A Hybrid Time Divisioning Scheme for Power Allocation in DMT-based DSL Systems

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***Abstract*— We propose an Iterative Waterfilling based multiuser Hybrid Time Divisioning (HTD) scheme for power allocation in DMT based DSL systems. The problem of finding the $(user, subchannel)$ pairs which should transmit alone and the pairs which should transmit along with other users, so as to result in the maximum aggregate data rate achievable using time divisioning, has been considered. The proposed scheme is a low complexity, sub-optimal solution to this problem. Results show that the HTD scheme can achieve better data rates than Iterative Waterfilling as well as Multiuser Discrete Bit Loading scheme (for high coupling coefficient values) with comparatively lesser complexity.**

***Index Terms*— Discrete multi-tone, time divisioning, digital subscriber line, dynamic spectrum management, multicarrier systems.**

I. INTRODUCTION

OVER recent years Digital subscriber line (DSL) technology has become an attractive broadband access service for residential and business areas. However the performance of DSL systems is constrained by crosstalk due to the electromagnetic coupling amongst the multiple twisted pairs making up a phone cable.

In existing power allocation algorithms [1], [4] for DMT based DSL systems, users transmit simultaneously. If the crosstalk channel gains between them are high, SINR values are affected, resulting in decreased data rates. This suggests the need for investigating a Time Divisioning approach. A possibility of using time divisioning for a 2 user, 2 subchannel case was studied in [2] and the utility of time divisioning was established. When we go for a time divisioning approach for the general M user, N subchannel case we have to find out the $(user, subchannel)$ pairs (denoted as (U, S)) which should transmit alone in a subchannel (i.e. no other user transmitting in that subchannel for a given time slot duration) and the (U, S) pairs which should transmit alongwith other users, so as to result in the maximum aggregate data rate achievable using time divisioning. We also need to determine the power that should be transmitted by different users in various subchannels. The Hybrid Time Divisioning (HTD) scheme proposed in this paper presents a low complexity, iterative waterfilling based sub-optimal solution to this problem.

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The paper is organized as follows. Section II reviews the DSL environment and models it as an interference channel. Section III introduces and explains the HTD scheme. Simulation results are presented in Section IV. We conclude the paper in Section V.

II. SYSTEM MODEL

Existing power allocation algorithms for DMT based DSL systems can be classified into three categories - Distributed, Semi-coordinated and Fully Coordinated. Distributed algorithms like Iterative Waterfilling (IW) [4] do not require the knowledge of channel crosstalk functions and power allocation is done by considering all the incoming interference from other users as channel noise. On the other hand, Fully Coordinated algorithms like Vectored DMT [5] require coordination among the receivers at the central office which is not feasible in the current loop topology of DSL systems. Semi-coordinated algorithms [1] [3] unlike the above two cases require the presence of a centralized Spectrum Management Center (SMC) which on the basis of some collected information such as channel crosstalk functions, controls the power allocation process. Since the proposed HTD scheme considers the presence of a Centralized Scheduler for calculating a threshold $\gamma_{\mathbf{t}}^*$ (to be discussed later), it also falls under the category of semi-coordinated algorithms.

We know that when the DMT technique is used with synchronized receivers, a DSL channel can be modeled as N independent, frequency non-selective subchannels, each of which is an interference channel of M users. With all users transmitting simultaneously the SINR value of user i in subchannel j is given by

$$\gamma_j^i = \frac{P_j^i}{N_0 W + \sum_{k=1 k \neq i}^{M} \beta_{ki}^j P_j^k} \tag{1}$$

where P_j^i denotes the power of user i in the jth subchannel, β_{ki}^j is the coupling coefficient value of user k in subchannel j of user i, W is the bandwidth of each subchannel and N_0 is the noise power per Hz. We assume a maximum power constraint for each user. Thus for the ith user we have $P_1^i + P_2^i + \cdots + P_N^i \leq P_{max}^i$. Further, a fixed total bandwidth is considered which is distributed equally amongst all subchannels of a user.

III. HYBRID TIME DIVISIONING SCHEME

In the proposed HTD scheme we divide all (U, S) pairs into two classes. One class comprises of pairs which transmit alone in a subchannel and each pair transmits for a fraction

$\alpha = \frac{1}{M}$ of total time. The maximum achievable data rate for such a pair say (i, j) is given by

$$R_j^i = \alpha W \log_2(1 + \gamma_{jA}^i) \tag{2}$$

where

$$\gamma_{jA}^i = \frac{P_j^i}{N_0 W} \tag{3}$$

We term these pairs as $Class\ A$ or CA pairs. It can be seen that the users which transmit alone for α fraction of time neither face any interference from other users nor create any interference for them thereby improving their own as well as the other users SINR values.

Pairs belonging to the second class transmit simultaneously, alongwith other users of the same class and are termed as $Class\ B$ or CB pairs. Since in a particular subchannel we can have users belonging to both classes we call the proposed scheme as Hybrid.

We define an $M \times N$ matrix Λ as $\Lambda_{ij} = 1$, if (i, j) is a CA pair and $\Lambda_{ij} = 0$, if (i, j) is a CB pair. The quantity $\sum_{i=1}^{M} \Lambda_{ij}$ gives the total number of users whose jth subchannels are in Class A. Class B users transmit simultaneously for a fraction $\Upsilon = [1 - \alpha \sum_{i=1}^{M} \Lambda_{ij}]$ of total time. Thus for a CB pair (k, l) the maximum achievable data rate can be given by

$$R_l^k = \Upsilon W \log_2(1 + \gamma_{lB}^k) \tag{4}$$

where

$$\gamma_{lB}^k = \frac{P_l^k}{N_0 W + \text{Interference from CB pairs}} \tag{5}$$

Having specified the rates and SINR's of Classes A and B, we need to come up with a scheme to decide how to divide all (U, S) pairs into these two classes. It is easy to see that a (U, S) pair with low SINR value should be put into Class A while a (U, S) pair with a high SINR value needs to be put in Class B. For this purpose we introduce a variable γ_t representing a threshold SINR value with which the SINR value of each (U, S) pair can be compared for selecting CA and CB pairs. This comparison is repeated with γ_t varying from 0 to a maxima γ_{max} (discussed later) to find out γ_t^* representing that value of γ_t which results in the maximum achievable aggregate data rate i.e. maximum value of R_{agg}, where R_{agg} is given by

$$R_{agg} = \sum_{i=1}^{M} \sum_{j=1}^{N} R_j^i \tag{6}$$

It is difficult to obtain a theoretical solution for γ_t^*. This is because the aggregate data rate R_{agg} cannot be expressed as a simple function of γ_t, since for different values of γ_t the division of (U, S) pairs into classes A and B changes. Hence, to determine γ_t^* we go for a repetitive comparison by increasing γ_t in steps of ϵ as described below.

A. Algorithm

The complete operation of the proposed HTD scheme is described as follows:

1) Initialize $\gamma_t = 0$.
2) Initialize the matrix Λ as $\Lambda_{ij} = 0\ \forall\ (i, j)$ pairs.
3) Allocate power to each (U, S) pair using IW [4], considering SINR values for the case of all users belonging to Class B (i.e. all users transmitting simultaneously). Store the resulting γ_j^i (same as γ_{jB}^i) values in an $M \times N$ matrix Δ defined as $\Delta_{ij} = \gamma_{jB}^i\ \forall\ (i, j)$. Let γ_{max} represent the maximum SINR value over all (U, S) pairs.
4) Consider subchannel j of user i. If $\gamma_{jB}^i \leq \gamma_t$, make $\Lambda_{ij} = 1$ and let (i, j) be a CA pair, while if $\gamma_{jB}^i \geq \gamma_t$, let $\Lambda_{i,j}$ continue to be 0 and let (i, j) continue to be a CB pair. Repeat the step for all (i, j) pairs $(i = 1, 2, \cdots M, j = 1, 2, \cdots N)$.
5) For every CA pair (i, j) assign $\gamma_j^i = \gamma_{jA}^i$ using (3).
6) Again allocate power to all (U, S) pairs using IW [4] as per the modified SINR values.
7) With this power allocation calculate R_{agg} using (6).
8) Reinitialize γ_j^i as $\gamma_j^i = \Delta_{ij}\ \ \forall\ (i, j)$.
9) Repeat 4, 5, 6, 7, 8 with values of γ_t increasing in steps of size say ϵ, till we reach γ_{max}. Denote γ_t^* as that value of γ_t which results in maximum R_{agg}.

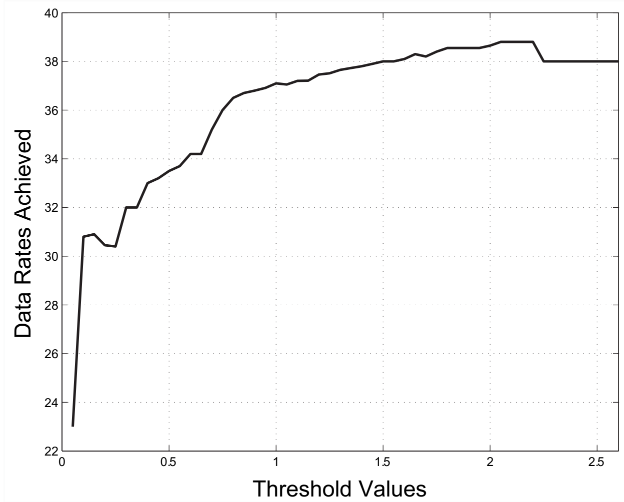

Fig. 1. Average data rate variation with γ_t.

Note that though the performance of our algorithm would depend upon coupling coefficient values but since we use IW [4] for power allocation, we do not require explicit knowledge of β values. Power allocation is done as per the SINR values which can be determined using physical measurements for each (U, S) pair. Fig. 1 shows a typical variation pattern of average data rates achieved per user with increasing γ_t values, for the case of 4 users each having 25 subchannels. We see that after $\gamma_t = 2.25$, average data rate becomes constant corresponding to the case of all (U, S) pairs behaving as CA pairs.

B. Why go for a Sub-Optimal Solution?

It can be observed that γ_t^* obtained in the previous section is not the optimal solution for getting the maximum aggregate data rate achievable using time divisioning. The maximum aggregate data rate can be obtained by comparing all the possible 2^{MN} combinations (MN pairs, each having 2 possibilities of either being a CA pair or a CB pair) and selecting the best out

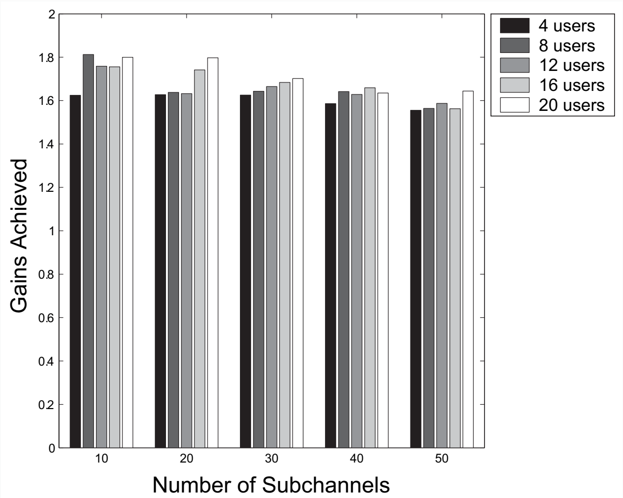

Fig. 2. Hybrid Scheme vs IW.

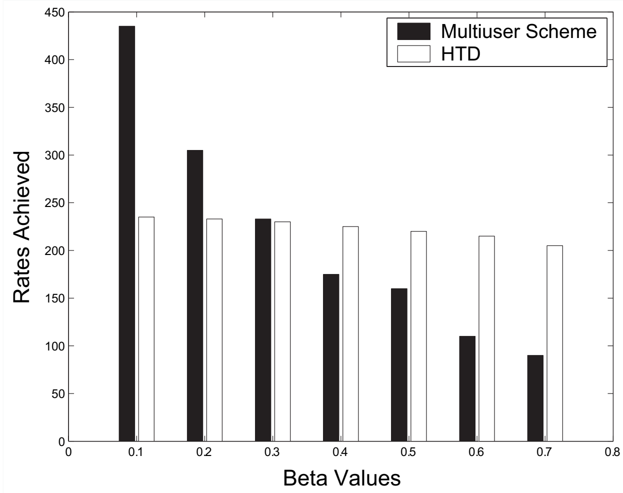

Fig. 3. Comparison with multiuser scheme.

of these. But for practical cases of say 50 users each with 256 subchannels this would require comparing 2^{50*256} possible combinations which is clearly infeasible.
But then why not at least go for a separate γ_t^* for each subchannel, instead of a common γ_t^* as assumed in HTD? Let us examine the problem in this approach. Each time we find out a γ_{ti}^* for the subchannel i and do power allocation by IW [4] as per modified $SINR$ values, γ_{tj}^*, $(j \neq i)$ values for other subchannels get altered. This leads to an iterative process for finding out an optimal set of γ_{ti}^*'s (one for each subchannel) and raises questions about its convergence.

Hence we chose for a suboptimal approach which assumes a common γ_t^* for all subchannels as this approach neither leads to any convergence issues, nor has a huge complexity. In fact we need to compute R_{agg} only $\frac{\gamma_{max}}{\epsilon}$ times, while each computation of R_{agg} involves only MN computations of data rate. We do not have a theoretical solution for γ_t^* and the accuracy in estimation of γ_t^* would depend upon the value of ϵ chosen.

IV. Simulation Results

For the purpose of simulations, the coupling coefficients have been taken to be uniformly distributed over 0 to 1. The values of N_0 , W and ϵ are assumed as 0.01 $\frac{W}{Hz}$, 150 Hz and 0.01 respectively. Further, a maximum power of 100 W is distributed amongst all the subchannels of a user. Fig. 2 shows the gains (mean gain for 20 different sets of coupling coefficient (β) values) obtained by HTD scheme over IW [4]. It has been plotted for 4, 8, 12, 16 and 20 users. In each case 10, 20, 30, 40 and 50 subchannels have been considered. We see that HTD performs consistently better than IW over all the cases considered.

The proposed scheme cannot perform worse than IW [4] as in the extreme case of very low coupling coefficients and high γ_j^i values, $\gamma_{\mathbf{t}}^*$ comes out to be 0 in which case all (U,S) pairs would become CB pairs and would transmit simultaneously thereby converging to IW [4] scheme. Similarly for very low γ_j^i values, $\gamma_{\mathbf{t}}^*$ becomes γ_{max} thereby making all (U,S) pairs as CA pairs which corresponds to the case of complete time divisioning.

In its original form, Multi User scheme in [1] returns the minimum power required for each user, in order to achieve a given data rate combination. What we would like to see, instead, is the maximum aggregate rate that is achievable for a given total power constraint per user. To this end, we implement a modified version of [1], allowing us to compare it with the proposed HTD scheme. Fig. 3 gives the variation in data rates obtained by the two schemes i.e. the HTD scheme and the modified form of Multi User scheme with increasing coupling coefficient values for the case of 5 users, each with 50 subchannels. It can be seen that the Multi User scheme achieves a higher data rate than HTD scheme for low β values. But for moderate and high values of β the converse is true and HTD yields better data rates than Multi User scheme.

V. Conclusion

The proposed HTD scheme gives a low complexity, sub-optimal solution to the problem of finding out the $(user, subchannel)$ pairs which should transmit alone in a subchannel and the pairs which should transmit along with other users so as to result in the maximum aggregate data rate achievable using time divisioning. Using simulations it was shown that the proposed scheme can achieve better data rates than Iterative Waterfilling as well as Multi User Discrete Bit Loading scheme (for high coupling coefficient values) with comparatively lesser complexity.

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