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Bit-Level Reduced Neighborhood Search for Low-complexity Detection in Large MIMO Systems

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***Abstract*—Neighborhood search algorithms (NSAs) reduce the detection complexity in a multiple-input multiple-output (MIMO) system equipped with a large number of antennas. These algorithms iteratively search for the optimal maximal likelihood (ML) vector in a chosen neighborhood and therefore, their performance depends on the probability that the desired solution vector belongs to the neighborhood. An efficient choice of neighborhood vectors which are likely to reduce the ML cost, can in-turn reduce the complexity of the NSAs. To enable this, we propose a novel MIMO detection framework by representing the transmit symbols as a polynomial function of its constituent bits. We use this framework to propose: i) a bit-level extension for the minimum mean squared error detector to initialize neighborhood search and ii) a metric-based selection criteria to reduce the neighborhood size. Combining the two ideas, we re-frame the NSAs, namely, likelihood ascent search and reactive tabu search, and numerically show that the proposed approach significantly reduces the complexity without affecting the bit error rate.**

***Index Terms*—Detection, large MIMO, neighborhood search.**

I. Introduction

DRIVEN by the demand for high data rates, multiple-input multiple-output (MIMO) systems with a large number of antennas are becoming an indispensable part of upcoming wireless standards such as 5G, IEEE 802.11ah, IEEE 802.11ax etc. Such systems are popularly referred to as large or massive MIMO systems. The realization of such a system brings a lot of new challenges [1], [2], and designing a reliable and low-complexity detector is one of them.

In the large MIMO literature, neighborhood search algorithms (NSAs) like likelihood ascent search (LAS) [3]–[5] and reactive tabu search (RTS) [6], [7] have been proposed to address the large MIMO detection problem. These algorithms begin with an initial solution vector (either obtained using a linear detector or generated randomly) and then iteratively search for a better solution vector in the neighborhood of the current solution vector. Their error performance and complexity are largely determined by the number of candidate vectors in the neighborhood. We note that with N_t transmit antennas and M-QAM constellation, the neighborhood consists of $(\sqrt{M}-1)^{\mathfrak{L}}\binom{2N_t}{\mathfrak{L}}$ candidate vectors [3], when the candidate neighbors differ with the current solution vector at $\mathfrak{L}$ elements for a real-equivalent MIMO system. For 1-LAS [4], the value of $\mathfrak{L}=1$ and for multistage LAS (MLAS) [3], $\mathfrak{L}\geq 2$.

Although the NSAs exhibit a good performance-complexity trade-off, their complexity can be significantly reduced, if only those vectors are included in the neighborhood, which are likely to reduce the optimal maximum-likelihood (ML) cost. We proposed this reduced neighborhood approach in [8] wherein we identified a few vectors (say $K \ll 2N_t$) which are likely to be in error and generated a reduced neighborhood set with only $(\sqrt{M}-1)^{\mathfrak{L}}\binom{K}{\mathfrak{L}}$ candidate vectors.

The approach in [8], while reducing the candidate vectors, only modifies the first term from $\binom{2N_t}{\mathfrak{L}}$ to $\binom{K}{\mathfrak{L}}$; it, however, does not touch the second term $(\sqrt{M}-1)^{\mathfrak{L}}$. This term arises due to multiple alternative candidate vectors available at the symbol level. For example, if we consider the current solution vector as $\mathbf{x}=[-1\ 3\ -3]$ and assume that the symbols of $\mathbf{x}$ takes values from the following constellation set $\{-3,-1,1,3\}$ with $\sqrt{M}=4$. Then there will be three candidate vectors if we flip only the first symbol of this vector ($\mathfrak{L}=1$) i.e., $[-3\ 3\ -3]$, $[1\ 3\ -3]$ and $[3\ 3\ -3]$. This number will grow rapidly if we simultaneously flip multiple symbols or use a higher-order constellation set. However, if we can map the symbols to their constituents bits, and can identify the bits which are likely to be in error, we can reduce the number of candidate vectors as a bit can either be -1 or 1.

Motivated by this observation, the current work proposes bit-level NSAs, and the **major contributions** of the work can be summarized as follows.

1) We first map the transmit symbol vector into its constituent bit vector by using a polynomial function and then formulate an equivalent detection problem.
2) We then derive the equivalent bit-level minimum mean squared error (MMSE) detector to generate an initial vector.
3) We next propose a metric to identify the bits which, if flipped, causes maximum reduction in the ML cost.
4) We combine the above two ideas to propose a novel bit-level extension of three existing NSAs: 1-LAS [4], MLAS [3], and RTS [6], [7]. The simulations show that the proposed bit-level approach can scale down the neighborhood size compared to the symbol-level approaches in [3]–[8], and that too without compromising the bit error rate (BER). We also explain how the proposed ideas can be extended to a coded system. The current approach is different from [9] which generates an ordered list to optimize the number of candidates to reduce the detection complexity.

II. Preliminaries

Consider an $N_r \times N_t$ MIMO system

$$\mathbf{y}_c = \mathbf{H}_c\mathbf{x}_c + \mathbf{n}_c, \tag{1}$$

where $\mathbf{x}_c$ and $\mathbf{y}_c$ are the transmit and receive complex symbol vectors, respectively. The elements of $\mathbf{x}_c$ belong to an M-QAM

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constellation and normalized such that $\mathbb{E}\{\|\mathbf{x}_c\|^2\} = 1$, where $\mathbb{E}$ is the expectation operator. The matrix $\mathbf{H}_c$ is the complex channel gain matrix where each element $h_{ij} \sim \mathcal{CN}(0, 1)$ and the vector $\mathbf{n}_c$ is the additive white Gaussian noise vector with each of its element $n_i \sim \mathcal{CN}(0, \sigma^2)$. The above complex system model can be represented as an equivalent real system as

$$\underbrace{\begin{bmatrix}\mathbf{y}_r \\ \mathbf{y}_i\end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix}\mathbf{H}_r & -\mathbf{H}_i \\ \mathbf{H}_i & \mathbf{H}_r\end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix}\mathbf{x}_r \\ \mathbf{x}_i\end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix}\mathbf{n}_r \\ \mathbf{n}_i\end{bmatrix}}_{\mathbf{n}}, \quad (2)$$

where the subscripts 'r' and 'i' denote the real and imaginary parts, respectively. The equivalent real transmit vector $\mathbf{x} = [x_1 \ , x_2, \dots, x_{2N_t}]^T$ its entries from $x_i \in \sqrt{M}$-PAM constellation. We will use now this real-valued system.

The optimal ML detector determines the transmit symbol vector $\hat{\mathbf{x}}$ among all possible $\sqrt{M}^{2N_t}$ transmit vectors which is nearest to the received vector $\mathbf{y}$ for the given channel matrix $\mathbf{H}$. Mathematically, this is stated as

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \Omega^{2N_t}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2. \quad (3)$$

The ML detector has exponential computational complexity and it cannot be practically used even for systems with small number of antennas. Various NSAs for low-complexity detection, that work at symbol level, have been investigated in the literature [3]–[7]. Aiming to further reduce the detection complexity of these symbol-level NSAs, we first formulate a bit-level system framework which will later be used to design bit-level NSAs.

III. Bit-level problem formulation

Let us express the entry x_i of the transmit symbol vector $\mathbf{x}$ as the polynomial function of its constituent bits given by $x_i = \sum_{j=0}^{l-1} v_j b_{ji} = \mathbf{v}^T \mathbf{b}_i$. Here $\mathbf{v}$ is the weighting vector defined as, $[v_0 \ v_1 \ \cdots \ v_{l-1}]^T$ with $v_j = 2^j$ and $j = 0$ to $l-1$ and $\mathbf{b}_i$ is the bit vector corresponding to a symbol x_i. Here l is the number of bits $(\log_2 \sqrt{M})$ required to represent each real symbol x_i. For example, for $\mathbf{x}_c \in$ 16-QAM, we have $l = 2$ and $\mathbf{v} = [2^0 \ 2^1]^T$ and each real symbol $x_i \in$ 4-PAM constellation $\Omega = \{-3, -1, 1, 3\}$. For the above constellation, the bit vectors $\mathbf{b}_i$ corresponding to each symbol x_i are given as $\left\{[-1 \ -1]^T, [1 \ -1]^T, [-1 \ 1]^T, [1 \ 1]^T\right\}$, respectively.

Using the above expansion, the relationship between the transmit symbol vector $\mathbf{x}$ and the bit vector $\mathbf{b}$ can be expressed by the following linear system of equation [10]

$$\mathbf{x} = (\mathbf{I}_{2N_t} \otimes \mathbf{v}^T)\mathbf{b} = \widetilde{\mathbf{Q}}\mathbf{b}, \quad (4)$$

where $\otimes$ denotes the standard Kronecker product and $\mathbf{b}$ represents the bit vector of size $2lN_t$ with each $b_i \in \{-1, 1\}$. We denote $(\mathbf{I}_{2N_t} \otimes \mathbf{v}^T)$ as the matrix $\widetilde{\mathbf{Q}}$, which has a rectangular structure with rank $\leq \ min \ (2N_r, 2lN_t)$. With the above definition of symbol vector, we rewrite (2) as

$$\mathbf{y} = \mathbf{H}\widetilde{\mathbf{Q}}\mathbf{b} + \mathbf{n} = \widetilde{\mathbf{H}}\mathbf{b} + \mathbf{n}. \quad (5)$$

Here, $\widetilde{\mathbf{H}} \in \mathbb{R}^{2N_r \times 2lN_t}$ is the equivalent bit-level channel matrix. The bit-level ML detection rule can now be stated as

$$\mathbf{b}_{ML} = \underset{\mathbf{b} \in \{-1,1\}^{2lNt}}{\operatorname{argmin}} \|\mathbf{y} - \widetilde{\mathbf{H}}\mathbf{b}\|^2. \quad (6)$$

We will use this bit-level formulation to next propose the bit-level reduced neighborhood search.

IV. Framework for bit-level reduced neighborhood search

Like the existing NSAs [8], the proposed bit-level reduced NSA requires an initial solution. We, next derive an equivalent bit-level MMSE detector, and then subsequently propose a metric to determine the likelihood of a bit being in error.

A. Initial bit-level MMSE solution vector

The MMSE detector for the proposed bit-level MIMO system can be expressed using the following standard expression

$$\hat{\mathbf{b}} = \mathbf{R}_{yb}^T \mathbf{R}_{yy}^{-1} \mathbf{y}, \quad (7)$$

where $\mathbf{R}_{yy}$ and $\mathbf{R}_{yb}$ are the covariance matrices given by $\mathbf{R}_{yy} = \mathbb{E}[\mathbf{y}\mathbf{y}^T] = P_b \widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^T + \sigma^2 \mathbf{I}_{2N_r}$, $\mathbf{R}_{yb} = \mathbb{E}[\mathbf{y}\mathbf{b}^T] = P_b \widetilde{\mathbf{H}}$. Here, $P_b = \mathbb{E}\left\{|b_i|^2\right\}$ represents the average bit power and $\mathbf{I}_N$ corresponds to an $N \times N$ identity matrix. Using $\mathbf{R}_{yy}$ and $\mathbf{R}_{yb}$ expressions, Eq. (7) can be expressed as $\hat{\mathbf{b}} = P_b \widetilde{\mathbf{H}}^T \left(P_b \widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^T + \sigma^2 \mathbf{I}_{2N_r}\right)^{-1} \mathbf{y}$, which is equivalent to

$$\hat{\mathbf{b}} = P_b \left(P_b \widetilde{\mathbf{H}}^T \widetilde{\mathbf{H}} + \sigma^2 \mathbf{I}_{2lN_t}\right)^{-1} \widetilde{\mathbf{H}}^T \mathbf{y}, \quad (8)$$

where the value of P_b can be determined using the transmit power constraint $\mathbb{E}\left\{\|\mathbf{x}\|^2\right\} = 1$. Substituting $x_i = \sum_{j=0}^{l-1} 2^j b_{ji}$ and $\mathbb{E}\left\{b_i b_j^*\right\} = 0 \, \forall i \neq j$, we have $P_b = \frac{1}{\sum_{j=0}^{l-1} 2^{2j}}$ and therefore, the initial bit-level MMSE solution vector is

$$\hat{\mathbf{b}} = \left(\widetilde{\mathbf{H}}^T \widetilde{\mathbf{H}} + \sigma^2 \sum_{j=0}^{l-1} 2^{2j} \mathbf{I}_{2lN_t}\right)^{-1} \widetilde{\mathbf{H}}^T \mathbf{y}, \quad (9)$$

which is mapped to $\{-1, 1\}$ to get the exact initial bit-level solution vector. We denote this initial solution vector as $\mathbf{b}^0$. This mapping is straightforward for 4-QAM. However, in general, $\mathbf{b}^0$ can be computed from $\hat{\mathbf{b}}$ such that $\mathbf{Q}\mathbf{b}^0 = \lceil \mathbf{Q}\hat{\mathbf{b}} \rfloor_\Omega$, where $\lceil \cdot \rfloor_\Omega$ denotes the element-wise rounding operation to the set Ω. We use this $\mathbf{b}^0$ to initialize an NSA.

B. Metric for constructing bit-level reduced neighborhood

Our objective is to determine the indices which, if flipped, will reduce the ML cost. Let us assume the bit vector for the rth iteration as $\mathbf{b}^r$ and we flip the kth bit of this bit vector. The corresponding ML cost can be expressed as

$$\sum_{j=1}^{2N_r} \left(y_j - \sum_{i=1, i \neq k}^{2lN_t} \widetilde{h}_{ji} b_i^r + \widetilde{h}_{jk} b_k^r\right)^2 = \sum_{j=1}^{2N_r} \left(m_j^r + 2\widetilde{h}_{jk} b_k^r\right)^2. \quad (10)$$

The RHS of (10) is obtained by adding and subtracting $\widetilde{h}_{jk} b_k^r$ to its LHS and by defining $m_j^r = y_j - \sum_{i=1}^{2lN_t} \widetilde{h}_{ji} b_i^r$ which stays constant during one iteration. We need to minimize (10) over the index k to find the optimal kth bit to be flipped that will cause the maximum reduction in the cost as follows

$$\begin{aligned} \hat{k} \quad &= \quad \min_k \sum_{j=1}^{2N_r} \left(m_j^r + 2\widetilde{h}_{jk} b_k^r\right)^2 \overset{(a)}{=} \min_k \sum_{j=1}^{2N_r} \left(\widetilde{h}_{jk}^2 P_b + b_k^r m_j^r \widetilde{h}_{jk}\right) \\ &\overset{(b)}{=} \quad \min_k P_b \left\|\widetilde{\mathbf{h}}_k\right\|^2 + b_k^r (\mathbf{m}^r)^T \widetilde{\mathbf{h}}_k. \end{aligned}$$

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Algorithm 1: Bit-level RN-LAS Algorithm

Input: $\mathbf{y}, \widetilde{\mathbf{H}}, \widetilde{\mathbf{Q}}, \Omega, K, nbr\_size$
Output: $\mathbf{b}, \mathbf{x}$
Initialization: $\mathbf{b}^0 \leftarrow$ MMSE detector output;
$Cost^0 \leftarrow \|\mathbf{y} - \widetilde{\mathbf{H}}\mathbf{b}^0\|^2$, r $\leftarrow 0$, $\mathfrak{L} \leftarrow 1$;
repeat
  $\mathbf{b}^{temp} \leftarrow \mathbf{b}^r$ & $Cost^{temp} \leftarrow Cost^r$;
  $\mathfrak{m}_k = P_b \left\|\widetilde{\mathbf{h}}_k\right\|^2 + b_k^r (\mathbf{m}^r)^T \widetilde{\mathbf{h}}_k \;\; \forall \;\; k = 1, 2, \ldots, 2lNt$;
  Ind $\leftarrow$ choose $\binom{K}{\mathfrak{L}}$ indices where $\mathfrak{m}_k$ is smallest;
  for $i \leftarrow 1$ **to** *length (Ind)* **do**
    for $j \leftarrow 1$ **to** $2lN_t$ **do**
      if $j \in$ *Ind* **then**
        $\mathbf{b}^i(j) = -\mathbf{b}^r(j)$;
      else
        $\mathbf{b}^i(j) = \mathbf{b}^r(j)$;
      end
    end
    $Cost^i \leftarrow \|\mathbf{y} - \widetilde{\mathbf{H}}\mathbf{b}^i\|^2$;
  end
  $\hat{i} = \underset{i}{\text{argmin}} \;\; Cost^i$;
  $r \leftarrow r + 1$;
  $b^r \leftarrow b^{\hat{i}}$ & $Cost^r \leftarrow Cost^{\hat{i}}$;
  if $Cost^r \geq Cost^{temp}$ **then**
    $\mathfrak{L} = \mathfrak{L} + 1$;
  end
until $\mathfrak{L} \leq nbr\_size$;
return $\mathbf{b} \leftarrow \mathbf{b}^{temp}$, $\mathbf{x} \leftarrow \widetilde{\mathbf{Q}}\mathbf{b}$.

Equality in (a) is because ${b_k^r}^2 = P_b$ and by dropping the terms constant with respect to k. Equality in (b) is obtained by reverting (a) back into vector notation with $\widetilde{\mathbf{h}}_k = [\widetilde{h}_{1k} \cdots \widetilde{h}_{2N_r k}]^T$. Let the likelihood metric $\mathfrak{m}_k = P_b \left\|\widetilde{\mathbf{h}}_k\right\|^2 + b_k^r (\mathbf{m}^r)^T \widetilde{\mathbf{h}}_k$, so the ideal bit to flip is the one where $\mathfrak{m}_k$ is minimum. We calculate the metric for current solution vector and select only a few K $(\ll 2lN_t)$ indices out of $2lN_t$.

V. Proposed Algorithms

The four main steps of any NSA are i) initialization, ii) neighborhood choice, iii) search operation, and iv) stopping criterion. We have already explained the initialization step in Section IV-A. For the neighborhood choice step, we select the K $(\ll 2lN_t)$ indices out of $2lN_t$ at which the likelihood metric $\mathfrak{m}_k$ takes smaller values. The selected set of indices are used to construct the neighborhood. For 1-LAS [4] and RTS [6], we flip the bits at the selected set of indices one by one and construct a bit-level reduced neighborhood. For MLAS [3], from the selected set of K indices, similar to [3], we flip bits in a group of $\mathfrak{L} = 2$ and/or 3 at a time to create a neighborhood of $\binom{K}{\mathfrak{L}}$ vectors.

For rest of the two steps – the search operation and the stopping criterion – we follow exactly the same strategy as in the case of respective original NSA i.e., LAS or RTS. We refer to this bit-level reduced neighborhood strategy as BL-RN algorithms. For an illustration, we have provided the pseudo-code of the proposed BL-RN-LAS in Algorithm 1. The pseudo code refers to BL-RN-1-LAS for $nbr\_size = 1$ and BL-RN-MLAS for $nbr\_size = 3$.

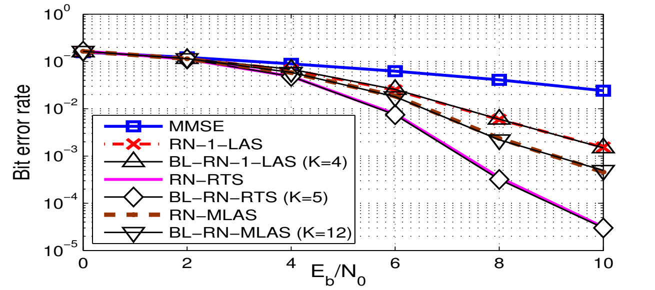

Fig. 1. BER performance of symbol- and bit-level reduced neighborhood algorithms for a 4-QAM 32×32 MIMO system.

Complexity Analysis: The complexity of NSAs [3]–[7] is dominated by the search operation, which depends on the number of candidate vectors. Since at bit-level, there is only one alternative, the number of candidate vectors depend on the value of K alone. This leads to only $\binom{K}{\mathfrak{L}}$ candidate vectors for the proposed bit-level approach when compared with $(\sqrt{M}-1)^{\mathfrak{L}}\binom{K}{\mathfrak{L}}$ and $(\sqrt{M}-1)^{\mathfrak{L}}\binom{2N_t}{\mathfrak{L}}$ candidate vectors for symbol-level reduced and original NSAs in [8] and [3]–[7], respectively. Thus the search complexity of the proposed BL-RN-1-LAS is of $O(K)$ in contrast to $O(\sqrt{M}K)$ of RN-1-LAS (symbol-level) [8] and $O(\sqrt{M}N_t)$ of 1-LAS [4]. For the three variants of MLAS i.e., BL-RN, symbol-level RN [8], and the original NSA [3], the search complexities are in the order of $O(K^3)$, $O(M^{1.5}K^3)$, and $O(M^{1.5}N_t^3)$, respectively. A similar complexity relationship holds for the RTS algorithms i.e., the proposed BL-RN-RTS algorithm and the symbol-level RN [8] and the original ones [6], [7].

VI. Simulation Results

We now numerically investigate the performance of the proposed BL-RN approach for three NSAs, namely 1-LAS [4], MLAS [6], and RTS [6]. We refer to these variants as BL-RN-1-LAS, BL-RN-MLAS, and BL-RN-RTS. We consider a 32×32 MIMO system with 4-QAM and 16-QAM modulations and compare the BL-RN algorithms performances with their respective symbol-level (SL) counterparts from [8], which have lower complexity that the original 1-LAS, MLAS, and RTS algorithms [3]–[7]. We prefix symbol-level variants of these algorithms from [8] with SL. The value of K for the proposed BL-RN algorithms is selected such that there is no degradation in the BER, when compared with the SL-RN counterparts [8]. The value of K for SL-RN algorithm is directly taken from [8].

The BER plots for 4- and 16-QAM are presented in Fig. 1 and Fig. 2(a) respectively, and the average number of vectors to be searched are shown in Fig. 2(b) and Fig. 2(c), respectively. We observe from Fig. 1 and Fig. 2(a) that the BL-RN algorithms do not degrade the BER when compared with the SL-RN algorithms. They, however, search significantly lesser number of vectors. For example for 4-QAM, we see from Fig. 2(b) that the proposed BL-RN-1-LAS algorithm searches 4 vectors per neighborhood when compared with 6 vectors of SL-RN-1-LAS leading to a 33% reduction in complexity. Similarly the BL-RN-MLAS algorithm requires $K = 12$ (220 vectors per neighborhood) and the SL-RN-MLAS algorithm requires $K = 16$ (560 vectors per neighborhood), which

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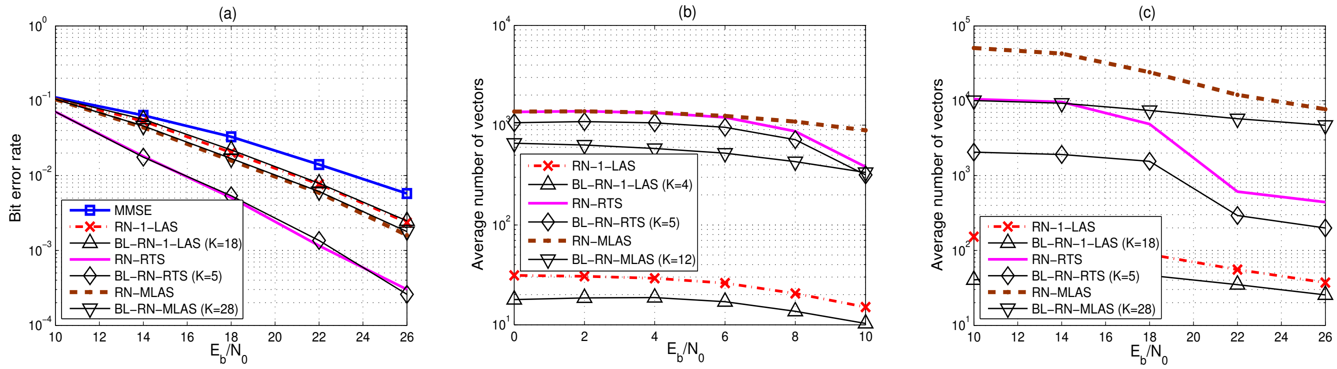

Fig. 2. BER of (a) a 32×32 16-QAM system; and the average number of vectors searched for a 32×32 system for (b) 4-QAM (c) 16-QAM.

Neighborhood Search Algorithms	4-QAM at $E_b/N_0 = 10$ dB RN	BL-RN
1-LAS	2.296	0.199
MLAS	13.906	4.402
RTS	7.351	5.717
16-QAM at $E_b/N_0 = 18$ dB		
1-LAS	2.118	0.668
MLAS	123.932	44.319
RTS	39.534	28.067

TABLE I
AVERAGE NUMBER OF ARITHMETIC OPERATIONS ($\times 10^4$) PER BIT FOR 32×32 MIMO SYSTEM.

reduces the complexity by 60%. A similar trend in complexity reduction can be observed for 16-QAM too (Fig. 2(c)).

We next examine the gains in terms of the average number of arithmetic operations (i.e., counting the additions and multiplications) for 4-QAM at $E_b/N_0 = 10$ dB and for 16-QAM at $E_b/N_0 = 18$ dB. The numbers for both SL-RN and BL-RN variants are shown in Table I. For 4-QAM, the BL-RN-1-LAS, BL-RN-MLAS and BL-RN-RTS algorithms require only 0.199, 4.402, and 5.717 number of arithmetic operations compared to 2.296, 13.906, and 7.351 number of arithmetic operations of SL-RN-1-LAS, SL-RN-MLAS and SL-RN-RTS. This decreases the overall computational complexity by 91%, 68% and 21%, respectively. Similar trends can be observed for 16-QAM. It is worthwhile to note that the proposed bit-level design has much lower computational complexity than the original 1-LAS, MLAS, and RTS algorithms [3]–[7].

Remark 1. **Way forward for the coded systems:** An inherent advantage of the BL-RN approach is that it can be used to realize the iterative detection and decoding scheme for soft decoding in coded large MIMO systems [11]. We next outline the key steps to realize this.

(a) The soft MMSE values $\hat{\mathbf{b}}$ in (9) are mapped to the initial vector $\mathbf{b}^r$ for $r = 0$, and are also used as a priori values later in step (c).
(b) We use the metric $\mathfrak{m}_k = P_b \left\|\widetilde{\mathbf{h}}_k\right\|^2 + b_k^r(\mathbf{m}^r)^T\widetilde{\mathbf{h}}_k$ to determine the candidate vectors and divide them into two disjoint sets $\mathbf{B}_i^+$ and $\mathbf{B}_i^-$, where $\mathbf{B}_i^+$ is the set of all bit vector with $b_i = +1$ and $\mathbf{B}_i^-$ with $b_i = -1$.
(c) The extrinsic log-likelihood ratios (LLRs) are calculated over the sets $\mathbf{B}_i^+$ and $\mathbf{B}_i^-$ using the standard LLR computation [11], which also uses the a priori values from step (a).
(d) We repeat the steps from (a) to (c) for a fixed number of iterations to improve upon the LLRs computed in step (c) by considering these LLRs as updated soft values.

Remark 2. The proposed outline is valid for the bit-level formulation in (4), and this mapping needs to be further explored for arbitrary bit to symbol-level encoding.

VII. Conclusion

The detection complexity of a neighborhood search algorithm is determined by its neighborhood size and how efficiently the vector search for the near-ML solution is carried out. In this work, we proposed a bit-level metric design which significantly reduces the detection complexity by transforming symbol-level vector search to the bit-level. The proposed detector locates the optimal bits to be flipped based on the derived metric. The bit-level framework can be easily extended to coded systems by computing the LLR values over the reduced neighborhood set.

References

[1] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742–758, 2014.
[2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, 2014.
[3] S. Mohammed, A. Chockalingam, and B. Sundar Rajan, "A low-complexity near-ML performance achieving algorithm for large MIMO detection," in *Proc. 2008 IEEE Int. Symp. Inf. Theory*, pp. 2012–2016.
[4] K. V. Vardhan, S. K. Mohammed, A. Chockalingam, and B. S. Rajan, "A low-complexity detector for large MIMO systems and multicarrier CDMA systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 3, pp. 473–485, 2008.
[5] P. Li and R. D. Murch, "Multiple output selection-LAS algorithm in large MIMO systems," *IEEE Commun. Lett.*, vol. 14, no. 5, 2010.
[6] B. Rajan, S. Mohammed, A. Chockalingam, and N. Srinidhi, "Low-complexity near-ML decoding of large non-orthogonal STBCs using reactive tabu search," in *Proc. 2009 IEEE Int. Symp. Inf. Theory*, pp. 1993–1997.
[7] N. Srinidhi, T. Datta, A. Chockalingam, and B. S. Rajan, "Layered tabu search algorithm for large-MIMO detection and a lower bound on ML performance," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 2955–2963, 2011.
[8] A. K. Sah and A. Chaturvedi, "Reduced neighborhood search algorithms for low complexity detection in MIMO systems," in *Global Communications Conference (GLOBECOM), 2015 IEEE*. IEEE, 2015, pp. 1–6.
[9] R. C. de Lamare, "Adaptive and iterative multi-branch mmse decision feedback detection algorithms for multi-antenna systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 10, pp. 5294–5308, 2013.
[10] J. Choi, "Iterative receivers with bit-level cancellation and detection for MIMO-BICM systems," *IEEE Trans. Signal Process.*, vol. 53, no. 12, pp. 4568–4577, 2005.
[11] B. M. Hochwald and S. Ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. Commun.*, vol. 51, no. 3, pp. 389–399, 2003.