

Optimal Bandwidth Allocation to Coding and Spreading in DS-CDMA Systems Using LMMSE Front-End Detector

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Abstract—In code-division multiple-access (CDMA) systems, it is interesting to study the optimal bandwidth allocation to coding and spreading in order to maximize the number of users that the system can accommodate. This optimal bandwidth allocation is referred to as the optimal allocation point (OAP). In this brief, a practical CDMA system with a fixed total-bandwidth expansion factor that employs convolutional codes and random spreading is considered. The receiver consists of a multiuser linear-minimum-mean-square-error (lmmse) detector front end followed by autonomous single-user decoders. Intuitive reasoning is applied for the existence of an OAP in such a system. Also, we present a theoretical formulation to estimate the OAP in the aforementioned system. Simulations confirm the correctness of results obtained. Further, the paper investigates system behavior at different values of information-bit signal-to-noise ratio (SNR). At high values of SNR, within the range considered, the system favors spreading only. However, at relatively lower SNRs, channel coding is required to improve the system performance, and it is important that we operate at the OAP obtained.

Index Terms—Bandwidth allocation, convolutional codes, direct-sequence code-division multiple-access (DS-CDMA), linear-minimum-mean-square-error (lmmse) receiver.

I. INTRODUCTION

IN A direct-sequence code-division multiple-access (DS-CDMA) system, both spreading and channel coding increase the bandwidth occupied by the modulated signal. When system constraints dictate a fixed total-bandwidth expansion factor, the natural question of how to allocate it between spreading and coding arises. We endeavor to determine optimal bandwidth allocation to coding and spreading in order to maximize the number of users that the system can accommodate. We call this optimal bandwidth allocation as optimal allocation point (OAP). It has been referred to as “coding–spreading tradeoff point” in [1] and [4]. The system behavior is expected to be dependent upon the information-bit signal-to-noise ratio (SNR).¹ This brief reports how the value of OAP changes with SNR. The results obtained here are important from the point of view of deciding which code rate to operate on, at different values of SNR, in a practical DS-CDMA system when we are

using convolutional encoders and random spreading together with linear-minimum-mean-square-error (lmmse) front-end detectors.

Before we set out to find the optimal allocation of bandwidth to “coding” and “spreading,” we need to define these terms precisely. In [3], Massey uses the concept of Fourier and Shannon bandwidth to define and differentiate these two forms of bandwidth expansion. Veeravali and Mantravadi make use of these definitions in [4] and predicts the existence of a coding–spreading tradeoff point while using ideal codes [1], random spreading, and an lmmse front-end receiver. We extend this work to the case of practical codes, such as convolutional codes in Section III. Here, we are able to conclude that OAP will occur at a lower code rate in the case of a convolutional code as compared to ideal codes.

The background needed for dealing with such a problem has already been developed in works like [1], [6], and [11]. Verdú and Shamai, in [6], discuss the spectral efficiency of CDMA systems under asymptotic conditions. However, they do not consider any specific channel coding scheme in conjunction with random spreading. As mentioned in [1], and further discussed in Section III, for practical coding schemes, finding the maximum number of users so that the code rate is as close as possible to the capacity is equivalent to finding the maximum number of users so that the probability of error remains below a certain desired threshold. In Section IV, we present an approach to obtain the maximum number of users that can be accommodated at a given code rate. The advantage of this approach over solutions presented in [1] and [11] is that it does not require the computation of the weight-distribution function (WDF) of convolutional codes. Besides, [11] uses a large-system assumption and relies only on numerical computation. Here, in Section V, we present simulation results for a non-asymptotic (finite bandwidth) system to verify the technique presented in Section IV. Further, the dependence of OAP on signal strength has been a relatively ignored although important aspect. Sections IV and V make some interesting observations on behavior of the system at different values of information-bit SNR. At high values of SNR, in the range considered, the system favors spreading only, but at relatively lower SNRs, channel coding can significantly improve the system performance.

II. SYSTEM MODEL

We consider the synchronous DS-CDMA system as shown in Fig. 1. Each user information bit undergoes channel coding and then spreading. The channel is assumed to be an AWGN channel. The receiver consists of an lmmse front-end detector followed by an autonomous single-user decoder.

Manuscript received September 6, 2003; revised November 14, 2004; accepted November 22, 2004. The editor coordinating the review of this paper and approving it for publication is N. Mandayam.

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Digital Object Identifier 10.1109/TWC.2005.858031

¹Throughout this brief, “SNR” means “information-bit SNR.”

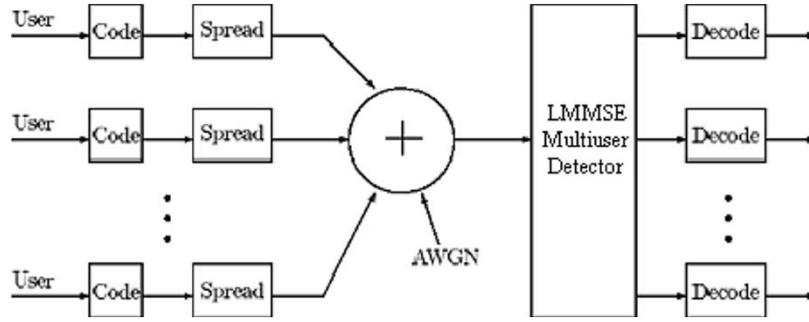


Fig. 1. System model.

We assume that all the K users have equal power, spreading factor N , and employ the same encoder. Although the spreading sequences are chosen at random, the lmmse detector has full knowledge of it. We use the standard discrete synchronous CDMA-system model [4], [9]. Let γ_b be the information-bit SNR and γ_s the code-symbol SNR. Also, Ω denotes the total bandwidth expansion factor available. If we denote the code rate by ν , then the following relations hold

$$\gamma_s = \nu\gamma_b \tag{1}$$

$$\Omega = \frac{N}{\nu}. \tag{2}$$

Note that at a fixed Ω , fixing ν implies fixing N . It has been shown in [2] and [5] that effective interference (multiple access interference plus noise) at the output of the lmmse detector can be assumed to be Gaussian when we use random spreading. This Gaussian assumption is valid for “asymptotic systems” and closely approximates the interference in the case of nonasymptotic systems. By “asymptotic systems,” we mean that both the number of users K and the spreading factor N approach infinity with the ratio K/N approaching a constant value α . Thus, the effective-single-user (ESU) channel seen by each user, i.e., the channel consisting of the spreading block, the AWGN channel, and the multiuser detector can be assumed to be a Gaussian channel.

Tse and Hanly [2] have computed an expression for signal-to-interference ratio (SIR) at the output of the lmmse detector when using random spreading in asymptotic systems as follows:

$$\text{SIR} = (1 - \alpha)\gamma_s - \frac{1}{2} + \sqrt{(1 - \alpha)^2\gamma_s^2 + (1 + \alpha)\gamma_s + \frac{1}{4}} \tag{3}$$

where

$$\alpha = \frac{K}{N}.$$

In this brief, however, we are concerned with nonasymptotic systems (finite Ω). Using simulations, it has been shown in [5] that the SIR expression in (3) closely approximates the SIR values for nonasymptotic systems using random spreading.

III. CONCEPTUAL FOUNDATION

Having understood the system model, let us develop the theoretical background required to deal with the problem and try to draw certain conclusions about it. In [3], Massey gives the definition of “coding” as a mapping for which the Fourier bandwidth and the Shannon bandwidth are equal. While using ideal codes, which comply with this definition, ESU channel capacity is the upper bound on the code rate of the encoder for reliable communication. In [4], Veeravali and Mantravadi make use of this to plot the curve of the maximum number of users K_{\max} that system can accommodate against code rate.

From the numerically computed plot of K_{\max} against ν in [4], using ideal codes and random spreading, the OAP can be seen to exist at a code rate close to one. If we use practical convolutional codes instead of ideal codes, then we should expect the OAP to exist at lower values of the code rate. This is because a practical coding scheme will have its Fourier bandwidth greater than its Shannon bandwidth [3]. Thus, a practical coding scheme has a certain spreading component in it, and hence, is actually equivalent to a higher rate ideal coding. To conclude, conceptually we expect the K_{\max} versus ν plot for convolutional codes to be something similar to what has been sketched in Fig. 2.

But we cannot plot K_{\max} against ν for practical convolutional codes like the way we did for ideal codes. It will require us to know how close to the capacity can the code rate ν get, in the case of convolutional codes keeping the communication reliable. This in turn will require us to quantify the amount of “spreading” present in them. As discussed in [1], K_{\max} at a code rate ν is equivalent to the maximum number of users such that the probability of information-bit error P_b remains below a threshold P_{thres} for reliable communication. Note that the probability of information-bit error is actually a function of encoder parameters and multiuser detector-output SIR, which in our multiuser CDMA model depends on K and N .

IV. THEORETICAL FORMULATION

In light of the discussion in the previous section, we need to know the expression for probability of information-bit error P_b in the case of the convolutional coding scheme. The exact expression is difficult to compute. We build upon the approach

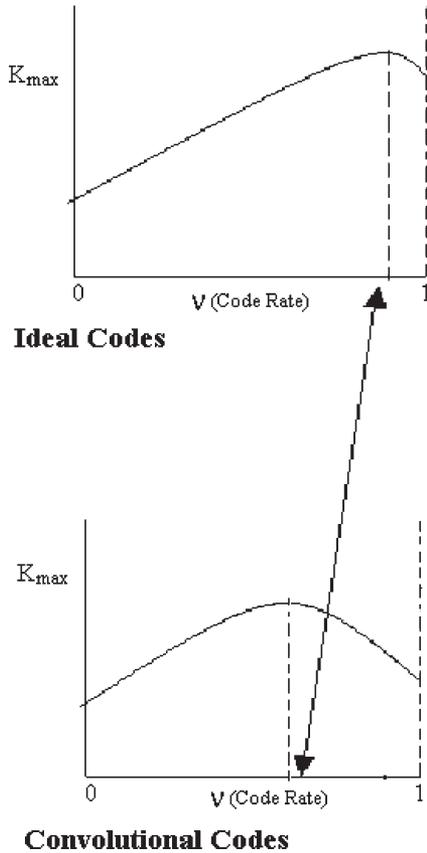


Fig. 2. Anticipated “left shift” of maxima in the case of convolutional codes as compared to ideal codes.

taken in [1] and use a union bound. For a rate $\nu = m/n$ convolutional encoder, the union-bound expression can be given by

$$P_b \leq \sum_{d=d_{free}}^{\infty} P(b|d)P(d)$$

where $P(d)$ is the probability of decoding to a codeword at a Hamming distance d from the transmitted codeword and $P(b|d)$ is the probability of bit error given that the error event with a code word at distance d has occurred. The free distance of the code is d_{free} . Again taking the union-bound approach, we can say that $P(b|d)$ is upper bounded by the expression given as follows:

$$P(b|d) \leq \sum_{l=1}^{w_d} P(b|l, d)$$

where w_d is the number of codewords at distance d . Also, $P(b|l, d)$ is the probability of bit error given that the error event with the l th codeword at distance d has occurred. On the average, $P(b|l, d)$ will be the ratio of the number of erroneous bits to the total number of bits transmitted. Keeping this in mind, we can write

$$P(b|l, d) \leq \beta_{dl}$$

where

$$\beta_{dl} = \frac{b_{dl}}{mL_{dl}} \tag{4}$$

where b_{dl} denotes the Hamming distance between the transmitted bits and the detected bits associated with the l th codeword at distance d . Also, L_{dl} is the number of trellis branches over which the l th codeword at distance d differs from the transmitted codeword. Thus, the final expression for the upper bound on P_b is given by²

$$P_b \leq \sum_{d=d_{free}}^{\infty} P(d) \sum_{l=1}^{w_d} \beta_{dl}. \tag{5}$$

In order to know the parameters in the above expression, we are required to compute the WDF of the code. An estimate of the performance can be obtained by considering only the codeword at free distance. Thus,

$$P_b \cong P(d_{free}) \sum_{l=1}^{w_{d_{free}}} \beta_{d_{free}l}. \tag{6}$$

In the case of hard decision decoding, we replace $P(d_{free})$ by the Chernoff bound given by [7]

$$P(d) \leq [4p(1-p)]^{\frac{d}{2}}$$

where p is replaced by its bound, which is given as

$$p = Q(\sqrt{SIR}) \leq e^{-\frac{SIR}{2}}.$$

The SIR here refers to the ratio at the output of the lmmse detector, that is, at the input of the decoder. The crossover probability for the binary symmetric channel is represented by p and can be written as above because the ESU channel can be effectively considered to be Gaussian, as explained in Section II.

For soft decision decoding [7]

$$P(d_{free}) = e^{-\frac{(SIR)d_{free}}{2}}.$$

In [1], Motani and Veeravali propose the use of a method given in [8] to calculate WDF so that factors such as $\beta_{d_{free}}$ and $w_{d_{free}}$ can be computed. But the computation of WDF becomes increasingly complex with the increase in constraint length of the convolutional encoder. In order to simplify the computation, we propose the following approximation.

- 1) $w_{d_{free}} = 1$.
- 2) $\beta_{d_{free}} = b/mL$.
- 3) $L =$ the constraint length of the encoder.
- 4) $b =$ code rate $\times d_{free}$.

This approximation is motivated by the following observations. The dominant term in (6) is $P(d_{free})$, since it is exponential. If we have a reasonable estimate of the remaining term, it should give us a working expression. To avoid the difficulty of calculating $w_{d_{free}}$, we assume it to be constant with a value of 1 for all the codes. This is motivated by the fact that $w_{d_{free}}$ is small for good codes (look at the list of convolutional codes given in [12]). Results obtained later show that it is not a bad

²A more detailed analysis taking a similar approach is done in [7] and [10].

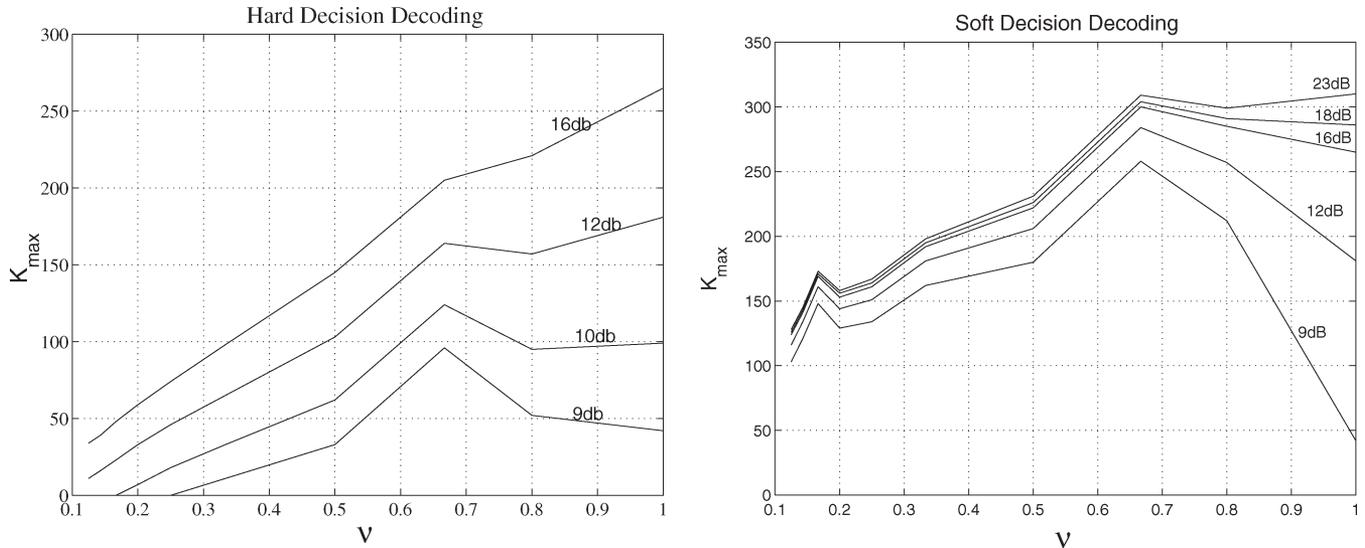


Fig. 3. Numerically computed curves for different values of information-bit SNR.

 TABLE I
 LIST OF ENCODERS

Rate	Constraint Length	Generators in Octal	d_{free}
1/8	5	37 33 25 25 35 33 27 37	32
1/7	5	35 27 25 27 33 35 37	28
1/6	5	37 35 27 33 25 35	24
1/5	5	37 27 33 25 35	20
1/4	5	25 27 33 37	16
1/3	5	25 33 37	12
1/2	5	23 35	7
2/3	4	236 155 337	7
4/5	2	237 274 156 255 337	3

approximation to make. Since $L_{d_{free}}$ is close to the constraint length of the encoder [10], we approximate it by L . The last equation gives a crude estimate of $b_{d_{free}}$.

Now at each code rate, for the given encoder design, we can compute minimum permissible SIR so that the expression in (6) does not exceed the set threshold. From (3), we have that at a fixed code rate (hence a fixed spreading factor N), SIR decreases with the increase in the number of users K . Hence, once we have the minimum SIR value, we can compute the maximum permissible number of users using the expression given in (3). This gives the maximum number of users K_{max} that system can accommodate at each code rate ν , i.e., at each distribution of bandwidth to coding and spreading. Hence, we can estimate the optimum allocation of bandwidth using this technique. This means that given a set of different rate encoders, we can use this technique to decide upon which encoder to use. Note that while choosing the optimal code rate, the constraint length of the encoders should be kept identical so that their decoding complexity is approximately the same. Otherwise, it would not make much sense to compare the different code-rate encoders.

Fig. 3 plots K_{max} against the code rate ν using the technique described above and uses the encoders listed in Table I. We have taken $P_{thres} = 5 \times 10^{-3}$ for hard as well as soft decision

decoding. Note that we have tried to keep the encoder constraint length approximately constant. The technique, because of the approximations involved, gives meaningful results only for information-bit SNRs of 9 dB and above while working with encoders listed in Table I. Hence, we consider information-bit SNR in the range of 9–23 dB for plotting the curves using this technique. Also, the total bandwidth expansion factor Ω is taken to be 300. Consistent with the intuitive conclusions made in Section III, the technique estimates that the OAP exists for convolutional codes and lies at a code rate that is much smaller than the optimal code rate (which is close to one) in the case of ideal codes [4]. The OAP is estimated to be at a code rate of 2/3 at low values of SNR, in the range considered, for hard as well as soft decision decoding. At relatively high values of SNR, the curve straightens and the system favors spreading only, in order to maximize the number of users. The maxima in the curve comes out to be more prominent when we do soft decision decoding, which is consistent with our understanding of the problem, as we now explain. For a code rate of one, whether we do soft or hard decision decoding, we will get the same result; but at intermediary values of the code rate, using soft decision decoding, we will be able to accommodate a greater number of users compared to when we use hard decision decoding. Thus, in Fig. 3, if we consider the case of 12-dB information-bit SNR, the curve of hard decision decoding is almost monotonically increasing, while that of soft decision decoding shows a clear maxima. In the next section, we go on to verify the obtained curves through simulations and comment on the nature of the curves.

V. SIMULATIONS

For simulation, we use the system design of Section II. The convolutional encoders used are the same as those used in the previous section. We assume that the spreading sequences are assigned to users at random when they enter the system but once assigned, the Immse detector has complete knowledge

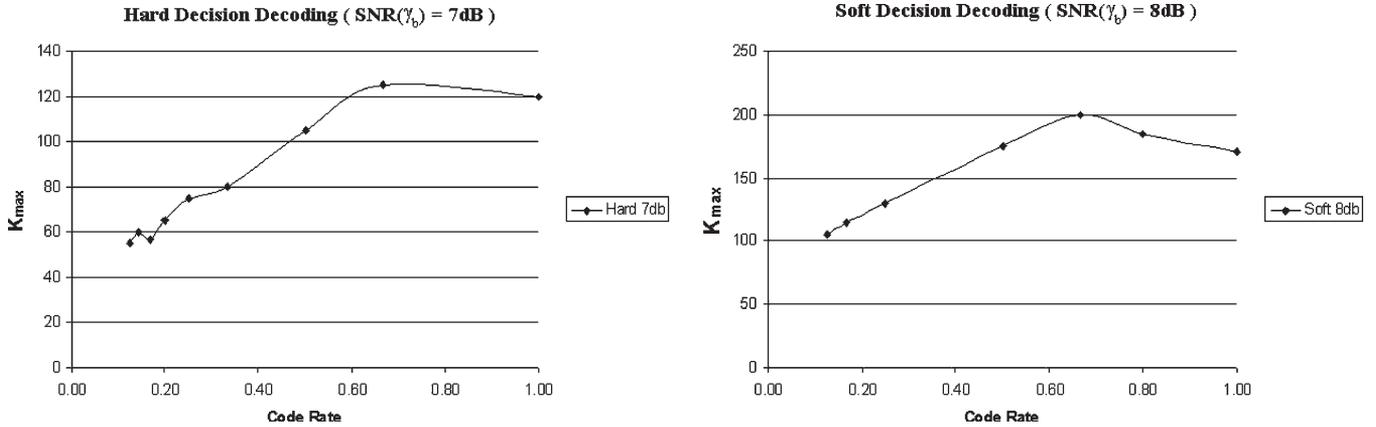


Fig. 4. Simulation results showing the presence of maxima at low values of SNR.

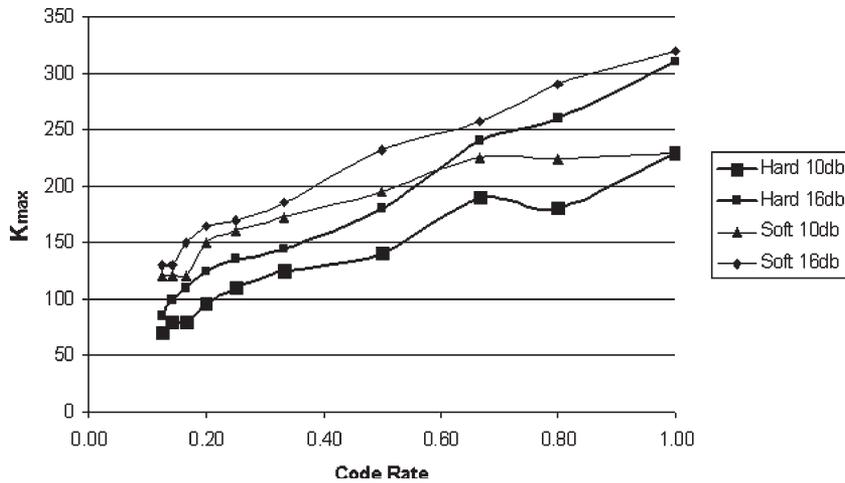


Fig. 5. Simulation results at relatively high SNR values of 10 and 16 dB for hard and soft decision decoding.

about them. The total-bandwidth expansion factor Ω is taken to be 300.

We have taken $P_{thres} = 5 \times 10^{-3}$ for hard as well as soft decision decoding. We find, through simulations, the maximum number of users at each code rate (that is, at each distribution of bandwidth to coding and spreading) such that the probability of information-bit error remains below P_{thres} . The user spreading code is not changed for a single run of the simulation program, during which the maximum number of users is determined using bit error rate averaged over 20 000 information bits. However, the system performance is averaged over 20 runs of the simulation program, and hence, over 20 spreading-code assignments. In order to capture the existence of OAP and the change in the nature of K_{max} versus ν curve as a function of SNR, it suffices to do simulations for SNR values of 7–16 dB. Figs. 4 and 5 sum up the simulation results. Simulations show that at relatively low values of information-bit SNR, in the range mentioned, the OAP exists and, as seen in Fig. 4, is located at code rate of $2/3$. This is consistent with the intuitive conclusions made in Section III about the existence of an OAP in the case of convolutional codes. At higher SNR, as seen in Fig. 5, the curve straightens and the system favors spreading only. The value of the OAP obtained and the pattern of the curves for different values of SNR is the same as anticipated by

the technique. This is true, although the simulation curves do not match the curves obtained in previous section for the exact value of information-bit SNR. This can be seen as a limitation of the numerical technique presented, as it involves approximate expressions. As expected, simulation results for higher values of SNR, such as 16 dB, show that the difference between the curves for soft and hard decision decoding decreases as we increase the SNR.

At low information-bit SNRs (around 8 dB), there exists an OAP, hence, it is important to operate at the optimum point so that the number of users in the system can be maximized. The curve straightens as the SNR increases and the system favors spreading only, in order to maximize the capacity in terms of the number of users. This behavior can be attributed to the interaction between convolutional encoder and lmmse detector. The lmmse detector makes use of knowledge of the spreading sequences in order to suppress multiple access interference. As the code rate increases, the encoder weakens, i.e., its capability to correct errors reduces. On the other hand, with increasing code rate, more bandwidth is allocated to spreading and the lmmse detector strengthens, i.e., it is able to suppress the multiple access interference more effectively as more degrees of freedom are available to it in the form of more chips. Hence at high SNR, with increasing spreading, the lmmse receiver is

able to suppress interference (multiple access interference plus noise) very efficiently, as it does not have to face much noise. Thus, at high SNR, it is able to accommodate more users by improving the SIR at the input of the decoder, even though the decoder weakens with increasing spreading. But at low SNR, since the noise is significant, lmmse is not able to suppress interference effectively, even with the increase in the spreading. Thus, with an increase in the code rate, beyond a point, the decoder weakens along with little improvement in the SIR at the input of the decoder (output of the lmmse), and thus, the number of users that can be accommodated falls. Hence, we get a maxima in the case of low SNR and a straight curve at high SNR.

VI. CONCLUSION

This brief gives intuitive reasoning for the existence of an optimum allocation point (OAP) of bandwidth between coding and spreading in DS-CDMA systems employing convolutional codes, random spreading, and a lmmse front-end detector by building upon the concepts involving ideal codes. We present a technique to predict the OAP. This means that given a set of different rate encoders, we can use this technique to decide which encoder to use. We also make important conclusions about optimal bandwidth allocation while operating at different SNRs. At low SNRs, system requires coding and the OAP is very prominent. Therefore, one should operate at this bandwidth allocation so as to maximize the performance of the system. At relatively higher SNRs, the system favors spreading only. The system behavior is dependent upon the kind of front-end detector employed. It would be interesting to plot similar curves with a matched filter or a decorrelator at the front end and to compare the system performance at different values of SNR.

ACKNOWLEDGMENT

This work was done at Department of Electrical Engineering, Indian Institute of Technology, Kanpur, India.

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