Effect of an independent antenna on the performance of a correlated dual-diversity predetection EGC receiver in Nakagami-m fading channel

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Abstract: The bit error rate (BER) performance improvement of a correlated dual-diversity predetection EGC receiver in a Nakagami-m fading channel when an additional antenna which is receiving signals independent of the correlated pair is available is analysed. Two percentage improvement criteria have been considered. Numerical results show that the percentage improvement is more for higher SNRs. One of them has been found to be almost independent of the correlation between the correlated pair of antennas.

1 Introduction

The performance of predetection equal gain combining (EGC) receivers in a Nakagami fading channel is known for an arbitrary number of independent branches [1, 2]. For postdetection EGC, in addition to maximal ration combining (MRC), the performance for an arbitrary number of correlated branches is known [3, 4]. In contrast, the performance of a predetection EGC receiver is available only for two correlated branches [5, 6] and more recently for L equally correlated branches [7].

In this paper, we derive the BER expression for a predetection EGC receiver which uses a three antenna configuration as shown in Fig. 1. We assume that the antennas have been placed in such a way that the signals received at antenna '1' and antenna '2' are correlated while the signal received at the third one is independent of this pair.

This configuration of antennas models situations where it is required to obtain enhanced receiver performance beyond what is possible using two antennas only but the physically available space does not permit the installation of three antennas which are separated sufficiently from each other so as to receive three independently fading signals. Since physical space is a scarce resource, such situations may frequently arise as the demand for wireless communications increases. An example is a base station site in the emerging congested urban scenario. The considered arrangement is intermediate to the following two situations: Three antennas where all the three received signals are correlated and three antennas with independent received signals. It is worth pointing out that the present work is not a special case of [7].

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2 Channel and receiver

The channel has been assumed to be slow, frequency non-selective, with Nakagami-m fading statistics. The complex low-pass equivalent of the signal received at the lth input branch over one symbol duration T_s can be expressed as

$$g_l(t) = \alpha_l e^{j\phi_l} s(t) + n_l(t),\ 0 \le t \le T_s, \quad l = 1, 2, 3 \qquad (1)$$

where $s(t)$ is the transmitted signal with energy E_s and $n_l(t)$ is the noise generated from a zero mean complex Gaussian random process with two sided power spectral density $2N_{ol}$. Random variable ϕ_l is uniformly distributed over $[0,2\pi]$, while α_l, which is independent of ϕ_l is Nakagami-m distributed with a density function given by

$$f(\alpha_l) = \frac{2\alpha_l^{2m-1}}{\Gamma(m)\Omega_l^m} \exp\left(-\frac{\alpha_l^2}{\Omega_l}\right), \quad \alpha_l \ge 0 \qquad (2)$$

where $\Omega_l = \frac{1}{m}E\left[\alpha_l^2\right]$ is the average signal energy at the lth branch.

Let the correlation coefficient of the correlated signal received at the antenna pair (shown in Fig. 5) be ρ. The joint density function of α_1 and α_2 is given by [6]

$$f(\alpha_1, \alpha_2) = \frac{4(\alpha_1\alpha_2)^m}{\Gamma(m)\Omega_1\Omega_2(1-\rho)(\sqrt{\Omega_1\Omega_2\rho})^{m-1}} \times I_{m-1}\left(\frac{2\sqrt{\rho}\alpha_1\alpha_2}{\sqrt{\Omega_1\Omega_2}(1-\rho)}\right),\ \alpha_1, \alpha_2 \ge 0 \qquad (3)$$

where $I_v(\cdot)$ is the modified Bessel function.

The EGC combiner cophases the received signals, and adds them algebraically. This is followed by a matched filter detector. For BPSK modulation, assuming a '1' was transmitted, the output decision variable for the system under consideration is given by [8]

$$D_1 = \sum_{l=1}^{3} (\alpha_l + w_l) \qquad (4)$$

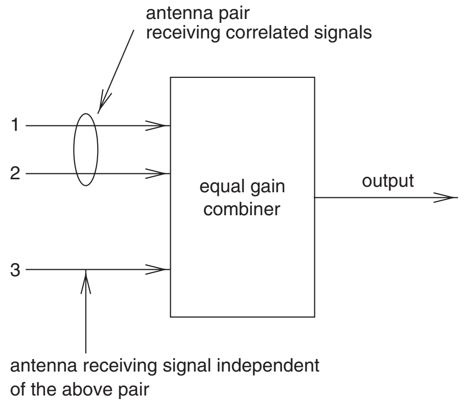

Fig. 1 *EGC receiver with three input antennas*

where w_l represents independent zero mean Gaussian random variable at the detector output corresponding to input noise components, with variances $\frac{N_{0l}}{2E_s}$ [5].

3 BER analysis

Using the characteristic function method [8], BER can be obtained from the expression

$$P_e = \Pr(D_1 < 0) = \frac{1}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\operatorname{Im}\{\Phi_{D_1}(j\omega)\}}{\omega} d\omega \tag{5}$$

where $\Phi_{D_1}(j\omega)$ is the characteristic function of the detector output decision variable D_1 and $\operatorname{Im}\{\cdot\}$ denotes its imaginary part. The characteristic function of D_1 is

$$\Phi_{D_1}(j\omega) = \Phi_{\alpha_1+\alpha_2}(j\omega)\Phi_{\alpha_3}(j\omega)\prod_{l=1}^{3}\Phi_{w_l}(j\omega) \tag{6}$$

where $\Phi_{w_l}(j\omega) = \exp\left(-\frac{N_{0l}\omega^2}{4E_s}\right)$ is the characteristic function of w_l. The characteristic function of correlated Nakagami-m random variables α_1 and α_2 is given by [6]

$$\Phi_{\alpha_1+\alpha_2}(j\omega) = \Phi_{\alpha_1,\alpha_2}(j\omega) = \frac{(1-\rho)^m}{\Gamma(m)}\sum_{k=0}^{\infty}\frac{\rho^k}{k!\Gamma(m+k)}A_k(\Omega_1)A_k(\Omega_2) \tag{7}$$

where

$$A_k(\Omega_l) = \Gamma(k+m)\,{}_1F_1\left(k+m;\frac{1}{2};-\frac{\omega^2\Omega_l(1-\rho)}{4}\right) + j\omega\sqrt{\Omega_l(1-\rho)}\,\Gamma\left(m+k+\frac{1}{2}\right) \times {}_1F_1\left(m+k+\frac{1}{2};\frac{3}{2};-\frac{\omega^2\Omega_l(1-\rho)}{4}\right),\quad l=1,2$$

where ${}_1F_1(\cdot\,;\cdot\,;\cdot)$ is the confluent hypergeometric function. The characteristic function of the independent Nakagami-m random variable α_3 is given by [1]

$$\Phi_{\alpha 3}(j\omega) = {}_1F_1\left(m;\frac{1}{2};-\frac{\Omega_3}{4}\omega^2\right) + j\omega\sqrt{\Omega_3}\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\,{}_1F_1\left(m+\frac{1}{2};\frac{3}{2};-\frac{\Omega_3}{4}\omega^2\right) \tag{8}$$

We need $\frac{\operatorname{Im}\{\Phi_{D_1}(j\omega)\}}{\omega}$ to evaluate (5). Using (7) and (8) in (6) we get

$$\begin{aligned}\frac{\operatorname{Im}\{\Phi_{D_1}(j\omega)\}}{\omega} &= \frac{(1-\rho)^m}{\Gamma(m)}\sum_{k=0}^{\infty}\frac{\rho^k}{k!\Gamma(m+k)} \times \exp(-a_0\omega^2)\frac{\operatorname{Im}\{A_k(\Omega_1)A_k(\Omega_2)\Phi_{\alpha 3}(j\omega)\}}{\omega}\\ &= \sum_{k=0}^{\infty}\frac{(1-\rho)^m\rho^k}{k!\Gamma(m)\Gamma(m+k)}\exp(-a_0\omega^2) \times \left\{(x_1x_2+\omega^2 y_1y_2)y_3 + (x_1y_2+x_2y_1)x_3\right\}\end{aligned} \tag{9}$$

where $a_0 = \frac{\sum_{l=1}^{3}N_{0l}}{4E_s}$,

$$x_i = \Gamma(k+m)\,{}_1F_1\left(m+k,\frac{1}{2};-\frac{\Omega_i(1-\rho)\omega^2}{4}\right),\quad i=1,2$$

$$y_i = \sqrt{\Omega_i(1-\rho)}\,\Gamma\left(\frac{1}{2}+k+m\right) \times {}_1F_1\left(\frac{1}{2}+k+m,\frac{3}{2};-\frac{\Omega_i(1-\rho)\omega^2}{4}\right),\quad i=1,2$$

$$x_3 = {}_1F_1\left(m,\frac{1}{2};-\frac{\Omega_3\omega^2}{4}\right)$$

$$y_3 = \frac{\sqrt{\Omega_3}\Gamma(m+\frac{1}{2})}{\Gamma(m)}\,{}_1F_1\left(m+\frac{1}{2},\frac{3}{2};-\frac{\Omega_3\omega^2}{4}\right)$$

Now putting (9) in (5), and interchanging the order of integration and summation we encounter four integrations. We denote these integrations by $I_{1,k}$, $I_{2,k}$, $I_{3,k}$, and $I_{4,k}$. They can be evaluated analytically using the formulas C.1 and C.5 in [1]. Let us define the lth branch SNR as [8] $\bar{\gamma}_1 = \Omega_l E_S/\frac{1}{L}\sum_{l=1}^{L}N_{0l}$, where L is the number of antennas. Substituting this in the integrations evaluated, the expression for BER is given by

$$P_e = \frac{1}{2} - \frac{(1-\rho)^m}{2\pi\Gamma(m)}\sum_{k=0}^{\infty}\frac{\rho^k}{k!\Gamma(m+k)} \times \left[I_{1,k} - I_{2,k} + I_{3,k} + I_{4,k}\right] \tag{10}$$

where,

$$I_{1,k} = 2q_1\sqrt{\frac{\pi\bar{\gamma}_3}{\xi}}F_A\left(\frac{1}{2};\frac{1}{2}-k-m,\frac{1}{2}-k-m,1-m;\ \frac{1}{2},\frac{1}{2},\frac{3}{2};\frac{\bar{\gamma}_1(1-\rho)}{\xi},\frac{\bar{\gamma}_2(1-\rho)}{\xi},\frac{\bar{\gamma}_3}{\xi}\right)$$

$$I_{2,k} = 4q_2\sqrt{\frac{\pi\bar{\gamma}_1\bar{\gamma}_2\bar{\gamma}_3}{\xi^3}}F_A\left(\frac{3}{2};1-k-m,1-k-m,1-m;\ \frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{\bar{\gamma}_1(1-\rho)}{\xi},\frac{\bar{\gamma}_2(1-\rho)}{\xi},\frac{\bar{\gamma}_3}{\xi}\right)$$

$$I_{3,k} = 2q_3\sqrt{\frac{\pi\bar{\gamma}_2}{\xi}}F_A\left(\frac{1}{2};\frac{1}{2}-k-m,1-k-m,\frac{1}{2}-m;\ \frac{1}{2},\frac{3}{2},\frac{1}{2};\frac{\bar{\gamma}_1(1-\rho)}{\xi},\frac{\bar{\gamma}_2(1-\rho)}{\xi},\frac{\bar{\gamma}_3}{\xi}\right)$$

$$I_{4,k} = 2q_4\sqrt{\frac{\pi\bar{\gamma}_1}{\xi}}F_A\left(\frac{1}{2};\frac{1}{2}-k-m,1-k-m,\frac{1}{2}-m;\ \frac{1}{2},\frac{3}{2},\frac{1}{2};\frac{\bar{\gamma}_1(1-\rho)}{\xi},\frac{\bar{\gamma}_2(1-\rho)}{\xi},\frac{\bar{\gamma}_3}{\xi}\right)$$

where $q_1 = \frac{[\Gamma(k+m)]^2\Gamma(m+\frac{1}{2})}{\Gamma(m)}$, $q_2 = \frac{(1-\rho)\left[\Gamma\left(\frac{1}{2}+k+m\right)\right]^2\Gamma(m+\frac{1}{2})}{\Gamma(m)}$, $q_3 = q_4 = \sqrt{(1-\rho)}\Gamma(k+m)\Gamma(\frac{1}{2}+k+m)$, $\xi = 3m + (1-\rho)(\bar{\gamma}_1+\bar{\gamma}_2)+\bar{\gamma}_3$ and F_A is the hypergeometric function of several variables.

3.1 Special cases

3.1.1 Correlated dual diversity: The BER expression for correlated dual-diversity predetection EGC system can be obtained from (10). This system is equivalent to the system under consideration in this paper with $L = 2$, $\Omega_3 = 0$, and $N_{03} = 0$. Under these conditions,

$$\begin{aligned} I_{1,k} &= 0 \\ I_{2,k} &= 0 \\ I_{3,k} &= 2q_3\sqrt{\frac{\pi\bar{\gamma}_2}{\xi_1}}F_A\left(\frac{1}{2};\frac{1}{2}-k-m, 1-k-m;\right. \\ &\quad \left.\frac{1}{2},\frac{3}{2};\frac{\bar{\gamma}_1(1-\rho)}{\xi_1},\frac{\bar{\gamma}_2(1-\rho)}{\xi_1}\right) \\ I_{4,k} &= 2q_4\sqrt{\frac{\pi\bar{\gamma}_1}{\xi_1}}F_A\left(\frac{1}{2};\frac{1}{2}-k-m, 1-k-m;\right. \\ &\quad \left.\frac{1}{2},\frac{3}{2};\frac{\bar{\gamma}_1(1-\rho)}{\xi_1},\frac{\bar{\gamma}_2(1-\rho)}{\xi_1}\right) \end{aligned} \quad (11)$$

where $\xi_1 = 2m + (1-\rho)(\bar{\gamma}_1 + \bar{\gamma}_2)$. Putting (11) in (10), the BER for correlated dual-diversity is

$$\begin{aligned} P_e &= \frac{1}{2} - \frac{(1-\rho)^{m+\frac{1}{2}}}{\sqrt{\pi\xi_1}\Gamma(m)}\sum_{k=0}^{\infty}\frac{\rho^k}{k!}\Gamma\left(\frac{1}{2}+k+m\right)\left(\sqrt{\bar{\gamma}_1}+\sqrt{\bar{\gamma}_2}\right) \\ &\quad \times \left[F_A\left(\frac{1}{2};\frac{1}{2}-k-m, 1-k-m;\right.\right. \\ &\quad \left.\left.\frac{1}{2},\frac{3}{2};\frac{\bar{\gamma}_1(1-\rho)}{\xi_1},\frac{\bar{\gamma}_2(1-\rho)}{\xi_1}\right)\right] \end{aligned} \quad (12)$$

which can be shown to be identical to (21) of [6] for BPSK modulation.

3.1.2 Independent reception: For the case when signals received at the three antennas are independent (i.e., $\rho = 0$), it can be shown that (10) reduces to (35) in [1] (with $m_1 = m_2 = m_3 = m$) which is the probability of error for BPSK signals for predetection EGC receiver with independent received signals using three antennas. For Rayleigh fading, when signals received at the three antennas are independent (i.e. $\rho = 0$, $m = 1$), it can be shown that (10) reduces to (20) in [8].

4 Numerical results and discussion

BER expression (10) has been numerically evaluated for different values of ρ and m as a function of average branch SNR (assuming $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_3 = \bar{\gamma}$) for $L = 3$ and $L = 2$ (correlated dual-diversity). This is shown in Fig. 2. The number of terms included in the numerical evaluation of the infinite series in (10) ensures an accuracy to the seventh significant digit. Clearly, as expected, performance of the system with $L = 3$ is better than the dual-diversity case. In Fig. 3 we plot $\text{BER}|_{L=2} - \text{BER}|_{L=3}$ i.e. the reduction in BER caused by the third (independent) antenna. We observe that the reduction in BER is more as ρ increases for a given m. The reason for this is the fact that for higher ρ the performance of a correlated dual-diversity receiver degrades and hence the contribution of the independent antenna improves the BER performance more than the case when ρ is low. The Figure also shows that the reduction in BER is more in the low SNR range and less in the high SNR range. We know that, in general, BER reduces with increase in SNR and hence this is not surprising.

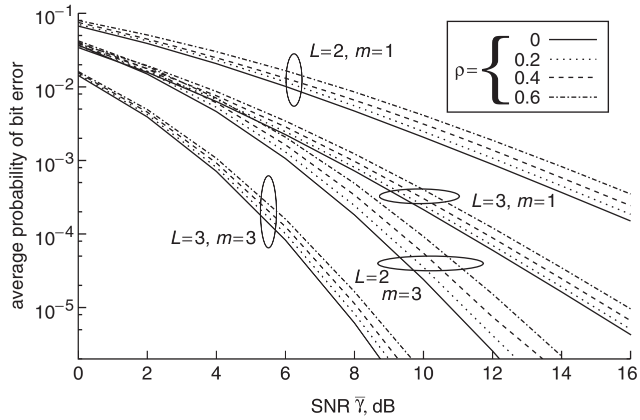

Fig. 2 *BER of a predetection EGC receiver in a Nakagami-m fading channel*

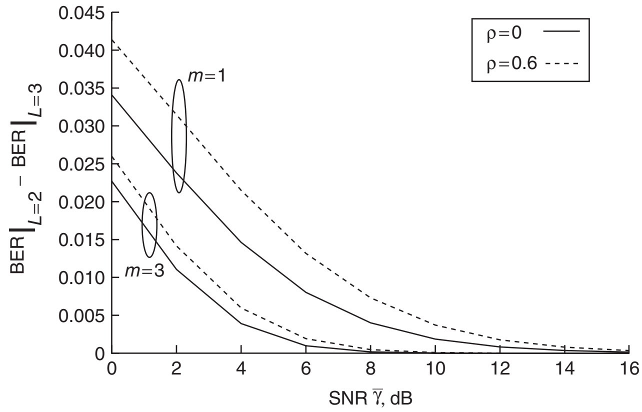

Fig. 3 *$\text{BER}|_{L=2}$—$\text{BER}|_{L=3}$ against SNR*

To obtain a better overall picture we also plot percentage improvement η_1 as defined below:

$$\eta_1 = \frac{\text{BER}|_{L=2} - \text{BER}|_{L=3}}{\text{BER}|_{L=2}} \times 100 \quad (13)$$

This plot has been shown in Fig. 4. We observe the η_1 increases with SNR. The curves for $\rho = 0$ and $\rho = 0.6$ have significant overlap implying that η_1 is almost independent of ρ. This is somewhat surprising and we investigate it further. Equation (13) can be rewritten as follows:

$$\eta_1 = \left(1 - \frac{\text{BER}|_{L=3}}{\text{BER}|_{L=2}}\right) \times 100 \quad (14)$$

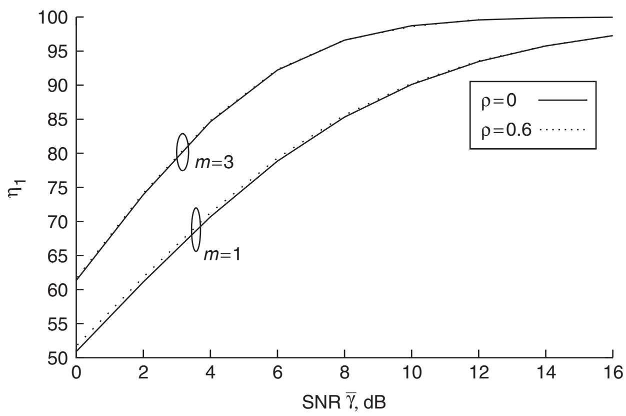

Fig. 4 *η_1 against SNR*

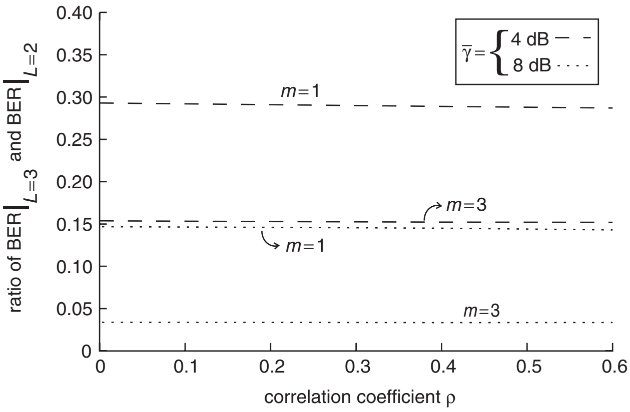

Fig. 5 *Ratio of $BER|_{L=3}$ and $BER|_{L=2}$ against ρ*

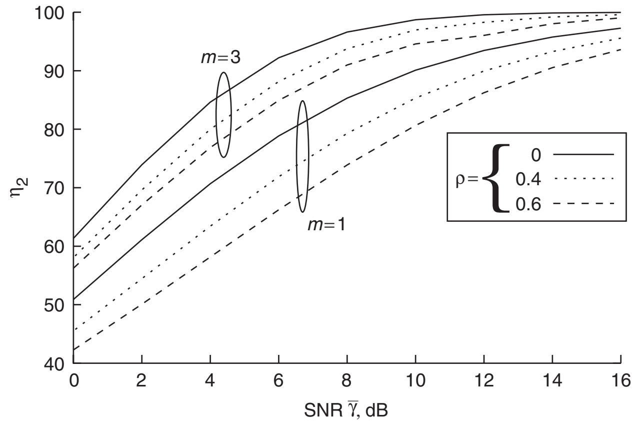

Fig. 6 *η_2 against SNR*

From the above equation it is clear that if η_1 is almost independent of ρ then the same should hold for the ratio of BERs for $L=3$ and $L=2$. In Fig. 5 we plot this ratio as a function of ρ, for several SNRs, and observe that this is indeed true. Hence, the underlying reason for the near independence of η_1 with respect to ρ is the fact that as ρ changes, BERs for $L=2$ and $L=3$ change by nearly the same factor. From Fig. 5 we also observe that for a given m this ratio decreases with increase in SNR, implying that with increase in SNR, $\mathrm{BER}|_{L=3}$ decreases at a faster rate than $\mathrm{BER}|_{L=2}$. This is also corroborated by Fig. 2.

Lastly, we plot the percentage improvement η_2 obtained by the three antenna system with respect to independent dual-diversity system ($L=2$, $\rho=0$) with η_2 defined as

$$\eta_2 = \frac{\mathrm{BER}|_{L=2,\rho=0} - \mathrm{BER}|_{L=3}}{\mathrm{BER}|_{L=2,\rho=0}} \times 100 \qquad (15)$$

where $\mathrm{BER}|_{L=3}$ can be computed for any value of ρ between 0 and 1. Note that for $\rho=0$, η_2 becomes identical to η_1. In Fig. 6 we have shown η_2 for $\rho=0$, $\rho=0.4$ and $\rho=0.6$. It can be observed from the curves that the maximum improvement occurs for $\rho=0$ and η_2 decreases with increase in ρ. This happens because $\mathrm{BER}|_{L=3}$ increases with ρ and hence the numerator of (15) decreases as ρ increses.

5 Conclusions

BER performance of a predetection EGC receiver using a three antenna configuration in a Nakagami-m fading channel has been analysed and a closed-form expression has been obtained. Numerical results have been plotted against average branch SNR. Results show that the percentage improvement in BER is more for higher SNRs. Further, percentage improvement η_1 is almost independent of the correlation between the correlated pair of antennas. The reason for this is that as ρ changes for a given SNR, BERs for $L=2$ and $L=3$ change by nearly the same factor.

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