these combinations from 1 to ${}^LC_M$, and the elements of each combination from 1 to M. Then $\theta_{(M)}^{(L)}(i,j)$ denotes the jth element of the ith combination. The range of i is $[1\ {}^LC_M]$, and that of j is $[1\ M]$. For example, for $L = 3$ and $M = 2$, one can verify that $\theta_{(2)}^{(3)}(1,1) = 1$, $\theta_{(2)}^{(3)}(1,2) = 2$, $\theta_{(2)}^{(3)}(2,1) = 1$, $\theta_{(2)}^{(3)}(2,2) = 3$, $\theta_{(2)}^{(3)}(3,1) = 2$, and $\theta_{(2)}^{(3)}(3,2) = 3$. Also, we define $\theta_{(\cdot)}^{(\cdot)}(p, 1:K) \triangleq \{\theta_{(\cdot)}^{(\cdot)}(p,1), \theta_{(\cdot)}^{(\cdot)}(p,2), \ldots, \theta_{(\cdot)}^{(\cdot)}(p,K)\}$.

Using (8) in (7) and then applying (9), $\Im\{\Phi_{D_1}(j\omega)\}/\omega$ can be written as in (10), where

$$a_0 = \frac{\sum_{i=1}^{L} N_{0i}}{4E_s}$$

$$x_k = {}_1F_1\left(\xi_k;\frac{1}{2};-c_k\omega^2\right)$$

$$y_k = {}_1F_1\left(\frac{1}{2}+\xi_k;\frac{3}{2};-c_k\omega^2\right).$$

Assuming equal noise power per branch, i.e., $N_{0l} = N_0$ for $l = 1, 2, \ldots, L$, the received average SNR per branch, $\bar{\gamma} = (2mE_s/N_0)$ [9], and $a_0 = (LN_0/4E_s)$. Substituting these in (10) and analytically solving the integration in (6) using [10, eq. (C.1)], the expression for the BER can be obtained as a function of $\bar{\gamma}$. Since the right-hand side of (9) depends on whether L is even or odd, we get two different expressions for BER, which have been given in (11) and (12), respectively, shown with (10) on the next page, where $F_A(\cdot;\cdot,\cdot,\ldots;\cdot,\cdot,\ldots;\cdot,\cdot,\ldots)$ is the hypergeometric function of several variables [13, 9.19], $\{z_k\}|_{k=1}^{R} \triangleq z_1, z_2, \ldots, z_R$, and

$$b = L + \frac{2\bar{\gamma}(1-\rho^2)}{m}\left(1 + \frac{L-2}{2(1+\rho^2)}\right).$$

The BER expression for the dual-diversity case can be easily obtained from (11), and is, as expected, identical to [3, eq. (21)] for BPSK modulation with $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$.

For binary frequency-shift keying (BFSK), the noise variance at the output of the detector is twice that of BPSK, and the above

BER expression can be easily modified to obtain an expression for BFSK.

In a recent publication [11], Karagiannidis has given an approach to obtain the performance of a predetection EGC receiver beyond dual diversity for a correlated, Nakagami-m fading channel. The method relies on Padé approximant theory and needs a closed-form expression for the moments. Furthermore, it does not give an explicit expression for the BER and needs numerical integration. On the contrary, the approach in this letter leads to an exact, closed-form expression in the form of an infinite series for an exponentially correlated, Nakagami-m fading channel.

$$\frac{\Im\{\Phi_{D_1}(j\omega)\}}{\omega} = \frac{2^{L(1-2m)}\pi^{\frac{L}{2}}(1-\rho^2)^m}{\Gamma(m)(1+\rho^2)^{m(L-2)}} \sum_{i_1,i_2,\ldots,i_{L-1}=0}^{\infty} \frac{\left(\frac{\rho}{4}\right)^{2\sum_{j=1}^{L-1} i_j} (1+\rho^2)^{-\left(i_1+2\sum_{j=2}^{L-2} i_j + i_{L-1}\right)\beta}}{\left[\prod_{j=1}^{L-1} i_j!\Gamma(m+i_j)\right]}$$
$$\times \begin{cases} \left[\prod_{k=1}^{L}\Gamma(2\xi_k)\right]\left\{\sum_{d=1}^{\frac{L}{2}}(-\omega^2)^{d-1}\sum_{p=1}^{{}^LC_{2d-1}}\prod_{r=1,r\neq\theta_{(2d-1)}^{(L)}(p,1:2d-1)}^{L} A_r x_r \prod_{n=1}^{2d-1} B_{\theta_{(2d-1)}^{(L)}(p,n)} y_{\theta_{(2d-1)}^{(L)}(p,n)}\right\} e^{-a_0\omega^2}, & L \text{ even} \\ \left[\prod_{k=1}^{L}\Gamma(2\xi_k)\right]\left\{\sum_{d=0}^{\frac{L-1}{2}}(\sqrt{-1}\omega)^{L-2d-1}\sum_{p=1}^{{}^LC_{2d}}\prod_{r=1}^{2d} A_{\theta_{(2d)}^{(L)}(p,r)} x_{\theta_{(2d)}^{(L)}(p,r)} \prod_{n=1,n\neq\theta_{(2d)}^{(L)}(p,1:2d)}^{L} B_n y_n\right\} e^{-a_0\omega^2}, & L \text{ odd} \end{cases} \tag{10}$$

$$P_e(\bar{\gamma}) = \frac{1}{2} - \frac{2^{L(1-2m)-1}\pi^{\frac{L}{2}-1}(1-\rho^2)^m}{\Gamma(m)(1+\rho^2)^{m(L-2)}} \sum_{i_1,i_2,\ldots,i_{L-1}=0}^{\infty} \frac{\left(\frac{\rho}{4}\right)^{2\sum_{j=1}^{L-1} i_j} (1+\rho^2)^{-\left(i_1+2\sum_{j=2}^{L-2} i_j + i_{L-1}\right)\beta}}{\left[\prod_{j=1}^{L-1} i_j!\Gamma(m+i_j)\right]}$$
$$\times \left[\prod_{k=1}^{L}\Gamma(2\xi_k)\right]\sum_{d=1}^{\frac{L}{2}}(-1)^{d-1}\sum_{p=1}^{{}^LC_{2d-1}}\left\{\prod_{r=1,r\neq\theta_{(2d-1)}^{(L)}(p,1:2d-1)}^{L} A_r\right\}\left\{\prod_{n=1}^{2d-1} B_{\theta_{(2d-1)}^{(L)}(p,n)}\right\} \frac{\Gamma\left(d-\frac{1}{2}\right)}{\left(\frac{mb}{2\bar{\gamma}}\right)^{d-\frac{1}{2}}}$$
$$\times F_A\left(d-\frac{1}{2}; \left\{\frac{1}{2}-\xi_r\right\}\Big|_{r=1,r\neq\theta_{(2d-1)}^{(L)}(p,1:2d-1)}^{L}, \left\{1-\xi_{\theta_{(2d-1)}^{(L)}(p,n)}\right\}\Big|_{n=1}^{2d-1}; \underbrace{\frac{1}{2},\frac{1}{2},\ldots,\frac{1}{2}}_{(L-2d+1)\text{ numbers}},\right.$$
$$\left.\underbrace{\frac{3}{2},\frac{3}{2},\ldots,\frac{3}{2}}_{(2d-1)\text{ numbers}}; \left\{\frac{2\bar{\gamma}c_r}{mb}\right\}\Big|_{r=1,r\neq\theta_{(2d-1)}^{(L)}(p,1:2d-1)}^{L}, \left\{\frac{2\bar{\gamma}c_{\theta_{(2d-1)}^{(L)}(p,n)}}{mb}\right\}\Big|_{n=1}^{2d-1}\right), \quad L \text{ even} \tag{11}$$

$$P_e(\bar{\gamma}) = \frac{1}{2} - \frac{2^{L(1-2m)-1}\pi^{\frac{L}{2}-1}(1-\rho^2)^m}{\Gamma(m)(1+\rho^2)^{m(L-2)}} \sum_{i_1,i_2,\ldots,i_{L-1}=0}^{\infty} \frac{\left(\frac{\rho}{4}\right)^{2\sum_{j=1}^{L-1} i_j} (1+\rho^2)^{-\left(i_1+2\sum_{j=2}^{L-2} i_j + i_{L-1}\right)\beta}}{\left[\prod_{j=1}^{L-1} i_j!\Gamma(m+i_j)\right]}$$
$$\times \left[\prod_{k=1}^{L}\Gamma(2\xi_k)\right]\sum_{d=0}^{\frac{L-1}{2}}(\sqrt{-1})^{L-2d-1}\sum_{p=1}^{{}^LC_{2d}}\left\{\prod_{r=1}^{2d} A_{\theta_{(2d)}^{(L)}(p,r)}\right\}\left\{\prod_{n=1,n\neq\theta_{(2d)}^{(L)}(p,1:2d)}^{L} B_n\right\}$$
$$\times \frac{\Gamma\left(\frac{L}{2}-d\right)}{\left(\frac{mb}{2\bar{\gamma}}\right)^{\frac{L}{2}-d}} F_A\left(\frac{L}{2}-d; \left\{\frac{1}{2}-\xi_{\theta_{(2d)}^{(L)}(p,r)}\right\}\Big|_{r=1}^{2d}, \{1-\xi_n\}\big|_{n=1,n\neq\theta_{(2d)}^{(L)}(p,1:2d)}^{L}; \underbrace{\frac{1}{2},\frac{1}{2},\ldots,\frac{1}{2}}_{2d\text{ numbers}},\right.$$
$$\left.\underbrace{\frac{3}{2},\frac{3}{2},\ldots,\frac{3}{2}}_{L-2d\text{ numbers}}; \left\{\frac{2\bar{\gamma}c_{\theta_{(2d)}^{(L)}(p,r)}}{mb}\right\}\Big|_{r=1}^{2d}, \left\{\frac{2\bar{\gamma}c_n}{mb}\right\}\Big|_{n=1,n\neq\theta_{(2d)}^{(L)}(p,1:2d)}^{L}\right), \quad L \text{ odd} \tag{12}$$

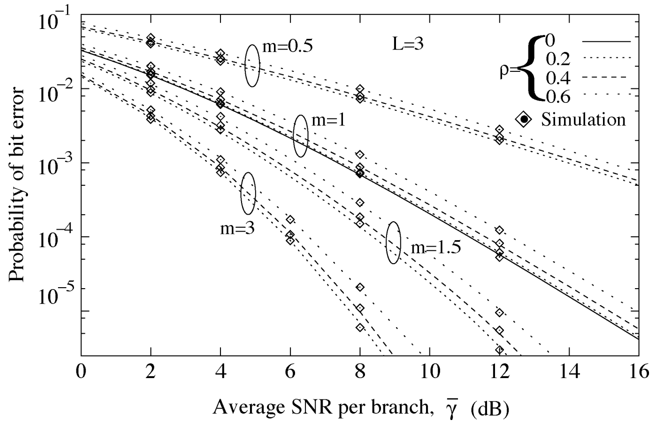

Fig. 1. BER performance of BPSK with $L = 3$ for different values of ρ and m.

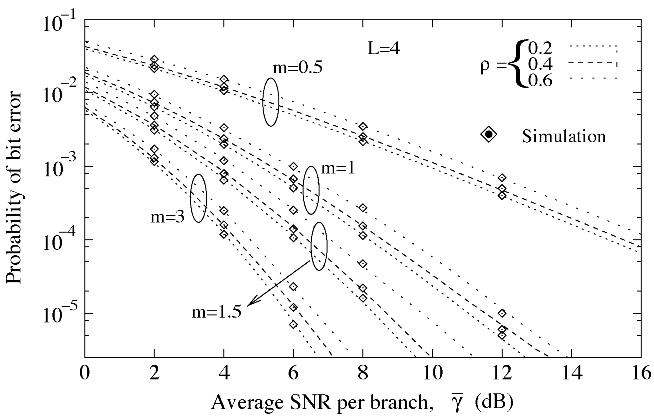

Fig. 2. BER performance of BPSK with $L = 4$ for different values of ρ and m.

IV. Numerical and Simulation Results

BER expressions (11) and (12) have been numerically evaluated and compared with simulation results. Probability of bit error versus average SNR per branch $\bar{\gamma}$ has been shown in Fig. 1 as a function of m and ρ for $L = 3$. The curve for $L = 3$, $m = 1$, and $\rho = 0$ has been included in Fig. 1 (solid line) to show that it matches with the previously published result in [8]. The number of terms included in the numerical evaluation of the infinite series ensures an accuracy at the sixth significant digit of the decimal point. Numerical results for $L = 4$ have been shown in Fig. 2. Simulations have been done using MATLAB. These results have also been plotted in both figures. It can be observed that the simulation results match with the numerical results. Although we do not show it in the figures, we have also compared our results with the results in [11, Fig. 2], and found them to match.

In order to check the convergence of the BER expression, a separate numerical evaluation has been done for $L = 3$ for an accuracy at the seventh significant digit. The number of terms required to sum to achieve this accuracy has been tabulated in Table I. To simplify the computation, an identical number of terms, denoted N, have been taken in the two infinite series after truncation. A similar procedure was followed in [5] to show the convergence of an expression containing multiple infinite series. The change in the required number of terms as a function of ρ, SNR, and m can be observed from Table I. It can be noted that for any m, the required number of terms increases with the increase in ρ for a constant SNR, while it decreases with an increase in SNR when ρ is constant.

TABLE I
Number of Terms Required to Achieve Accuracy at 7th Significant Digit for $L = 3$

SNR(dB)	ρ	$m = 1$		$m = 2$	
		N	BER	N	BER
0	0.2	6	0.0335436	7	0.0189083
	0.4	10	0.0356960	10	0.0200482
	0.6	15	0.0400461	18	0.0223376
8	0.2	5	0.0007325	5	0.0000424
	0.4	8	0.0008846	8	0.0000567
	0.6	15	0.0012723	18	0.0000980

V. Conclusion

Using the Gil-Palaez lemma-based approach, we have derived a closed-form BER expression for a predetection EGC receiver for coherent BPSK modulation in exponentially correlated, Nakagami-m fading channels for an arbitrary number of branches. The expression has been numerically evaluated and plotted for different values of correlation coefficients and fading parameters. Numerical results have been corroborated with simulations. The formulation presented in this letter can be extended to the case of arbitrary correlation, using the approach given in [6].

Appendix
Characteristic Function for $L \geq 3$

To find the characteristic function, we use the infinite series form of the pdf in (4), as obtained in [5, eq. (9)]. Then the characteristic function is given by

$$\Phi_{\alpha_1,\alpha_2,\ldots,\alpha_L}(j\omega, j\omega, \ldots, j\omega) = \frac{1}{2^{L(m-1)}\Gamma(m)(1-\rho^2)^{m(L-1)}} \times \sum_{i_1,i_2,\ldots,i_{L-1}=0}^{\infty} \frac{\left(\frac{\rho}{2(1-\rho^2)}\right)^{2i_1+2i_2+\ldots+2i_{L-1}}}{\prod_{j=1}^{L-1} i_j!\Gamma(m+i_j)} \times \int_0^\infty \alpha_1^{2i_1+2m-1} e^{-\frac{\alpha_1^2}{2(1-\rho^2)}} e^{j\omega\alpha_1} d\alpha_1 \times \prod_{k=2}^{L-1} \int_0^\infty \alpha_k^{2i_{k-1}+2i_k+2m-1} e^{-\frac{(1+\rho^2)\alpha_k^2}{2(1-\rho^2)}} e^{j\omega\alpha_k} d\alpha_k \times \int_0^\infty \alpha_L^{2i_{L-1}+2m-1} e^{-\frac{\alpha_L^2}{2(1-\rho^2)}} e^{j\omega\alpha_L} d\alpha_L. \quad (13)$$

The integrations in (13) can be evaluated using [12, eq. (4.5.24)]. Then

$$\begin{aligned}
&\Phi_{\alpha_1,\alpha_2,\ldots,\alpha_L}(j\omega, j\omega, \ldots, j\omega) \\
&\quad= \frac{1}{2^{L(m-1)}\Gamma(m)(1-\rho^2)^{m(L-1)}} \\
&\quad\times \sum_{i_1,i_2,\ldots,i_{L-1}=0}^{\infty} \frac{\left(\frac{\rho}{2(1-\rho^2)}\right)^{2i_1+2i_2+\ldots+2i_{L-1}}}{\prod_{j=1}^{L-1} i_j!\Gamma(m+i_j)} \\
&\quad\times \Gamma(2i_1+2m)\left(1-\rho^2\right)^{i_1+m} e^{-\frac{(1-\rho^2)\omega^2}{4}} \\
&\quad\times D_{-(2i_1+2m)}\left(-j\sqrt{1-\rho^2}\omega\right) \\
&\quad\times \prod_{k=2}^{L-1} \Gamma(2i_{k-1}+2i_k+2m) \\
&\quad\times \left[\frac{(1-\rho^2)}{(1+\rho^2)}\right]^{i_{k-1}+i_k+m} e^{-\frac{(1-\rho^2)\omega^2}{4(1+\rho^2)}} \\
&\quad\times D_{-(2i_{k-1}+2i_k+2m)}\left(-j\sqrt{\frac{(1-\rho^2)}{(1+\rho^2)}}\omega\right) \\
&\quad\times \Gamma(2i_{L-1}+2m)(1-\rho^2)^{i_{L-1}+m} e^{-\frac{(1-\rho^2)\omega^2}{4}} \\
&\quad\times D_{-(2i_{L-1}+2m)}\left(-j\sqrt{1-\rho^2}\omega\right)
\end{aligned} \tag{14}$$

where the parabolic cylinder function $D_p(z)$ is defined in [13, eq. (9.240)] as

$$D_p(z) = 2^{\frac{p}{2}} e^{-\frac{z^2}{4}} \left\{ \frac{\sqrt{\pi}\, {}_1F_1\left(-\frac{p}{2}; \frac{1}{2}; \frac{z^2}{2}\right)}{\Gamma\left(\frac{1-p}{2}\right)} - \frac{\sqrt{2\pi} z\, {}_1F_1\left(\frac{1-p}{2}; \frac{3}{2}; \frac{z^2}{2}\right)}{\Gamma\left(-\frac{p}{2}\right)} \right\} \tag{15}$$

where ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function. The final expression for the characteristic function can be obtained by substituting (15) in (14), and has been given in (8).

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