An Upper Bound on the Performance of K-best Detection for MIMO Systems

Abhay Kumar Sah† and A. K. Chaturvedi‡
Department of Electrical Engineering
Indian Institute of Technology Kanpur, Kanpur, India 208016
Email: {abhaysah†, akc‡}@iitk.ac.in

***Abstract*—K-best detection is known to be a useful breadth first tree search detection technique for multiple input multiple output (MIMO) systems. In the process of the tree search a number of possible transmitted symbols are searched and the error performance of the K-best detection depends on this number. More the number of visited symbols, better the error performance. However this relationship is not straightforward and needs to be analyzed. In this paper, a tight upper bound on the error performance of K-best detection for MIMO systems with linear modulation scheme has been derived. In particular upper bounds for M-ary pulse amplitude modulation (M-PAM) and for 4-ary quadrature amplitude modulation (4-QAM) with Rayleigh fading channel have been established. This upper bound requires the K-best error performance for single-input single-output (SISO) systems. Hence we first derive an exact expression for M-PAM as well as for 4-QAM with K-best detection for SISO systems and use this to establish the upper bound. Finally we compare the derived upper bound with the simulations. It is found that the upper bound is close to the results obtained through simulations.**

I. Introduction

Nowadays MIMO systems have become an essential part of various wireless standards such as 4G, WiMax, 802.11n, HSPA+. The popularity of MIMO systems is due to its ability to provide high data rate reliable communication. This reliability of communication or the error performance depends on the choice of detection technique. Maximum likelihood (ML) detection [1] has best error performance but its computational complexity increases exponentially with the number of transmit antennas and the constellation size. In the literature, K-best detection [2] has been seen as a potential alternative to achieve near ML error performance at the cost of polynomial type complexity. The advantage of K-best is not limited to this only but it can be also used as high throughput detector [3] and can be combined with other low complexity techniques to further reduce the complexity [4].

K-best detection primarily belongs to the family of breadth first tree search (BFTS) [5] algorithms. The principle of this technique lies in searching for K minimum Euclidean cost symbols starting from a particular antenna and forming a tree. This is done repeatedly until we are finished with all the antennas and at last we select the minimum over these K possibilities. It is evident from the principle itself that the error performance of K-best detection depends on the choice of K and this dependency of error performance over K needs to be analyzed. In this paper an upper bound on the symbol error rate (SER) of K-best detection for MIMO systems with linear modulation has been derived. This upper bound is found to be a function of SER performance of K-best detection for SISO systems. Though K-best detection is meaningless for SISO systems but to establish the upper bound it is necessary to find its error performance. Therefore we first derive an exact expression for one dimensional constellation i.e. M-PAM with K-best detection for SISO systems over AWGN as well as Rayleigh channel and use it to establish the upper bound. We have also analyzed the two dimensional constellation geometries for different values of K. Particularly for 4-QAM, we first derive the expressions for different values of K and then use these expressions to derive the upper bound. The trend of SER performances for different values of K has been investigated and it shows that the derived upper bound is close to the simulation results.

The rest of this paper is structured as follows: Section II describes the system model and Section III first describes the K-best detection and then a general upper bound on the error performance of K-best is established. To derive the upper bound for specific cases such as M-PAM and 4-QAM the exact expressions for K-best detection with SISO systems have been established in Section IV and V respectively. Section VI compares the theoretical results with simulations and finally Section VII concludes the paper.

II. System Model

A MIMO system uses N_t number of transmit antennas for transmission and N_r number of receive antennas for reception $(N_t \leq N_r)$ [1]. The input-output relationship can be mathematically modeled as

$$Y = \mathbf{H}X + N \tag{1}$$

where $Y = (y_1, y_2, \cdots, y_{N_r})^T$ is $(N_r{\times}1)$ received signal vector and y_i represents data received at i^{th} receive antenna. $X = (x_1, x_2, \cdots, x_{N_t})^T$ is $(N_t{\times}1)$ transmitted signal vector and each $x_i \in \Omega$ represents data transmitted through i^{th} transmit antenna and Ω a set of constellation symbols such as M-PAM, M-QAM. $\mathbf{H}$ denotes $(N_r{\times}N_t)$ channel matrix with each

This work was supported by Tata Consultancy Services Limited.

coefficient $h_{ij} \sim \mathcal{CN}(0,1)$ and $N = (n_1, n_2, \cdots, n_{N_r})^T$ represents $(N_r \times 1)$ i.i.d. additive white gaussian noise (AWGN) vector with each $n_i \sim \mathcal{CN}(0, \sigma^2)$.

Utilizing QR decomposition the channel matrix can be decomposed as a product of two matrices $\mathbf{Q}$ and $\mathbf{R}$, where $\mathbf{Q}$ is $(N_r \times N_t)$ orthogonal matrix and $\mathbf{R}$ is $(N_t \times N_t)$ upper triangular matrix. Hence using the properties of orthogonal matrices the system model (1) can be rewritten as

$$Z = \mathbf{R}X + \widetilde{N} \tag{2}$$

where $Z = \mathbf{Q}^H Y$ is equivalent received vector of order $(N_t \times 1)$ and $\widetilde{N} = \mathbf{Q}^H N$ is equivalent noise vector with same distribution and order as $(N_t \times 1)$.

III. An Upper Bound on K-best error performance

In this section, we first briefly discuss the K-best detection [2] and then propose an approach for an upper bound on its error performance.

A. K-best Detection

This is a kind of sequential detection technique where symbols are detected through bottom to top antenna. Here, we denote each antenna by its position that means the top most antenna has given number 1 and the bottom most has number N_t. At first ML detection is performed on N_t^{th} antenna and K closest symbols are selected. We formed a tree by connecting these K symbols to a virtual root node and then keeping track of these K closest symbols expand the tree with all M possible constellation symbol. Now, ML detection is performed only on these MK symbols at $(N_t - 1)^{th}$ antenna and again K closest symbols are retained and prune rest part of the tree. This is done repeatedly until we reach at the 1^{st} antenna and the minimum Euclidean cost vector among these K paths is declared as final solution. The Euclidean cost for a particular vector X is given by

$$cost(X) = \sum_{j=1}^{N_t} (z_j - \sum_{k=1}^{N_t} r_{j,k} x_k)^2 \tag{3}$$

B. Upper Bound on K-best error performance

To establish a theoretical upper bound on SER, we have constructed the following sets, defined as follows

$$\mathcal{B} = \{\hat{X} : \hat{X} = \underset{X \in \Omega^{N_t}}{\mathrm{argmin_K}} \|Z - \mathbf{R}X\|^2\} \tag{4}$$

$$\mathcal{B}_i = \{\hat{x_i} : \hat{x_i} = \underset{x_i \in \Omega}{\mathrm{argmin_K}} |\ddot{z}_i - r_{ii} x_i|^2\}; i = 1 \cdots N_t \tag{5}$$

where $\ddot{z}_i = (z_i - \sum_{j=i+1}^{N_t} r_{ij} x_j)\ \forall x_j \in \mathcal{B}_j$. The $\mathrm{argmin_K}(\cdot)$ returns first K solutions which minimizes the function in ascending order according to their cost. Explicitly, $\mathcal{B}$ is a set containing K nearest possible transmit vectors to the received vector, while $\mathcal{B}_i$ contains the K closest symbols transmitted from i^{th} transmit antenna. Let $X = (x_1, x_2, \cdots, x_{N_t})^T$ be a transmit vector then the probability of event when the transmitted signal vector lie inside the set $\mathcal{B}$ can be expressed as

$$P(X \in \mathcal{B}) = P(x_{N_t} \in \mathcal{B}_{N_t}) P(x_{N_t-1} \in \mathcal{B}_{N_t-1} | x_{N_t} \in \mathcal{B}_{N_t}) \cdots P(x_1 \in \mathcal{B}_1 | (x_2 \in \mathcal{B}_2, x_3 \in \mathcal{B}_3, \cdots x_{N_t} \in \mathcal{B}_{N_t})) \tag{6}$$

Hence probability of event when transmitted signal vector does not lie inside the K best solution set is given by

$$\begin{aligned} P(X \notin \mathcal{B}) &= 1 - P(X \in \mathcal{B}) \\ &= 1 - [P(x_{N_t} \in \mathcal{B}_{N_t}) P(x_{N_t-1} \in \mathcal{B}_{N_t-1} | x_{N_t} \in \mathcal{B}_{N_t}) \\ &\quad \cdots P(x_1 \in \mathcal{B}_1 | (x_2 \in \mathcal{B}_2, x_3 \in \mathcal{B}_3, \cdots x_{N_t} \in \mathcal{B}_{N_t}))] \\ &\leq 1 - [P(x_{N_t} \in \mathcal{B}_{N_t}) P(x_{N_t-1} \in \mathcal{B}_{N_t-1}) P(x_1 \in \mathcal{B}_1)] \end{aligned} \tag{7}$$

Statistically, $P(x_i \in \mathcal{B}_i)$ is same for all $i \in 1, 2, \cdots N_t$. Let α denotes the probability of error for K-best detection in SISO systems i.e. the probability of event that the transmitted symbol lie outside the set $\mathcal{B}_i$. Mathematically, $P(x_i \notin \mathcal{B}_i) = \alpha$ or in other words $P(x_{N_t} \in \mathcal{B}_{N_t}) = P(x_{N_t-1} \in \mathcal{B}_{N_t-1}) \cdots = P(x_1 \in \mathcal{B}_1) = 1 - \alpha$. Hence (7) can be written as $P(X \notin \mathcal{B}) \leq 1 - (1-\alpha)^{N_t}$. Now, we are looking for the probability of error in MIMO with K-best detection. There are two cases: first case is when the transmitted vector belongs to the set $\mathcal{B}$ yields same performance as ML and the second case is when the transmitted vector does not belong to the set $\mathcal{B}$ then there is definitely a loss in performance as compared to ML. Thus, the error probability for K-best is given by

$$\begin{aligned} P_{e,K-best} &= P(X \notin \mathcal{B}) + P(X \in \mathcal{B}) P_{e,ML} \\ &= P(X \notin \mathcal{B}) + (1 - P(X \notin \mathcal{B})) P_{e,ML} \\ &= P_{e,ML} + P(X \notin \mathcal{B})(1 - P_{e,ML}) \\ &\leq P_{e,ML} + (1 - (1-\alpha)^{N_t})(1 - P_{e,ML}) \end{aligned} \tag{8}$$

Since this upper bound does not consider the modulation scheme or the nature of wireless channel, the (8) can be consider as general upper bound on the SER of K-best detection for MIMO systems. The value of α depends on the choice of modulation scheme, signal power and the value of K. The detailed derivation for M-PAM is provided in Section IV and derivation for 4-QAM is provided in Section V. Here we do not provide the expression for $P_{e,ML}$, the expression for error probability for different cases in MIMO systems can be found in [6]–[8].

Let us examine the upper bound for $\alpha = 0$. Equation (8) becomes $P_{e,K-best} \leq P_{e,ML}$ but the error rate can not be less than the ML error rate which implies $P_{e,K-best} = P_{e,ML}$. We can observe that as the value of α approaches zero the K-best error performance approaches ML performance and for very small values of α it can achieve near ML performance.

IV. K-best Detection for SISO with M-PAM

This section first considers K-best Detection for SISO systems with M-PAM for AWGN channel, then extends the results to Rayleigh fading channel. Let us consider the set constructed in (5), $\mathcal{B}_{N_t} \subseteq \Omega$ taking $r_{N_t N_t} = 1$. This can be viewed as the case that it contains all the K symbols whose Euclidean cost

Fig. 1. The M-PAM Constelation.

is less than the other symbols in SISO systems with AWGN channel. For M-PAM, $\Omega = \{\pm\frac{d}{2}, \pm\frac{3d}{2}, \cdots \pm \frac{(M-1)d}{2}\}$ is a set of M different symbols and each symbol is separated by a distance d. The geometrical interpretation for M-PAM is well known and shown in Fig. 1. Our objective is to find the probability of event that the actual transmitted symbol does not lie in the set $\mathcal{B}_{N_t}$ i.e. $P(x \notin \mathcal{B}_{N_t})$. For a particular symbol (say s) it is easy to visualize from the figure that the decision boundaries depend on the value of K. For a particular value of K, on the basis of these decision boundaries we can categorize the symbols into the following three categories:

1) Symbols bounded by the right: There are K number of symbols from left which are bounded by right side only. These points are outside the set $\mathcal{B}_{N_t}$ only when the noise n is greater than or equal to $Kd/2$. This can be given by

$$P(s \notin \mathcal{B}_{N_t}) = P\left(n \geq \frac{Kd}{2}\right) \tag{9}$$

2) Symbols bounded by the left: In this scenario, K number of symbols are bounded by left side only. Using the symmetric properties of noise, the error probability can be expressed as

$$P(s \notin \mathcal{B}_{N_t}) = P\left(n \leq \frac{Kd}{2}\right) = P\left(n \geq \frac{Kd}{2}\right) \tag{10}$$

3) Symbols bounded by both side: There are $M - 2K$ number of symbols which are bounded by both side and these point lies within the set $\mathcal{B}_{N_t}$ when the absolute value of noise is bounded by $Kd/2$. Hence using the properties of noise the error probability can be expressed as

$$\begin{aligned} P(s \notin \mathcal{B}_{N_t}) &= P\left(n \leq \frac{Kd}{2}\right) + P\left(n \geq \frac{Kd}{2}\right) \\ &= 2P\left(n \geq \frac{Kd}{2}\right) \end{aligned} \tag{11}$$

Following the probability of symbol error in these three categories one can see that there are $2K$ number of symbols in the corners with equal probability and $(M-2K)$ number of symbols in the middle. Assuming all M symbols are equally likely (i.e. $P_{s_i} = 1/M \quad \forall \quad i = 1 \cdots M$) and noise has zero mean Gaussian distribution with variance $\sigma^2 = N_0/2$. The probability of the event $x \notin \mathcal{B}_{N_t}$ can be found as follows

$$\begin{aligned} P(x \notin \mathcal{B}_{N_t}) &= \frac{1}{M}\sum_{i=1}^{M} P_{e|s_i} \\ &= \left[\frac{2K}{M}P\left(n \geq \frac{Kd}{2}\right) + \frac{2(M-2K)}{M}P\left(n \geq \frac{Kd}{2}\right)\right] \\ &= \frac{2(M-K)}{M}P\left(n \geq \frac{Kd}{2}\right) = \frac{2(M-K)}{M}Q\left(\frac{Kd}{\sqrt{2N_0}}\right) \end{aligned} \tag{12}$$

where $Q(x) = \frac{1}{2\pi}\int_0^\infty \exp -\frac{t^2}{2}dt$. The relationship between average symbol energy of the constellation E_s and d for M-PAM can be found in [9] as $d = \sqrt{\frac{12E_s}{M^2-1}}$. Using this (12) can be written as

$$P(x \notin \mathcal{B}_{N_t}) = \frac{2(M-K)}{M}Q\left(K\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right) \tag{13}$$

Equation (13) describes the symbol error probability of K-best for SISO with M-PAM over AWGN channel. The above result could be generalize for Rayleigh fading channel considering $r_{N_tN_t} \sim \mathcal{CN}(0,1)$. This will give instantaneous value of α as

$$\alpha_{inst} = \frac{2(M-K)}{M}Q\left(K\sqrt{\frac{6\gamma_s}{(M^2-1)}}\right) \tag{14}$$

where $\gamma_s = |r_{N_tN_t}|^2\frac{E_s}{N_0}$ is instantaneous SNR. To compute the α this instantaneous error probability must be integrated with respect to the distribution as follows

$$\alpha = \int_0^\infty \frac{2(M-K)}{M}Q\left(K\sqrt{\frac{6\gamma_s}{(M^2-1)}}\right)P_{\gamma_s}(\gamma_s)d\gamma_s \tag{15}$$

where $P_{\gamma_s}(\gamma_s) = \frac{1}{\bar{\gamma}_s}\exp\left(-\frac{\gamma_s}{\bar{\gamma}_s}\right)$ for Rayleigh distribution. The (15) can be solved for Rayleigh distribution using the result from [10], stated as

$$\int_0^\infty Q(a\sqrt{\gamma_s})P_{\gamma_s}(\gamma_s)d\gamma = \frac{1}{2}\left(1 - \sqrt{\frac{a^2\bar{\gamma}_s/2}{1+a^2\bar{\gamma}_s/2}}\right) \tag{16}$$

where $\bar{\gamma}_s = \frac{E_s}{N_0}$. Using (16), the above integral (15) can be solved and expressed as

$$\alpha = \frac{M-K}{M}\left(1 - \sqrt{\frac{3K^2\bar{\gamma}_s}{(M^2-1)+3K^2\bar{\gamma}_s}}\right) \tag{17}$$

where, $\bar{\gamma}_s = \frac{E_s}{N_0}$. One can notice that for $K = 1$ the results obtained in (13) and (17) are matched with SER expressions for M-PAM in SISO for AWGN and Rayleigh fading channel respectively [10].

V. K-BEST DETECTION FOR SISO WITH 4-QAM

The results derived in Section IV can be extended for two dimensional constellations. This is not straightforward, because the decision boundaries for two dimensional constellation depend on the constellation size and the chosen value of K. Therefore, we have considered 4-QAM and derived the value of α using the geometry of constellation and decision boundaries for different values of K. For 4-QAM modulation, we can have four values of K i.e. $K = 1, 2, 3$ and 4. The corresponding decision boundaries are shown in Fig. 2. The all 4 cases are as follows:

1) $K = 1$: This can be seen as conventional symbol error probability for 4-QAM. The detail derivation can be found in [10] and expressed as

$$\alpha = \left[1 - \sqrt{\frac{\bar{\gamma}_s}{2+\bar{\gamma}_s}}\right] - \frac{1}{4}\left[1 - \sqrt{\frac{\bar{\gamma}_s}{2+\bar{\gamma}_s}}\left(\frac{4}{\pi}\tan^{-1}\sqrt{\frac{2+\bar{\gamma}_s}{\bar{\gamma}_s}}\right)\right] \tag{18}$$

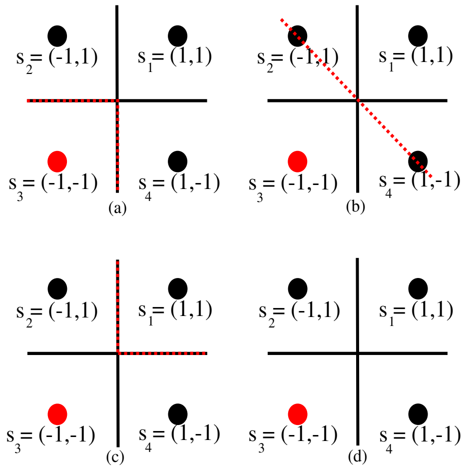

Fig. 2. Decision boundaries for 4-QAM when s_3 has been transmitted for different value of K: (a) $K = 1$, (b) $K = 2$, (c) $K = 3$ and (d) $K = 4$.

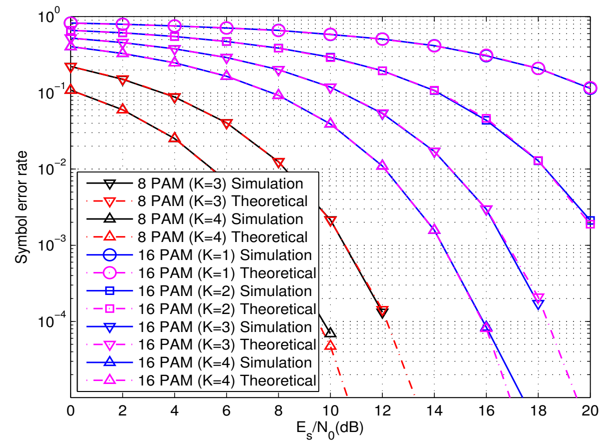

Fig. 3. SER comparison for analytical result with simulations for different value of K and M in M-PAM modulation for SISO systems with AWGN.

2) $K = 2$: We first consider the $\mathcal{B}_{N_t}$ for $r_{N_tN_t} = 1$ and using the geometry of Fig. 2(b) the probability of symbol error for s_3 is given by

$$P(s_3 \notin \mathcal{B}_{N_t}) = P\left(Re\{n\} + Im\{n\} \geq d\right) \tag{19}$$

As we know the fact: if two independent gaussian random variables $\theta_1 \sim \mathcal{N}(0, N_0/2)$ and $\theta_2 \sim \mathcal{N}(0, N_0/2)$ then $\theta_1 + \theta_2 \sim \mathcal{N}(0, N_0)$. Now using the symmetry of distribution it can be seen that $P(s_i \notin \mathcal{B}_{N_t})$ for $i = 1, 2$ and 4 is same as above. Assuming all 4 symbols are equally likely, the probability of event $x \notin \mathcal{B}_{N_t}$ can be given by

$$P(x \notin \mathcal{B}_{N_t}) = \frac{1}{4}\sum_{i=1}^{4} P_{e|s_i} = Q\left(\frac{d}{\sqrt{N_0}}\right) \tag{20}$$

The relationship between average symbol energy E_s and the minimum distance between the symbols d for M-QAM can be found in [9] as $d = \sqrt{\frac{6E_s}{M-1}}$. Using this, above can be expressed for 4-QAM as $Q\left(\sqrt{\frac{6E_s}{2N_0}}\right)$. This can be generalize for Rayleigh fading channel i.e. $r_{N_tN_t} \sim \mathcal{CN}(0, 1)$. Following the similar steps as (14), (15) and using the integral in (16) the value of α for $K = 2$ can be expressed as

$$\alpha = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_s}{1 + \bar{\gamma}_s}}\right) \tag{21}$$

3) $K = 3$: Considering the $\mathcal{B}_{N_t}$ with $r_{N_tN_t} = 1$ and using the geometry of Fig. 2(c) the event $s_3 \notin \mathcal{B}_{N_t}$ occurs when the real and imaginary part of the noise is greater than $d/2$. Mathematically, $P(s_3 \notin \mathcal{B}_{N_t}) = P(Re\{n\} \geq d/2)P(Im\{n\} \geq d/2)$. Using the symmetry of problem it turns out that this probability is same for all 4 symbols. Assuming all the symbols have equal probability of occurrence, the probability of event $x \notin \mathcal{B}_{N_t}$ can be expressed as

$$P(x \notin \mathcal{B}_{N_t}) = Q^2\left(\frac{d}{\sqrt{2N_0}}\right) = Q^2\left(\sqrt{\frac{E_s}{N_0}}\right) \tag{22}$$

Generalizing this for Rayleigh fading channel taking $r_{N_tN_t} \sim \mathcal{CN}(0, 1)$ and averaging with respect to Rayleigh distribution we get α as $\int_0^\infty Q^2\left(\sqrt{\bar{\gamma}_s}\right) P_{\gamma_s}(\gamma_s) d\gamma_s$. The solution for the integral for $P_{\gamma_s}(\gamma_s) = \frac{1}{\bar{\gamma}_s}\exp\left(-\frac{\gamma_s}{\bar{\gamma}_s}\right)$ can be found in [10]. Using this, we can find α for this case as

$$\alpha = \frac{1}{4}\left[1 - \sqrt{\frac{\bar{\gamma}_s}{2 + \bar{\gamma}_s}}\left(\frac{4}{\pi}\tan^{-1}\sqrt{\frac{2 + \bar{\gamma}_s}{\bar{\gamma}_s}}\right)\right] \tag{23}$$

4) $K = 4$: There is no decision boundary in this case because no matter how much noise is added the set have always all the four symbols. Thus the probability of event $x \notin \mathcal{B}_{N_t}$ is 0 which implies $\alpha = 0$.

Keeping in mind the value of K and the geometry of constellation, this procedure can be extended for any other two dimensional constellation.

VI. Simulation Results

This section compares the derived analytical results with the simulated results for different values of K and M for M-PAM and M-QAM modulations. In Fig. 3, (13) has been plotted for 8-PAM and 16-PAM with AWGN channel for different values of K and compared with their respective simulated curves. From the figure one can observe that the analytical results exactly match with the simulated results. Fig. 4 compares (17) with the simulations for 8-PAM and 16-PAM with Rayleigh fading channel for different values of K and the results are found to be exactly matching with the simulations.

Further, the upper bound derived for MIMO K-best detection in (8) has been plotted and compared for 2×2 and 4×4 MIMO systems for M-PAM modulation with Rayleigh fading channel. In Fig. 5, 8-PAM and 16-PAM is compared for $K = 4$, while in Fig. 6, 8-PAM and 16-PAM have been compared for different values of K. Fig. 7 compares (8) for 4-QAM modulation with the simulated results for 2×2 and 3×3 MIMO systems with Rayleigh fading. One can observe from these three figures that the analytical upper bound is close to the simulated results.

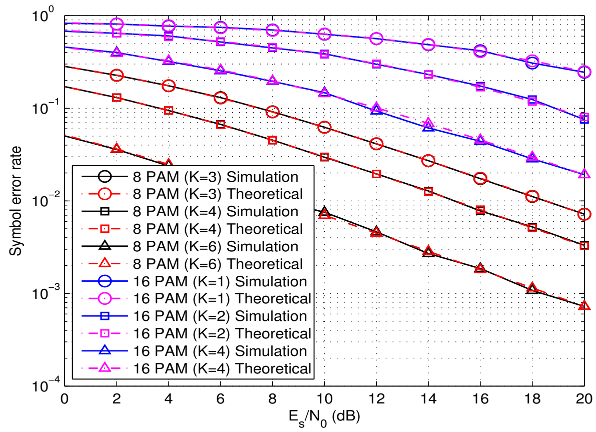

Fig. 4. SER comparison for analytical result with simulations for different value of K and M in M-PAM modulation for SISO systems with Rayleigh fading.

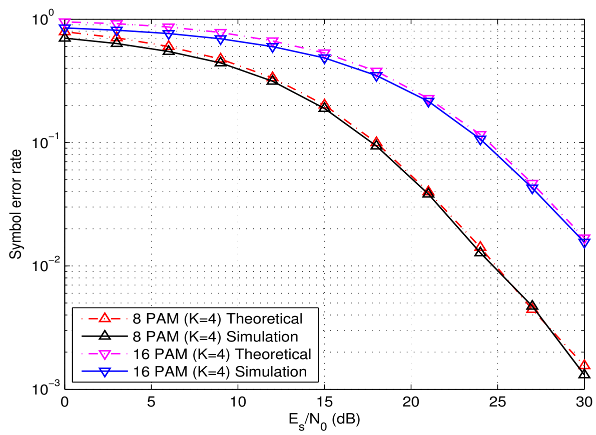

Fig. 5. SER comparison for analytical result with simulations for different value of M in M-PAM modulation for 2×2 MIMO systems with Rayleigh fading.

VII. Conclusion

This paper establishes an upper bound on the SER performance of K-best detection for MIMO systems. The established result is general in nature and is applicable for linear modulation schemes. In particular, the paper derives an upper bound for M-PAM and 4-QAM modulation for Rayleigh fading channel. Derivation of the upper bound requires the performance of K-best detection for SISO systems. Therefore, we also derive an exact expression for K-best detection for SISO systems for AWGN as well as Rayleigh fading channel. Finally, the upper bound has been plotted and has been found to be close to the simulation results.

References

[1] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications*. Cambridge University press, 2007.

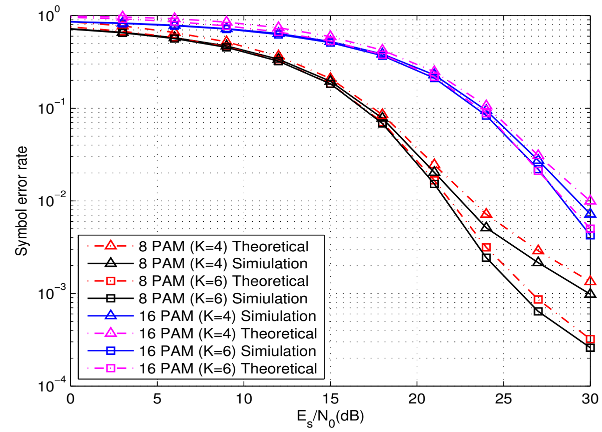

Fig. 6. SER comparison for analytical result with simulations for different value of K and M in M-PAM modulation for 4×4 MIMO systems with Rayleigh fading.

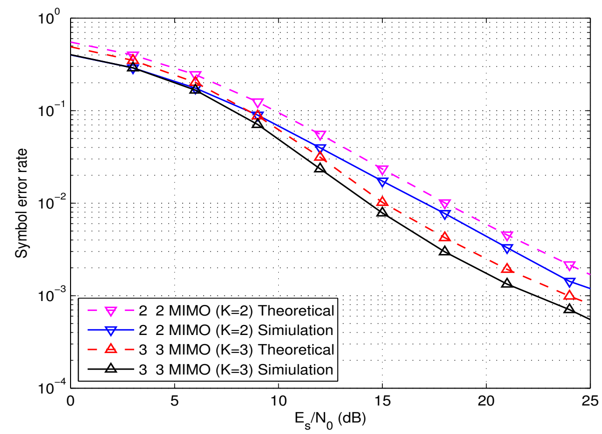

Fig. 7. SER comparison for analytical result with simulations for 2×2 and 3×3 MIMO systems with Rayleigh fading for 4-QAM modulation.

[2] Z. Guo and P. Nilsson, "Algorithm and implementation of the K-best sphere decoding for MIMO detection," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 491–503, 2006.

[3] M.-Y. Huang and P.-Y. Tsai, "Toward multi-gigabit wireless: Design of high-throughput MIMO detectors with hardware-efficient architecture," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 2, pp. 613–624, 2014.

[4] Q. Wen, Q. Zhou, C. Zhao, and X. Ma, "Fixed-point realization of lattice-reduction aided MIMO receivers with complex K-best algorithm," in *ICASSP, IEEE International Conference*, 2013, pp. 5031–5035.

[5] Y. Jia, C. Andrieu, R. Piechocki, and M. Sandell, "Depth-first and breadth-first search based multilevel SGA algorithms for near optimal symbol detection in MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 7, no. 3, pp. 1052–1061, 2008.

[6] P. Dighe, R. Mallik, and S. Jamuar, "Analysis of transmit-receive diversity in rayleigh fading," *IEEE Transactions on Communications*, vol. 51, no. 4, pp. 694 – 703, april 2003.

[7] J. Romero-Jerez, J. Pea-Martin, and A. Goldsmith, "Bit error rate analysis in mimo channels with fading and interference," in *Vehicular Technology Conference, 2009 IEEE 69th VTC Spring*, 2009, pp. 1–5.

[8] M. Kang and M.-S. Alouini, "A comparative study on the performance of mimo mrc systems with and without cochannel interference," *IEEE Transactions on Communications*, vol. 52, no. 8, pp. 1417–1425, 2004.

[9] J. G. Proakis, *Digital Communications*. McGraw-Hill, 1995.

[10] Marvin K. Simon and Mohamed-Slim Alouini, *Digital Communication over Fading Channels, 2nd Edition*. John Wiley and Sons, 2004.