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A Common Transceiver Design for Non-Regenerative Asymmetric and Symmetric Two-way Relaying With Relaxed Antenna Constraints

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***Abstract*—We investigate a novel multiple-input multiple-output common transceiver design for conventional symmetric and recently proposed asymmetric two-way relaying (TWR). In conventional symmetric TWR, a user exchanges data with a base station. Both base station and user can cancel back-propagating interference (BI). In asymmetric TWR, the base station performs two-way relaying with two different users – a *transmit-only* user and a *receive-only* user, which experiences BI. The existing asymmetric TWR transceiver designs constraints the number of relay antennas to cancel the BI. The proposed transceiver relaxes these antenna constraints and works seamlessly for both asymmetric and symmetric TWR. Further, the design also enables TWR communication between multiple users and a base station. The proposed design is also shown to have lower complexity than the existing designs. For the proposed transceiver, we maximize its sum rate using geometric programming for different TWR scenarios. We demonstrate using exhaustive numerical simulations that the sum-rate of the proposed design not only matches the best known designs in asymmetric and symmetric TWR literature, but also outperforms them for certain antenna configurations.**

***Index Terms*—Asymmetric two-way relaying (ATWR), geometric program (GP), generalized QR decomposition (GQRD), joint power allocation.**

I. Introduction

Cooperative communication is being investigated to increase the coverage and rate, and to reduce the power consumption in wireless systems [1]–[6]. Under the ambit of cooperative communication, symmetric two-way relaying (STWR) where two nodes exchange data using a half-duplex relay is being studied extensively [7]–[9]. In the first channel use in STWR, two source nodes transmit their data signals to the relay, which receives the sum of two signals. In the second channel use in STWR, the relay amplifies and forwards the sum-signal back to both source nodes. Since each node knows its first-channel-use self-signal, it can cancel self/back-propagating interference (BI) from its received signal and detect its desired signal. In STWR, two channel uses are thus required by two source nodes to exchange data between them self, which is half the number of channel uses required in conventional half-duplex one-way relaying [10]–[12]. This increases the spectral efficiency of STWR when compared with one-way relaying [7]–[9].

It is assumed in STWR that two source nodes exchange data via a relay. *In cellular systems, a user usually does not exchange data with a base station (BS)* [13]–[16]. Consider two examples: 1) a *transmit-only* user TUE that only sends data to the BS e.g., user uploading data to a cloud; and 2) a *receive-only* user RUE that only receives data from BS e.g., user watching a Youtube video. We observe that neither TUE nor RUE exchanges data with the BS. The BS therefore cannot use STWR to serve TUE or RUE. If the BS serves both users with spectrally-inefficient one-way relaying, it will require four channel uses (two for each user). By using the principles of asymmetric TWR (ATWR) [13]–[16], the BS serves both users in two channel uses as discussed next.

In the first channel use of ATWR as shown in Fig. 1, both BS and TUE transmit data signals to the relay. The BS transmits downlink data demanded by the RUE, while the TUE transmits uplink data which it wants to send to the BS. The relay receives a sum of these two signals. In the second channel use of ATWR, the relay amplifies and broadcasts its received signal to both BS and RUE. The BS in ATWR requires only two channel uses to serve both TUE and RUE. We observe that the BS has necessary side-information, that is its self-data, to cancel BI from its received sum-signal. The RUE does not have necessary side-information, that is TUE's transmit data, to cancel BI from its received signal.

This asymmetric two-way relaying scenario is considered earlier in [13]–[16]. In references [13], [14], the RUE overhears TUE in the first channel use and uses this side-information to cancel BI from its receive signal in the second channel use. The authors in [13], [14] design relay precoders to optimize sum-rate and minimize MSE, respectively. These studies consider single-antenna nodes except relay which has multiple antennas. It is non-trivial to extend their precoder design for the scenario when the BS and users also have multiple antennas. The transceiver herein, is contrast, is designed for all MIMO nodes. Also the RUE in [13], [14] first overhears and then jointly processes both overheard and desired signals, which complicates its receiver design. The RUE in the proposed transceiver neither requires overhearing nor joint processing which simplifies its receiver design. The proposed approach is also relevant for practical cellular systems wherein a user does not normally overhear another user's transmission.

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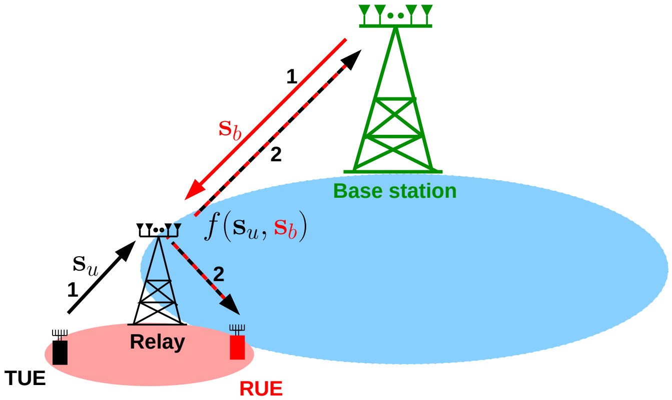

Fig. 1: *Illustration of ATWR*: In the first channel use (labeled '1'), both BS and TUE transmit to the relay. In the second channel use (labeled '2'), the relay broadcasts a function of the signal received in the first channel use to both BS and RUE.

Further, the designs in [13], [14] consider only two asymmetric users in the system whereas the proposed design can also work for multiple such asymmetric users.

References [15], [16] assume that the RUE cannot overhear the TUE and designed transceivers to cancel the BI and optimize quality-of-service constrained sum-rate in [15] and sum-rate alone in [16]. Transceiver designed in references [15], [16] impose the condition that the number of relay antennas $N_r \geq 2N_u$, where N_u is the number of user antennas. This condition is required to cancel the back-propagating interference experienced by the RUE. The proposed transceiver, in contrast, works for $N_r < 2N_u$ antennas and therefore has *relaxed antenna constraints* than the designs in [15], [16].

Many studies have similarly investigated multiple-input multiple-output (MIMO) transceiver designs for one-way relaying and STWR [4]–[9], [17]–[28]. References [4]–[6] develop unified transceiver designs for MIMO point-to-point and relay communication scenarios. Xing *et al.* in [4] derive optimal linear and non-linear transceiver structures under per-antenna power constraints for multi-hop MIMO AF relay communications. They show that the transceiver designs under per-antenna power constraints can be transformed into equivalent optimization under weighted sum power constraint. Reference [5] proposes a generic transceiver design framework for multi-hop communications and show that the framework includes various transceiver designs as its special cases e.g., linear transceiver designs with additively Schur-convex/concave objective functions and non-linear transceiver designs with multiplicatively Schur-convex/concave objective functions. Xing *et al.* in [6] show that for MIMO systems various optimization problems with matrix variables can be transformed into matrix-monotonic optimization problems After deriving the optimal transceiver solutions, the authors also analyze set of Pareto optimal solutions based on multi-objective optimization theory.

The authors in [7], [8] constructed space-time codes for MIMO STWR. References [9], [17] constructed zero-forcing (ZF) relay precoders and analyzed their sum-rate. Wang *et al.* constructed joint source and relay precoders in [19] to minimize sum-MSE. The authors in [18], [20] constructed source and relay precoders to maximize sum-rate. Transceiver designs to minimize the MSE, and maximize the sum-rate using MSE duality are developed in [22] and [23], respectively. Application of STWR to OFDM systems is considered in [26]. References [27] proposed a transceiver that uses generalized singular value decomposition and zero-forcing to orthogonalize the two end-to-end MIMO STWR channels.

Both ATWR and STWR are extended to include multiple users in [29]–[32]. Here receiving users experience both BI and co-channel interference (CCI) from multiple users transmitting/receiving on the same spectral resource.

Most of the transceiver designs reported in STWR and ATWR literature are designed exclusively for either symmetric or asymmetric scenarios. For example, STWR transceiver designs in [7]–[9], [17]–[28] inherently assume that both source nodes can cancel their BI; these designs are therefore unsuitable for ATWR wherein the RUE cannot cancel its BI. Similarly, ATWR transceivers in [15], [16], as mentioned before, require $N_r \geq 2N_u$ antennas to cancel RUE's BI. These designs cannot work with $N_r < 2N_u$ antennas, and are therefore restrictive for STWR, for which the existing transceivers work for all N_r values. Further, the single-user MIMO STWR and ATWR designs in [7]–[9], [17]–[28], [32] are not applicable for multi-user STWR and ATWR as they perform joint processing at the receiver nodes.

We next list the **main contributions** of this paper.

1) We propose a novel transceiver design that overcomes the antenna constraints of existing designs which limits their applicability. The proposed transceiver requires $N_b \geq N_r \geq N_u$ antennas when compared with $N_r \geq 2N_u$ antennas. The relaxed antenna constraints of the proposed design enables it to work for symmetric, asymmetric and multi-user TWR scenarios. The proposed transceiver design utilizes generalized QR decomposition (GQRD) [33], which jointly decomposes two matrices into a set of unitary matrices and upper triangular matrices. For $N_r \geq 2N_u$, we show that with the GQRD of MIMO channel matrices, along with a novel anti-diagonal permutation matrix, the proposed transceiver creates two spatially orthogonal end-to-end MIMO channels by cancelling the BI experienced by the RUE. This configuration is therefore suitable for ATWR. For $N_r < 2N_u$, the proposed design creates two end-to-end MIMO channels which experience BI, which makes this configuration suitable for STWR where RUE can cancel its BI.

The proposed transceiver design is therefore more general than the existing designs in [7]–[9], [16]–[28], [32] which construct separate transceivers for symmetric, asymmetric and multi-user TWR scenario. This design consequently provides flexibility to a system designer as a single design now works for different TWR scenarios. We note that the references [4]–[6] design unified transceivers for point-to-point and multi-hop one-way relaying scenarios whereas the current work constructs a common transceiver for STWR and ATWR. In ATWR, the receiving user experience BI which needs to be canceled – the proposed common design is constructed with this constraint. This is unlike references [4]–[6] where the receiving user does not experience interference.

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2) For the proposed transceiver design, we optimally allocate power at the BS and relay to maximize its sum-rate. We will show that the sum-rate can be maximized as a series of convex geometric program (GP). We note that the optimization programs, similar to the proposed transceiver design, is applicable for symmetric, asymmetric and multi-user TWR scenarios. The proposed design is based on GQRD of MIMO channel matrices and employs geometric programming to optimize sum rate. This is different from [4]–[6] wherein the authors construct optimal designs using Majorization theory that transforms the objective as a function of the diagonal elements of mean-square-error matrix.

3) We also show that the proposed design has significantly lower computational complexity than the designs in in [15], [16].

4) We numerically demonstrate that the proposed design has nearly similar sum-rate as that of existing transceiver designs. We show that the proposed design also outperforms the existing STWR designs for certain antenna configurations.

To summarize, *the proposed design has following advantages over existing transceivers i) overcomes their antenna inflexibility which limits their applicability to different TWR scenarios; ii) has lower computational complexity; and iii) matches their sum-rate, and even outperforms them for certain antenna configurations*. To the best of our knowledge, such a low-complexity common design for symmetric, asymmetric and multi-user TWR scenarios with optimal sum-rate power allocation has not been considered earlier in the literature.

The rest of the paper is organized as follows. We discuss the system model and the GQRD-based transceiver design in Section II and Section III, respectively. The optimization program to jointly allocate power at the BS and relay to maximize sum-rate is formulated in Section IV. We numerically compare the sum-rate of the proposed design with the existing ones in Section V, and conclude the paper in Section VI.

Our notation is as follows. Bold upper- and lower-case letters are used to denote matrices and column vectors, respectively. The symbols $\text{Tr}(\mathbf{A})$, $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^*$ and $[\mathbf{A}]_{i,j}$ respectively the trace, transposition, conjugate-transposition, complex-conjugate of the elements and the (i,j)th element of matrix $\mathbf{A}$. The $\mathbf{A} \odot \mathbf{B}$ denotes the Hadamard product of two matrices $\mathbf{A}$ and $\mathbf{B}$. Further the symbol $\mathbf{I}_N$ denotes an $N \times N$ identity matrix, and $\text{diag}(x_1, \cdots, x_n)$ denotes a diagonal matrix with x_1 to x_n as its diagonal elements. The $\boldsymbol{x}_n$ denotes nth component of a vector $\boldsymbol{x}$. The notation $\boldsymbol{x} \succeq 0$ implies that all elements of the vector $\boldsymbol{x}$ are ≥ 0. The symbol $\mathbb{E}(\cdot)$ represents the expectation operator.

II. System model

A relay in two-way relaying is usually based on either regenerative technology [34], [35] or non-regenerative technology [36]–[38]. A non-regenerative relay is easy to implement and also has lower delay. We also assume a non-regenerative amplify-and-forward relay in this work. We first describe the system model for amplify-and-forward ATWR which we will later extend to STWR. We consider MIMO ATWR where a transmit-only user TUE sends data to the BS and another receive-only user RUE demands data from the BS. Both users communicate with the BS via a half-duplex relay; the direct links between the users and the BS are assumed to be weak due to shadowing and are ignored in this study. The BS and the relay have N_b and N_r antennas respectively, while the users TUE and RUE have N_u antennas each. In the first channel use of ATWR, commonly known as multiple access (MAC) phase, both BS and TUE simultaneously transmit their respective signals to the relay. The relay receives a sum-signal $\mathbf{y}_r \in \mathbb{C}^{N_r \times 1}$ given as

$$\mathbf{y}_r = \mathbf{H}_u \mathbf{s}_u + \mathbf{H}_b \mathbf{s}_b + \mathbf{n}_r. \tag{1}$$

Here matrices $\mathbf{H}_u \in \mathbb{C}^{N_r \times N_u}$ and $\mathbf{H}_b \in \mathbb{C}^{N_r \times N_b}$ denote the MIMO channels of the TUE→RS and BS→RS links, respectively. The vector $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$ is circularly-symmetric complex Gaussian noise at the relay with $\mathbb{E}(\mathbf{n}_r \mathbf{n}_r^H) = \sigma_r^2 \mathbf{I}_{N_r}$. The BS and user TUE generate their respective transmit vectors, $\mathbf{s}_b \in \mathbb{C}^{N_b \times 1}$ and $\mathbf{s}_u \in \mathbb{C}^{N_u \times 1}$ by precoding normalized complex source vectors as following.

$$\mathbf{s}_b = \mathbf{B}_b \tilde{\mathbf{s}}_b \text{ and } \mathbf{s}_u = \mathbf{B}_u \tilde{\mathbf{s}}_u. \tag{2}$$

The matrices $\mathbf{B}_b$ and $\mathbf{B}_u$ are precoders used by the BS and TUE, respectively. We assume that with N_u antennas, user TUE sends N_u independent source streams to the BS. Similarly with N_u antennas, user TUE demands N_u independent source streams from the BS. We therefore have $\mathbb{E}(\tilde{\mathbf{s}}_b \tilde{\mathbf{s}}_b^H) = \mathbb{E}(\tilde{\mathbf{s}}_u \tilde{\mathbf{s}}_u^H) = \mathbf{I}_{N_u}$. The precoded vectors satisfy the total power constraint:

$$\text{Tr}(\mathbb{E}(\mathbf{s}_i \mathbf{s}_i^H)) = \text{Tr}(\mathbf{B}_i \mathbf{B}_i^H) \triangleq \text{Tr}(\boldsymbol{\Sigma}_i) \leq P_i, \tag{3}$$

for $i \in \{u, b\}$. The terms P_u and P_b denote the maximum transmit power of the TUE and BS, respectively.

In the second channel use of ATWR, commonly known as broadcast (BC) phase, the relay generates its transmit signal $\mathbf{s}_r \in \mathbb{C}^{N_r \times 1}$ by amplifying its received signal using a precoder matrix $\mathbf{W} \in \mathbb{C}^{N_r \times N_r}$ such that

$$\mathbf{s}_r = \mathbf{W}\mathbf{y}_r. \tag{4}$$

The precoder $\mathbf{W}$ is designed to satisfy the inequality

$$\begin{aligned} P_r &\geq \text{Tr}\left(\mathbb{E}(\mathbf{s}_r \mathbf{s}_r^H)\right) = p_r(\mathbf{W}) \\ &= \text{Tr}\left(\mathbf{W}(\mathbf{H}_u \boldsymbol{\Sigma}_u \mathbf{H}_u^H + \mathbf{H}_b \boldsymbol{\Sigma}_b \mathbf{H}_b^H + \sigma_r^2 \mathbf{I}_{N_r})\mathbf{W}^H\right), \end{aligned} \tag{5}$$

to meets the relay transmit power constraint. Here the symbol P_r denotes the maximum transmit power of the relay. The signals received by the RUE and BS, $\mathbf{y}_u$ and $\mathbf{y}_b$ respectively, in the BC phase are given as

$$\begin{aligned} \mathbf{y}_u &= \mathbf{G}_u \mathbf{s}_r + \mathbf{n}_u \\ &= \underbrace{\mathbf{G}_u \mathbf{W} \mathbf{H}_u \mathbf{s}_u}_{\text{BI-RUE}} + \mathbf{G}_u \mathbf{W} \mathbf{H}_b \mathbf{s}_b + \underbrace{\mathbf{G}_u \mathbf{W} \mathbf{n}_r + \mathbf{n}_u}_{\triangleq \tilde{\mathbf{n}}_u}, \\ \mathbf{y}_b &= \mathbf{G}_b \mathbf{s}_r + \mathbf{n}_b \\ &= \mathbf{G}_b \mathbf{W} \mathbf{H}_u \mathbf{s}_u + \underbrace{\mathbf{G}_b \mathbf{W} \mathbf{H}_b \mathbf{s}_b}_{\text{BI-BS}} + \underbrace{\mathbf{G}_b \mathbf{W} \mathbf{n}_r + \mathbf{n}_b}_{\triangleq \tilde{\mathbf{n}}_b}. \end{aligned} \tag{6}$$

The terms labelled BI-RUE and BI-BS denote the BI experienced by the RUE and BS, respectively. The matrices

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$\mathbf{G}_u \in \mathbb{C}^{N_u \times N_r}$ and $\mathbf{G}_b \in \mathbb{C}^{N_b \times N_r}$ are the MIMO channels of the RS→RUE and RS→BS links, respectively. The terms $\mathbf{n}_u \in \mathbb{C}^{N_u \times 1}$ and $\mathbf{n}_b \in \mathbb{C}^{N_b \times 1}$ are circularly-symmetric complex Gaussian noise vectors at the RUE and BS with $\mathbb{E}(\mathbf{n}_u \mathbf{n}_u^H) = \sigma_u^2 \mathbf{I}_{N_u}$ and $\mathbb{E}(\mathbf{n}_b \mathbf{n}_b^H) = \sigma_b^2 \mathbf{I}_{N_b}$.

Since the BS knows its self-data transmitted in the MAC phase, it can cancel BI with the availability of necessary channel state information. The precoder $\mathbf{W}$ is designed such that it will cancel the BI experienced by the RUE. Both TUE and BS (after cancelling BI) linearly combine their respective signals using matrices $\mathbf{D}_u \in \mathbb{C}^{N_u \times N_u}$ and $\mathbf{D}_b \in \mathbb{C}^{N_u \times N_b}$ such that

$$\tilde{\mathbf{y}}_b = \mathbf{D}_b \mathbf{y}_b = \mathbf{D}_b \mathbf{G}_b \mathbf{W} \mathbf{H}_u \mathbf{s}_u + \mathbf{D}_b \mathbf{G}_b \mathbf{W} \mathbf{n}_r + \mathbf{D}_b \mathbf{n}_b, \quad (7)$$

$$\tilde{\mathbf{y}}_u = \mathbf{D}_u \mathbf{y}_u = \mathbf{D}_u \mathbf{G}_u \mathbf{W} \mathbf{H}_b \mathbf{s}_b + \mathbf{D}_u \mathbf{G}_u \mathbf{W} \mathbf{n}_r + \mathbf{D}_u \mathbf{n}_u. \quad (8)$$

Remark 1: Extension of ATWR system model for STWR: In STWR, as discussed before, a user (say UE) exchanges data with the BS i.e., sends data to the BS and also receives data from the BS. In the MAC phase of STWR, therefore, the UE, similar to the TUE, will precode and transmit its signal to the relay. The BS transmit signal in the MAC phase of STWR is meant for the UE. In the BC phase in STWR, the UE unlike the RUE in ATWR, will have its MAC phase transmit data and will be able to cancel its BI. The UE, after cancelling the BI, will process its data similar to the RUE in ATWR. To conclude, the system model for ATWR and STWR are similar except for the fact that the UE in STWR can cancel its BI.

III. Common transceiver design for ATWR and STWR

In this section, we design linear precoders ($\mathbf{B}_u$, $\mathbf{B}_b$ and $\mathbf{W}$) and linear receivers ($\mathbf{D}_u$ and $\mathbf{D}_b$) to satisfy the following objectives.

1) Cancels the RUE's BI in ATWR.
2) Works for STWR without restrictive constraints on the number of relay antennas.

The existing state-of-the-art designs only partially satisfies these objectives e.g., the designs in [16], [32] cancels RUE's BI but imposes the constraint that $N_r \geq 2N_u$ antennas. These design only satisfy the first objective. Similarly, the STWR designs e.g., in [7]–[9], [17]–[22] work for all N_r but assume that the receiving nodes can cancel their BI; these designs therefore satisfy the second objective alone. In the sequel we construct a GQRD-based transceiver that requires $N_b \geq N_r \geq N_u$ antennas and satisfies both of the above objectives. Additionally, we show that the proposed transceiver works for multi-user TWR also. For the proposed transceiver design we assume that the relay i) has complete information of all channel matrices i.e., $\mathbf{H}_u$, $\mathbf{H}_b$, $\mathbf{G}_u$ and $\mathbf{G}_b$; and ii) designs the precoder/receiver matrices and distributes them to other nodes. The availability of complete channel information at the relay and design/distribution of matrices by the relay is frequently assumed in the literature [19], [21].

A. GQRD Precoder Design

We begin this section by outlining the principles of GQRD-based transceiver design.

We start by using the following theorem from [33].

Theorem 3.1: Let $\mathbf{A}$ be an $n \times m$ matrix, $\mathbf{B}$ an $n \times p$ and assume that $p \leq n \leq m$. Then there are orthogonal matrices $\mathbf{Q}$ $(n \times n)$ and $\mathbf{U}(m \times m)$ such that

$$\mathbf{Q}^H \mathbf{A} \mathbf{U} = \mathbf{R}, \ \mathbf{Q}^H \mathbf{B} = \mathbf{S}, \quad (9)$$

where

$$\mathbf{R} = [\underbrace{\mathbf{0}}_{m-n} \ \underbrace{\widetilde{\mathbf{R}}}_{n}] n \quad (10)$$

and

$$\mathbf{S} = \underbrace{\begin{bmatrix} \widetilde{\mathbf{S}} \\ \mathbf{0} \end{bmatrix}}_{p} \begin{matrix} p \\ n-p \end{matrix}, \quad (11)$$

with $\widetilde{\mathbf{R}}$ and $\widetilde{\mathbf{S}}$ being upper triangular matrices.

By applying the above theorem to MAC-phase channels, we have with $\mathbf{A} = \mathbf{H}_b$ and $\mathbf{B} = \mathbf{H}_u$

$$\mathbf{Q}_h^H \mathbf{H}_b \mathbf{U}_h = \mathbf{R}_h \text{ and } \mathbf{Q}_h^H \mathbf{H}_u = \mathbf{S}_h, \quad (12)$$

where

$$\mathbf{R}_h = [\underbrace{\mathbf{0}}_{N_b - N_r} \ \underbrace{\widetilde{\mathbf{R}}_h}_{N_r}] \ N_r \quad (13)$$

and

$$\mathbf{S}_h = \underbrace{\begin{bmatrix} \widetilde{\mathbf{S}}_h \\ \mathbf{0} \end{bmatrix}}_{N_u} \begin{matrix} N_u \\ N_r - N_u \end{matrix}. \quad (14)$$

Here both $\widetilde{\mathbf{R}}_h$ and $\widetilde{\mathbf{S}}_h$ are upper-triangular matrices. Similarly, by applying this theorem to BC-phase channels with $\mathbf{A} = \mathbf{G}_b^H$ and $\mathbf{B} = \mathbf{G}_u^H$ we have

$$\mathbf{U}_g^H \mathbf{G}_b \mathbf{Q}_g = \mathbf{R}_g^H \text{ and } \mathbf{G}_u \mathbf{Q}_g = \mathbf{S}_g^H, \quad (15)$$

where

$$\mathbf{R}_g^H = \underbrace{\begin{bmatrix} \mathbf{0} \\ \widetilde{\mathbf{R}}_g^H \end{bmatrix}}_{N_r} \begin{matrix} N_b - N_r \\ N_r \end{matrix}, \quad (16)$$

and

$$\mathbf{S}_g^H = [\underbrace{\widetilde{\mathbf{S}}_g^H}_{N_u} \ \underbrace{\mathbf{0}}_{N_r - N_u}] \ N_u. \quad (17)$$

Here both $\widetilde{\mathbf{R}}_g^H$ and $\widetilde{\mathbf{S}}_g^H$ are lower-triangular matrices. We note that with the above decomposition, the matrices $\mathbf{Q}_g$, $\mathbf{U}_g$, $\mathbf{Q}_h$ and $\mathbf{U}_h$ are fixed unitary matrices which reduce MIMO channels into upper-triangular matrices. With this decomposition, we choose the MAC-phase transmit precoder of the BS and the transmit user (TUE/UE) respectively as:

$$\mathbf{B}_b = [\overline{\mathbf{U}_h}] \boldsymbol{\Lambda} \text{ and } \mathbf{B}_u = \rho \mathbf{I}. \quad (18)$$

Here we use the notation $[\overline{\mathbf{A}}]$ to denote the last N_u columns of a matrix $\mathbf{A}$. Here $\boldsymbol{\Lambda} \in \mathbb{R}_+^{N_u \times N_u}$ is a diagonal power allocation

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matrix at the BS such that $\mathbf{\Lambda}\mathbf{\Lambda}^H = \text{diag}(\lambda_1, \cdots \lambda_{N_u})$. The scalar $\rho = P_u/N_u$ satisfies the transmit power constraint of the transmit user. Similarly, the BC-phase receiver for the BS and the receive user (RUE/UE) are respectively chosen as:

$$\mathbf{D}_b = [\underline{\mathbf{U}_g^H}] \text{ and } \mathbf{D}_u = \mathbf{I}, \tag{19}$$

where the notation $[\underline{\mathbf{A}}]$ denotes the last N_u rows of a matrix $\mathbf{A}$. We also choose the relay precoder as

$$\mathbf{W} = \mathbf{Q}_g \mathbf{\Delta} \mathbf{Q}_h^H, \tag{20}$$

where $\mathbf{\Delta}$ is an anti-diagonal permutation and power allocation matrix such that $\mathbf{\Delta}\mathbf{\Delta}^H = \text{diag}(\delta_1, \cdots, \delta_{N_r})$. With the precoder/decoder in (18) and (19), the BC-phase receive signals at the BS in (6) can be expressed as

$$\begin{aligned}\tilde{\mathbf{y}}_b &= [\underline{\mathbf{U}_g^H}]\mathbf{G}_b\mathbf{Q}_g\mathbf{\Delta}\mathbf{Q}_h^H\mathbf{H}_u\mathbf{s}_u + [\underline{\mathbf{U}_g^H}]\mathbf{G}_b\mathbf{Q}_g\mathbf{\Delta}\mathbf{Q}_h^H\mathbf{n}_r + [\underline{\mathbf{U}_g^H}]\mathbf{n}_b \\ &\overset{(a)}{=} \rho[\underline{\widetilde{\mathbf{R}}_g^H}]\mathbf{\Delta}\mathbf{S}_h\tilde{\mathbf{s}}_u + [\underline{\widetilde{\mathbf{R}}_g^H}]\mathbf{\Delta}\mathbf{Q}_h^H\mathbf{n}_r + [\underline{\mathbf{U}_g^H}]\mathbf{n}_b \\ &\overset{(b)}{=} \rho[\underline{\widetilde{\mathbf{R}}_g^H}]\mathbf{\Delta}\mathbf{S}_h\tilde{\mathbf{s}}_u + \underbrace{[\underline{\widetilde{\mathbf{R}}_g^H}]\mathbf{\Delta}\mathbf{n}_r + \mathbf{n}_b}_{\tilde{\mathbf{n}}_b}.\end{aligned} \tag{21}$$

The equality in (a) is due to (12) and (15), and the equality in (b) is because $\mathbf{Q}_h^H\mathbf{n}_r$ and $[\underline{\mathbf{U}_g^H}]\mathbf{n}_b$ have same statistical properties as $\mathbf{n}_r$ and $\mathbf{n}_b$, respectively. The lower-triangular $\mathbf{R}_g^H$, anti-diagonal $\mathbf{\Delta}$ and upper-triangular $\mathbf{S}_h$ result in the reflected lower-triangular structure of the end-to-end channel between $\widetilde{\mathbf{y}}_b$ and $\tilde{\mathbf{s}}_u$ as

$$\begin{bmatrix} \tilde{y}_{b,1} \\ \tilde{y}_{b,2} \\ \vdots \\ \tilde{y}_{b,N_u-1} \\ \tilde{y}_{b,N_u} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & * \\ 0 & & & * & * \\ \vdots & & \iddots & \vdots & \vdots \\ 0 & * & \cdots & * & * \\ * & * & \cdots & * & * \end{bmatrix} \begin{bmatrix} \tilde{s}_{u,1} \\ \tilde{s}_{u,2} \\ \vdots \\ \tilde{s}_{u,N_u-1} \\ \tilde{s}_{u,N_u} \end{bmatrix} + \tilde{\mathbf{n}}_b. \tag{22}$$

With this channel structure, the last transmit stream ($\tilde{s}_{u,N_u}$) is free from inter-stream interference and is detected first. The rest of the transmit streams are then detected by successive interference cancellation (SIC) from *previously detected* streams.

The BC-phase receive signal of the receive user (RUE/UE) is:

$$\begin{aligned}\tilde{\mathbf{y}}_u &= \mathbf{G}_u\mathbf{W}\mathbf{H}_b\mathbf{s}_b + \mathbf{G}_u\mathbf{W}\mathbf{n}_r + \mathbf{n}_u, \\ &= \mathbf{G}_u\mathbf{Q}_g\mathbf{\Delta}\mathbf{Q}_h^H\mathbf{H}_b[\overline{\mathbf{U}_h}]\mathbf{\Lambda}_u\tilde{\mathbf{s}}_b + \mathbf{G}_u\mathbf{W}\mathbf{n}_r + \mathbf{n}_u, \\ &= \mathbf{S}_g^H\mathbf{\Delta}[\overline{\widetilde{\mathbf{R}}_h}]\mathbf{\Lambda}_b\tilde{\mathbf{s}}_b + \underbrace{\mathbf{S}_g^H\mathbf{\Delta}\mathbf{n}_r + \mathbf{n}_u}_{\tilde{\mathbf{n}}_u}\end{aligned} \tag{23}$$

The end-to-end user channel also has a similar structure as that of the BS, and therefore the receive user (RUE/UE) also employs SIC to detect its desired data. With SIC, the SNR of the nth receive stream of the BS and receive user are

$$\begin{aligned}\text{SNR}_{b,n}(\boldsymbol{\delta}) &= \frac{\rho^2\delta_{\tilde{n}}(|[\widetilde{\mathbf{R}}_g^H]_{\tilde{n},\tilde{n}}[\widetilde{\mathbf{S}}_h]_{\tilde{j},\tilde{j}}|^2)}{\sigma_r^2\sum_{l=1}^{N_r}\delta_l[\widetilde{\mathbf{R}}_g^H \odot \widetilde{\mathbf{R}}_g^T]_{n,l} + \sigma_b^2}, \\ \text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda}) &= \frac{\delta_n\lambda_{\tilde{j}}(|[\widetilde{\mathbf{S}}_g^H]_{n,n}[\widetilde{\mathbf{R}}_h]_{j,j}|^2)}{\sigma_r^2\sum_{l=1}^{N_r}\delta_l[\widetilde{\mathbf{S}}_g^H \odot \widetilde{\mathbf{S}}_g^T]_{n,l} + \sigma_u^2}.\end{aligned} \tag{24}$$

Here $n = 1$ to N_u, $\tilde{n} = N_r - N_u + n$, $j = N_r - n + 1$ and $\tilde{j} = N_u - n + 1$. Further $\boldsymbol{\delta} = [\delta_1, \cdots, \delta_{N_r}]$ and $\boldsymbol{\lambda}_i = [\lambda_{i,1}, \cdots, \lambda_{i,N_u}]$. These expressions can be derived from (21) and (23) after simple algebraic manipulations. Note that the coefficients of the power-distribution variables, δ_l for $l = 1$ to N_r, λ_{1n} and λ_{2n} for $n = 1$ to N, are non-negative. We will use this fact to solve the sum-rate optimization as a sequence of geometric programs. The proposed GQRD transceiver design is summarized below in Algorithm 1.

Algorithm 1: GQRD Transceiver Design.

1: Choose the following precoders at the BS and the transmit user respectively.
   - $\mathbf{B}_b = \mathbf{U}_h\mathbf{\Lambda}_b$ and $\mathbf{B}_u = \rho\mathbf{I}$.

2: Choose relay precoder $\mathbf{W} = \mathbf{Q}_g\mathbf{\Delta}\mathbf{Q}_h^H$.

3: Choose the following receivers at the BS and receive user respectively.
   - $\mathbf{D}_b = \mathbf{U}_g^H$ and $\mathbf{D}_u = \mathbf{I}$.

B. BI cancelling property of the proposed design for ATWR

The objective of this section is to demonstrate that the proposed relay precoder will cancel RUE's BI for ATWR scenario. To this end we state the following lemma.

Lemma 3.2: The proposed precoder will cancel the RUE's BI if the number of antennas at the relay satisfies the following constraint: $N_r \geq 2N_u$.

Proof: To prove the lemma, we reproduce the RUE's receive signal from (6):

$$\begin{aligned}\mathbf{y}_u &= \mathbf{G}_u\mathbf{W}\mathbf{H}_u\mathbf{s}_u + \mathbf{G}_u\mathbf{W}\mathbf{H}_b\mathbf{s}_b + \tilde{\mathbf{n}}_u \\ &= \mathbf{G}_u\mathbf{Q}_g\mathbf{\Delta}\mathbf{Q}_h^H\mathbf{H}_u\tilde{\mathbf{s}}_u + \mathbf{G}_u\mathbf{Q}_g\mathbf{\Delta}\mathbf{Q}_h^H\mathbf{H}_b\mathbf{U}_h\mathbf{\Lambda}_u\tilde{\mathbf{s}}_b + \tilde{\mathbf{n}}_u, \\ &\overset{(a)}{=} \underbrace{\rho\mathbf{S}_g^H\mathbf{\Delta}\mathbf{S}_h}_{\text{BI-UE}}\tilde{\mathbf{s}}_u + \mathbf{S}_g^H\mathbf{\Delta}\mathbf{R}_h\mathbf{\Lambda}_b\tilde{\mathbf{s}}_b + \tilde{\mathbf{n}}_u\end{aligned} \tag{25}$$

The equality in (a) is due to (12), (15) and (18). With $N_r \geq 2N_u$, the last N_u rows (resp. columns) of matrix $\mathbf{S}_h$ (resp. $\mathbf{S}_g^H$) are zero. With these structure of the matrices, and anti-diagonal power allocation matrix $\mathbf{\Delta}$ we have

$$\text{BI-UE} = \rho\mathbf{S}_g^H\mathbf{\Delta}\mathbf{S}_h = \rho\begin{bmatrix}\widetilde{\mathbf{S}}_g & \mathbf{0}\end{bmatrix}\mathbf{\Delta}\begin{bmatrix}\widetilde{\mathbf{S}}_h \\ \mathbf{0}\end{bmatrix} = \mathbf{0} \tag{26}$$

■

The anti-diagonal structure of the power allocation matrix suitably aligns the all-zeros columns and rows of $\mathbf{S}_g$ and $\mathbf{S}_h$ to nullify the BI-UE term. The anti-diagonal structure is therefore important in cancelling the BI-UE term.

Remark 2: The condition $N_r \geq 2N_u$, as seen from Lemma 3.2, is required only in ATWR to cancel the BI experienced by the RUE. In STWR, this condition is not required as the UE can itself cancel the BI.

Remark 3: Comment on the number of antennas: Recall that $N_b \geq N_r \geq N_u$ for STWR and $N_b \geq N_r \geq 2N_u$ for ATWR. This is a practical assumption as both BS and relay usually have more number of antennas than a user.

C. Multi-user transceiver design

We now extend the ATWR system with a *single multi-antenna* TUE/RUE to a scenario with *multiple single-antenna* TUEs/RUEs. In the multi-user ATWR as shown in Fig. 2, N_u single-antenna TUEs, TUE-1 to TUE-N_u, want to send data to the BS via the relay and N_u single-antenna RUEs, RUE-1 to RUE-N_u, want to receive data from the BS via the relay. We first describe the transceiver design for RUEs.

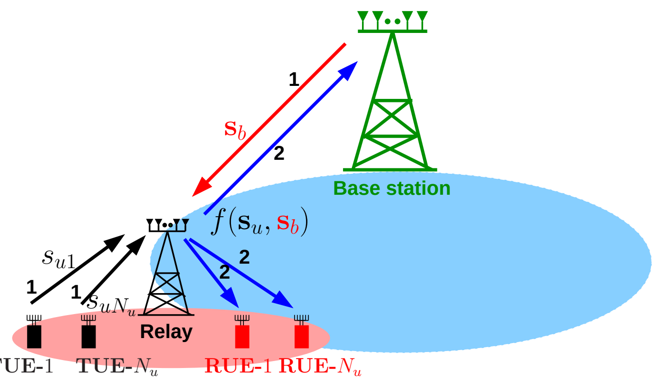

Fig. 2: *Illustration of multi-user ATWR*: In the MAC phase (labeled '1'), both BS and TUEs transmit to the relay. In the broadcast phase (labeled '2'), the relay broadcasts a function of the signal received in the first channel use to both BS and RUEs.

For this system model, we assume that the N_u-stream vector $\mathbf{s}_b$ transmitted earlier by the BS to a single RUE is now meant for N_u single-antenna RUEs. The RUEs, similar to single-user ATWR,[1] will experience BI from the MAC-phase signal of the TUEs. They will now also experience CCI from the $N_u - 1$ streams of the data vector $\mathbf{s}_u$ transmitted by the BS to the other $N_u - 1$ RUEs in the system. We now show that the proposed GQRD transceiver (summarized in Algorithm 1) works for the multi-user ATWR scenario also.

With $N_r \geq 2N_u$ antennas, the relay precoder will cancel the BI as before in single-user ATWR. The CCI experienced by the RUEs needs to cancelled either by the BS or by the relay. With the end-to-end channel structure for the BS→Relay→RUE-n links $\forall n$, given in (22), we propose that the BS employs zero-forcing dirty-paper coding (ZF-DPC) [39] to pre-cancel the CCI. The GQRD transceiver, with ZF-DPC, will ensure interference-free channel for all RUEs. The SNR of the receive signal of the nth RUE will be same as that of the nth stream in the single-user ATWR scenario (cf. (24)).

We now focus on the TUE-n→RS→BS links $\forall n$. In the proposed GQRD design for single-user ATWR, as TUE employs identity precoder, its N_u independent streams can equivalently assumed to be transmitted by N_u single-antenna TUEs. The BS will detect these N_u streams from N_u single-antenna users, as in the single-user scenario, with nth user experiencing the same SNR as that of the nth stream in single-user ATWR.

The proposed precoder, with ZF-DPC at the BS, can thus enable multi-user ATWR communication between a BS, N_u single-antenna TUEs and N_u single-antenna RUEs via relay.

[1]We refer the ATWR model with a TUE and a RUE as the single-user ATWR to differentiate it from the multi-user ATWR.

The power allocation to maximize sum-rate for multiple users can too be performed by solving the optimization discussed in the next section. The overall transceiver design procedure for three different scenarios viz: 1) ATWR; 2) STWR; and 3) multi-user ATWR is summarized in Algorithm 2.

Remark 4: Reference [40] developed a unified framework to design transceivers for various MIMO wireless systems. The transceiver framework is based on linear minimum mean square error (LMMSE) metric and uses quadratic matrix programming to construct the transceivers. The main differences between the current work and [40] is that the proposed transceiver design uses non-linear SIC to detect its desired data whereas the designs in [40] employs LMMSE receiver. Further, the proposed transceiver framework, as shown later in Remark 6, can easily be extended to include per-stream quality-of-service (QoS) constraints in the transceiver design. The QoS constraints are based on the per-stream rate demanded by the users.

Algorithm 2: Unified Transceiver Design Procedure

```
if Asymmetric two-way relaying then
    Set 2N_u ≤ N_r ≤ N_b;
    Precode the BS and relay transmit signal;
    Cancel BI experienced by the BS receiver;
    Perform beamforming and SIC at the BS receiver;
    Perform SIC at the RUE.
if Symmetric two-way relaying then
    Set N_u ≤ N_r ≤ N_b;
    Precode the BS and relay transmit signal;
    Cancel BI experienced by the BS and UE receivers;
    Perform beamforming and SIC at the BS receiver;
    Perform SIC at the UE receiver.
if Multi-user two-way relaying then
    Set 2N_u ≤ N_r ≤ N_b;
    Precode and employ DPC at the BS transmitter;
    Precode the relay transmit signal;
    Cancel BI experienced by the BS receiver;
    Perform beamforming and SIC at the BS receiver.
```

Before moving on to the next section where allocate optimal power for sum-rate maximization, we summarize the main result of the paper in Theorem 3.3.

Theorem 3.3: Design of a common transceiver for ATWR and STWR wherein relay requires $2N_u \leq N_r \leq N_b$ for ATWR scenario and $N_u \leq N_r \leq N_b$ for STWR scenario. In this design, we assume that the relay i) has complete information of all channel matrices i.e., $\mathbf{H}_u$, $\mathbf{H}_b$, $\mathbf{G}_u$ and $\mathbf{G}_b$; and ii) designs the precoder/receiver matrices and distributes them to other nodes.

IV. Geometric Programming Approach for Joint Power Allocation to Maximize Sum-Rate

We now allocate optimal power jointly at the BS and relay to maximize the system sum-rate. This power allocation, as mentioned before, is applicable for the three relaying scenarios. Geometric programming is a convex optimization tool that

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is used for allocating optimal power in wireless networks [32], [41]–[44]. We next prove two lemmas that will be used to show that the sum-rate optimization can be cast as a GP.

Lemma 4.1: The transmit power of the BS defined in (3), is a posynomial in optimization variable $\boldsymbol{\lambda}$.[2]

Proof: The transmit power of the BS with the transmit precoder $\mathbf{B}_b$ designed in (18) is

$$\text{Tr}(\mathbf{B}_b\mathbf{B}_b^H) = \text{Tr}(\mathbf{U}_h^H \boldsymbol{\Lambda}\boldsymbol{\Lambda}^H \mathbf{U}_h) \overset{(a)}{=} \sum_{j=1}^{N} \lambda_j. \tag{27}$$

The equality in (a) is due to the circular property of the trace operator. The BS transmit power is a posynomial in $\boldsymbol{\lambda}$, as the coefficients of optimization variable $\lambda_j, \forall j$ are positive. ■

The transmit power of the BS, for notational convenience, is henceforth denoted as $p_b(\boldsymbol{\lambda})$.

Lemma 4.2: The transmit power of the relay, defined in (5), is a posynomial in $\boldsymbol{\lambda}$ and $\boldsymbol{\delta}$, where $\boldsymbol{\delta} = [\delta_1, \cdots, \delta_{N_u}]$.

Proof: Refer to Appendix A. The transmit power of the relay is now denoted as $p_r(\boldsymbol{\delta}, \boldsymbol{\lambda})$. ■

A. Sum-rate maximization

We now maximize the sum-rate subject to the transmit power constraints of the BS and relay. We first define the sum-rate $R(\boldsymbol{\delta}, \boldsymbol{\lambda})$

$$\begin{aligned} &\overset{(a)}{=} \frac{1}{2} \sum_{n=1}^{N_u} \log_2(1 + \text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda})) + \log_2(1 + \text{SNR}_{b,n}(\boldsymbol{\delta})) \\ &= \frac{1}{2} \log_2 \prod_{n=1}^{N_u} (1 + \text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda}))(1 + \text{SNR}_{b,n}(\boldsymbol{\delta})) \end{aligned} \tag{28}$$

The factor of half is due to the half-duplex constraint. The sum-rate optimization can now be cast as

$$\begin{aligned} \mathbf{P}_1 : \underset{\{\boldsymbol{\lambda}, \boldsymbol{\delta}\} \succeq 0}{\text{Minimize}} \quad & \frac{1}{\prod_{n=1}^{N_u} (1 + \text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda}))(1 + \text{SNR}_{b,n}(\boldsymbol{\delta}))} \\ \text{subject to} \quad & p_b(\boldsymbol{\lambda}) \leq P_b, p_r(\boldsymbol{\delta}, \boldsymbol{\lambda}) \leq P_r \end{aligned} \tag{29}$$

In a GP, the objective function is a posynomial and inequality constraints are upper-bounded posynomials. We note that the posynomials are closed under addition and multiplication but not under division [45]. The objective in problem $\mathbf{P}_1$ is not a posynomial as it is a ratio of two posynomials; problem $\mathbf{P}_1$ therefore cannot be cast as a GP. An approach to solve this problem is to approximate the denominator posynomial as a monomial. The ratio of a posynomial and a monomial is a posynomial; the problem $\mathbf{P}_1$, with the posynomial objective and upper-bounded posynomial constraints, then becomes a GP. To this end, we now reproduce a lemma from [41].

Lemma 4.3: Let $g(\mathbf{x}) = \sum_i u_i(\mathbf{x})$ is a posynomial. Then

$$g(\mathbf{x}) \geq \widetilde{g}(\mathbf{x}) = \prod_i \left(\frac{u_i(\mathbf{x})}{\alpha_i}\right)^{\alpha_i}. \tag{30}$$

If, in addition, $\alpha_i = u_i(\mathbf{x}_0)/g(\mathbf{x}_0), \forall i$, for any fixed positive $\mathbf{x}_0$, then $\widetilde{g}(\mathbf{x}_0) = g(\mathbf{x}_0)$, and $\widetilde{g}(\mathbf{x}_0)$ is the best local monomial approximation to $g(\mathbf{x}_0)$ near $\mathbf{x}_0$ in the sense of first order Taylor approximation. Note that $u_i(\mathbf{x})$ are monomial terms.

Proof: Refer Lemma 1 of [41]. ■

We first use the approximation monomial obtained using the above lemma to solve problem $\mathbf{P}_2$ and improve this approximation using an iterative algorithm – Algorithm 3.

Algorithm 3: Sum-rate maximization

Input: A fixed threshold value ν, and initial feasible $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$.

Output: A close-to-optimal $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$.

Set $p = 1$ and calculate $R^0(\boldsymbol{\delta}, \boldsymbol{\lambda})$ using initial feasible $\boldsymbol{\delta}$;

while $\left(|R^p(\boldsymbol{\delta}, \boldsymbol{\lambda}) - R^{p-1}(\boldsymbol{\delta}, \boldsymbol{\lambda})| \geq \nu\right)$ **do**

- **Evaluate** the denominator posynomial in the objective of problem $\mathbf{P}_1$ with a given feasible optimization variables $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$;
- **Set** α_i for each ith term in this posynomial as $\alpha_i = \frac{\text{value of } i\text{th term in posynomial}}{\text{value of posynomial}}$;
- **Approximate** this posynomial into a monomial by using (30) with weights α_i;
- **Assign** $p \leftarrow p + 1$;
- **Solve** the GP to calculate $R^p(\boldsymbol{\delta}, \boldsymbol{\lambda})$ and a feasible $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$.

Remark 5: An initial feasible $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$ in Algorithm 3 can be calculated by making high-SNR approximation on the objective i.e., approximate (28) as $\frac{1}{2}\log_2 \prod_{n=1}^{N_u} (\text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda}))(\text{SNR}_{b,n}(\boldsymbol{\delta}))$. The objective now becomes a ratio of two posynomials and is therefore a posynomial.

Remark 6: Though in this work we focus on sum-rate maximization, but the proposed GP framework can easily be extended to include the per-stream QoS constraints, expressed in terms of per-stream rate, demanded by the user as following.

$$\begin{aligned} \mathbf{P}_2 : \underset{\{\boldsymbol{\lambda}, \boldsymbol{\delta}\} \succeq 0}{\text{Minimize}} \quad & \frac{1}{\prod_{n=1}^{N_u} (1 + \text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda}))(1 + \text{SNR}_{b,n}(\boldsymbol{\delta}))} \\ \text{subject to} \quad & \log_2(1 + \text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda})) \geq r_{un} \\ & \log_2(1 + \text{SNR}_{b,n}(\boldsymbol{\delta})) \geq r_{bn} \\ & p_b(\boldsymbol{\lambda}) \leq P_b, p_r(\boldsymbol{\delta}, \boldsymbol{\lambda}) \leq P_r \end{aligned} \tag{31}$$

The first and the second constraint impose the per-stream rate constraints demanded by the downlink and the uplink user, respectively. The per-stream rates demanded by the downlink and the uplink users are denoted as r_{un} and r_{bn}, respectively. The per-stream rate constraints are express as upper-bounded posynomials as follows.

$$\begin{aligned} \mathbf{P}_2 : \underset{\{\boldsymbol{\lambda}, \boldsymbol{\delta}\} \succeq 0}{\text{Minimize}} \quad & \frac{1}{\prod_{n=1}^{N_u} (1 + \text{SNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda}))(1 + \text{SNR}_{b,n}(\boldsymbol{\delta}))} \\ \text{subject to} \quad & \text{ISNR}_{u,n}(\boldsymbol{\delta}, \boldsymbol{\lambda}) \leq 1/(2^{r_{un}} - 1) \\ & \text{ISNR}_{b,n}(\boldsymbol{\delta}) \leq 1/(2^{r_{bn}} - 1) \\ & p_b(\boldsymbol{\lambda}) \leq P_b, p_r(\boldsymbol{\delta}, \boldsymbol{\lambda}) \leq P_r \end{aligned} \tag{32}$$

[2] GP terminology from [45]: A *monomial* is a function $f : \mathbf{R}_{++}^n :\rightarrow \mathbf{R}$ with domain $\mathbf{R}_{++}^n$, defined as $f(\mathbf{x}) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$, where $c > 0$ and $a_j \in \mathbf{R}$. A sum of monomials that is, $f(\mathbf{x}) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}$, where $c_k > 0$ is called a *posynomial*. Here $\mathbf{R}_{++}^n$ denotes the set of n-dimensional positive real vectors.

The above optimization can be solved using Algorithm 3 as the per-stream rate constraints are also upper-bounded posynomials.

V. Numerical results

We now use Monte Carlo simulations to study the performance of the proposed GQRD design, and compare it with existing state-of-the-art designs. We assume that the individual elements of channel matrices $\mathbf{H}_i$ and $\mathbf{G}_i$ are independent and identically distributed Gaussian random variables with zero mean and variance h_i^2 and g_i^2 respectively. We fix the maximum transmit power and noise variance at all the nodes to unity, and define the average SNR of the user/BS$\rightarrow$Relay link as $\eta_h = h_u^2 = h_b^2$ and Relay$\rightarrow$user/BS links as $\eta_g = g_u^2 = g_b^2$. We further assume that all transmit nodes employ Gaussian signalling. We average the simulation results over 5000 statistically independent channel realizations.

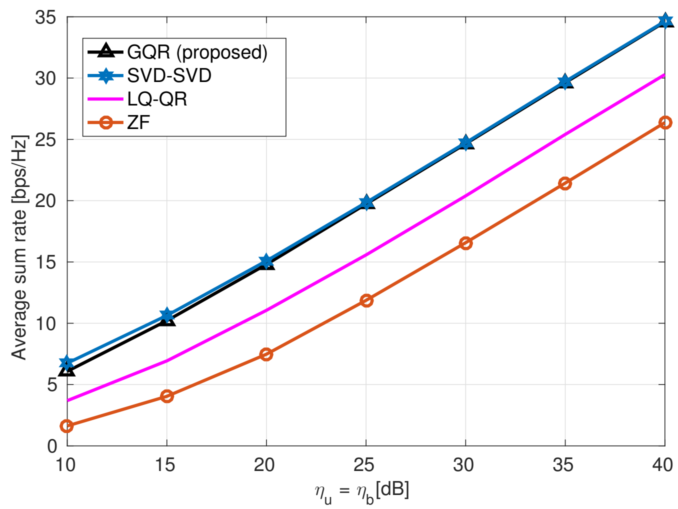
A. Single-user ATWR

We first compare the sum-rate of the proposed transceiver with the existing single-user ATWR designs. We consider following transceivers for this comparison: i) LQ-QR [16]; ii) SVD-SVD [32]; and iii) ZF-ZF [17]. We note that among the transceiver designs available in the ATWR literature, the SVD-SVD design provides the best sum-rate.[3] We also note that LQ-QR, SVD-SVD, and ZF-ZF designs, similar to the current design, require complete channel information at the relay to construct relay precoder. The SVD design, similar to the proposed design, distributes precoding matrices to nodes. Since ZF and LQ-QR designs do not precode at the source nodes, the relay does not distribute precoders to the source nodes. From Fig. 3a, where we have set $\eta_u = \eta_b$ and have fixed the number of BS, relay and user antennas as $N_b = 6, N_r = 6$ and $N_u = 3$ respectively, we see that the proposed GQRD design yields nearly the same average sum-rate as that of the SVD-SVD design. We also observe that the GQRD design vastly outperforms the other two ATWR designs. In Fig. 3b where we now fix $\eta_b = 30$ dB instead of varying it as in Fig. 3a, we see that the GQRD design mimics its behavior in Fig. 3a wherein it: 1) has the same sum-rate as that of the SVD-SVD design; and 2) outperforms LQ-QR and ZF-ZF designs for all SNR values.

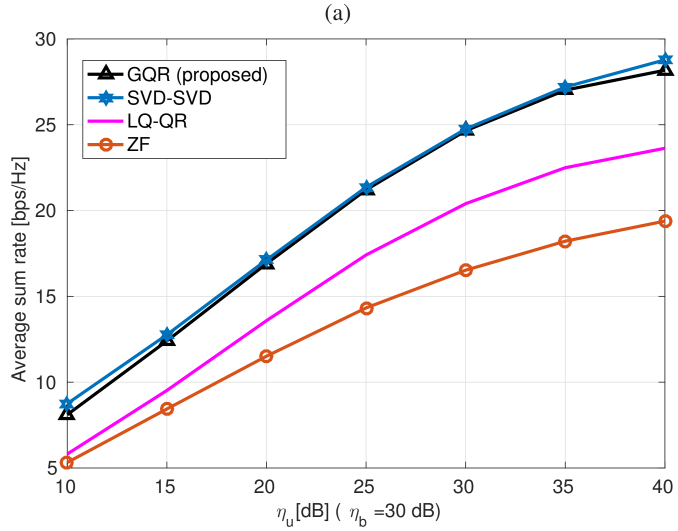
We now compare the computational complexity of the proposed GQRD design with that of the best-performing SVD-SVD design. The computational complexity for the proposed design, according to the results in [46], is $4N_bN_r^2$ flops. The computational complexity for the SVD-SVD design, by using the R-SVD algorithm in [47], is $12N_bN_r^2{+}40N_r^3{+}50N_u^3$ flops. We plot the number of flops required for these two designs in Fig. 4. We see that for $N_b = 20$, the GQRD design requires $\approx 9\times$ lesser flops than the SVD-SVD design. The proposed design thus matches the performance of the SVD-SVD design with lower design complexity.

The proposed GQRD transceiver and the existing LQ-QR, SVD-SVD and ZF-ZF ATWR transceivers require $N_r \geq 2N_u$ antennas to cancel the BI experienced by the receiving user. The existing ATWR transceiver designs, however, cannot work with $N_r < 2N_u$ antennas. We note that ATWR transceivers, including the proposed GQRD design, can also be used for STWR scenario wherein they will cancel the BI experienced by receiving user, but will require $N_r \geq 2N_u$ antennas. The existing STWR designs, in contrast, work with $N_r < 2N_u$ antennas as the receiving user therein can itself cancel the BI. Imposing $N_r \geq 2N_u$ antenna requirement is therefore restrictive for STWR. The proposed GQRD design, in contrast to the existing ATWR designs, works also with $N_r < 2N_u$ antennas and is thus more relevant for the STWR scenario when compared with existing ATWR designs. We will show that the proposed GQRD transceiver not only matches the sum-rate of the existing STWR designs that provide close-to-optimal performance, but also outperforms them for certain antenna configurations. Before doing that, let us briefly digress

[3]Optimal sum-rate transceiver design for ATWR is still an open problem.



(a)



(b)

Fig. 3: Average sum-rate comparison of the proposed GQRD transceiver design with other state-of-the-art *single-user* ATWR designs for two different scenarios :a) $\eta_u = \eta_b$ dB; and b) $\eta_b = 30$ dB. For both these figures, the number of BS and relay antennas $N_b = N_r = 6$, and the number of user antennas $N_u = 3$.

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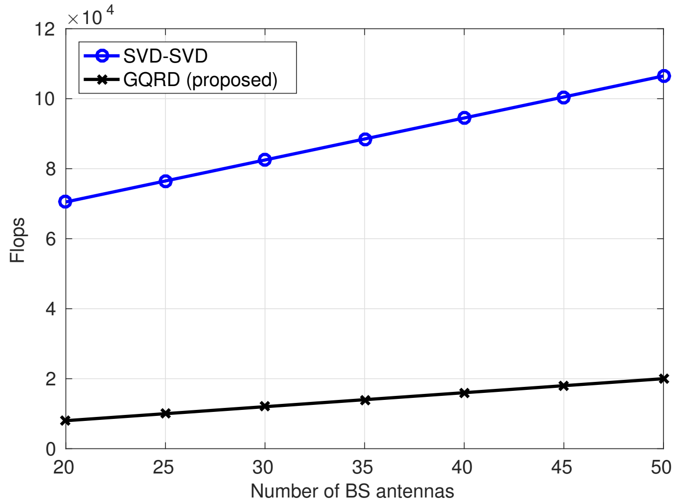


Fig. 4: Computational complexity comparison of the proposed GQRD transceiver design with SVD-SVD design. Here we vary the number of BS antennas (N_b) and fix $N_r = 10$ and $N_u = 5$.

to compare the sum-rate of the GQRD design with existing multi-user ATWR transceiver designs.

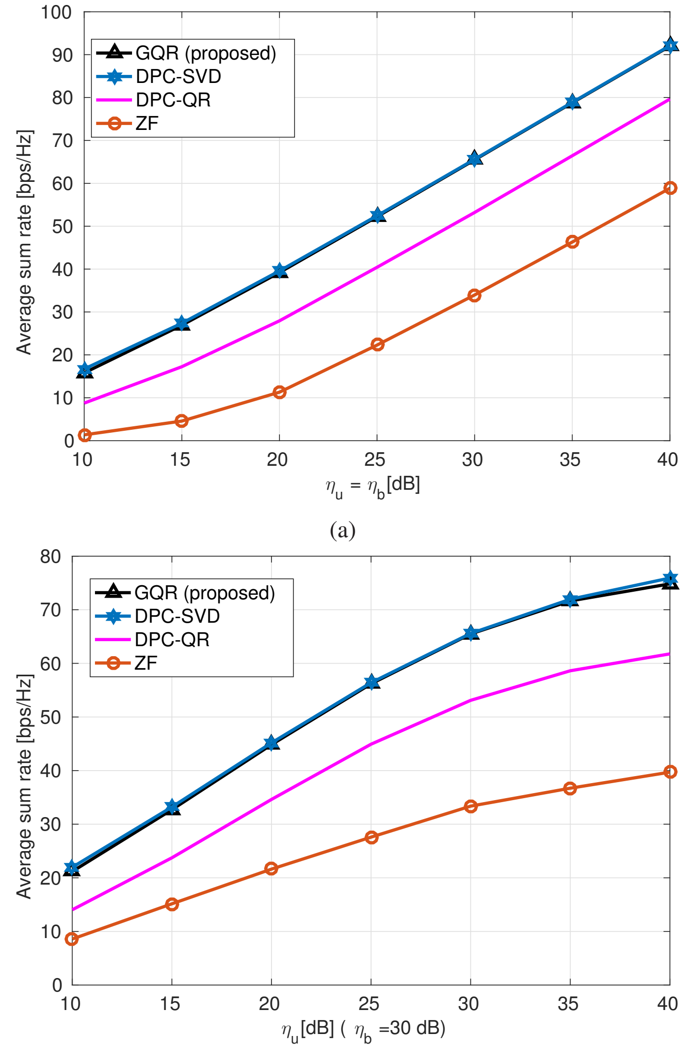
B. Asymmetric multi-user two-way relaying

The proposed GQRD design also works for multi-user ATWR scenario with single-antenna users by using ZF-DPC at the BS. We now compare its sum-rate with the existing multi-user ATWR transceiver designs: i) the best-performing DPC-SVD [32]; ii) DPC-QR [48]; and iii) ZF [9]. (Optimal sum-rate design for asymmetric multi-user TWR is also an open problem.) For this comparison, we consider a cellular system with $N_b = 16$ BS antennas, with $N_r = 16$ relay antennas and $N_u = 8$ single-antenna users. We see from Fig. 5a that the proposed GQRD design has the same sum-rate as that of the best-performing DPC-SVD design, and has much better performance than the DPC-QR and ZF designs. It is easy to see that at $\eta_u = \eta_b = 25$ dB, the GQRD design has 12 bps/Hz higher sum-rate than the DPC-QR design. Similarly in Fig. 5b where we fix $\eta_b = 30$ dB and vary η_u alone, the GQRD plot overlaps that of the SVD-LQ and provides better sum-rate than other two designs.

The existing multi-user ATWR designs, similar to the single-user ATWR designs, require $N_r \geq 2N_u$ antennas to cancel the BI experienced by the receiving user. These designs do not work with $N_r < 2N_u$ antennas. The proposed GQRD design works with $N_r < 2N_u$ antennas and is therefore more flexible. We next investigate the sum-rate of GQRD design for STWR scenario where we specifically consider $N_r < 2N_u$ antenna configuration.

C. Symmetric single-user two-way relaying

We compare the sum-rate of the GQRD with the following STWR designs that are shown to have close-to-optimal performance: i) channel-diagonalizing (CH-DIAG) [27]; ii) channel-triangularizing (CH-TRI) [49] and iii) channel-alignment (CH-ALIGN) [50]. We observe from Fig. 6 that the GQRD



(a)

(b)

Fig. 5: Average sum-rate comparison of the proposed GQRD transceiver design with other state-of-the-art *multi-user* ATWR designs for two different scenarios :a) $\eta_u = \eta_b$ dB; and b) $\eta_b = 30$ dB. For both these figures, the number of BS and relay antennas $N_b = N_r = 16$, and the number of single-antenna users $N_u = 8$.

transceiver has same sum-rate as the other designs, and *consequently provides close-to-optimal sum-rate for the STWR also*. The above STWR transceivers employ same number of antennas at both source nodes who want to exchange data via a relay. These designs when incorporated in a cellular system will constrain the BS to have same number of antennas as that of the user. In a cellular system, a BS usually has more number of antennas than the user. The GQRD transceiver is designed such that it can have $N_b \geq N_u$. We now study the transmit beamforming gains provided by the GQRD design with these additional BS antennas.

In Fig. 7a where we compare the GQRD design with CH-DIAG design for a fixed $N_u = N_r = 2$ and variable BS antennas, it is evident that the additional BS antennas considerably improve the sum-rate. Figure 7b further substantiates this improvement where the sum-rate of the GQRD design is compared with that of the CH-DIAG design for different N_b

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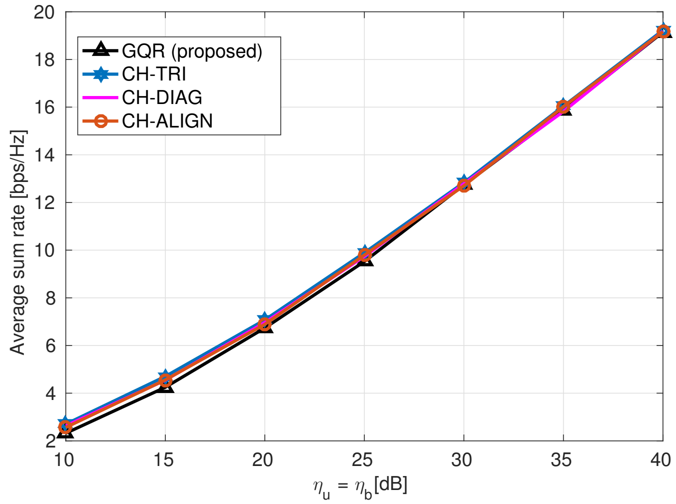

Fig. 6: Average sum-rate comparison of the proposed GQRD transceiver design for STWR with other state-of-the-art designs. Here we fix $N_b = N_r = N_u = 2$.

values and $\eta_u = \eta_b = 30$ dB and $N_u = N_r = 2$. The sum-rate of the CH-DIAG design is the same for different N_b values as it cannot use additional BS antennas. The sum-rate difference between the two designs increases with increasing number of BS antennas. With $N_b = 5$, the GQRD design yields 3 bps/Hz higher bit-rate than the CH-DIAG design.

D. Convergence of the proposed algorithm

We now investigate the number of iterations required for the algorithm to converge for a particular channel realization. We observe from Fig. 8 that the algorithm converges within 5 GP iterations for an exit condition of $\nu = 1 \times 10^{-10}$ at $\eta_u = \eta_b = 10$ dB. The algorithm has a similar convergence behavior for other channel realizations and SNR values as well.

VI. Conclusion

We proposed a common transceiver design for asymmetric, symmetric and multi-user cellular two-way relaying. The existing two-way relaying literature constructs separate transceivers for these relaying scenarios, and also constrains the number of antennas at different nodes. The proposed transceiver, designed using generalized QR decomposition of channel matrices, has relaxed antenna constraints, and consequently works for varied relaying scenarios. We also showed that the sum-rate of the proposed transceiver can be maximized using geometric programming by suitably approximating the objective, and by improving this approximation using an iterative algorithm. The optimal sum-rate of the proposed transceiver is shown to not only match that of the existing designs but also outperform them for certain antenna configurations. We investigated the computational complexity of the proposed design and showed it to be significantly lower than the best-performing design available in the literature. The proposed design thus matches (even exceeds) the performance of existing designs, relaxes their antenna constraints, and thus provides a single low-complexity transceiver option to system designer for multiple cellular relaying scenarios.

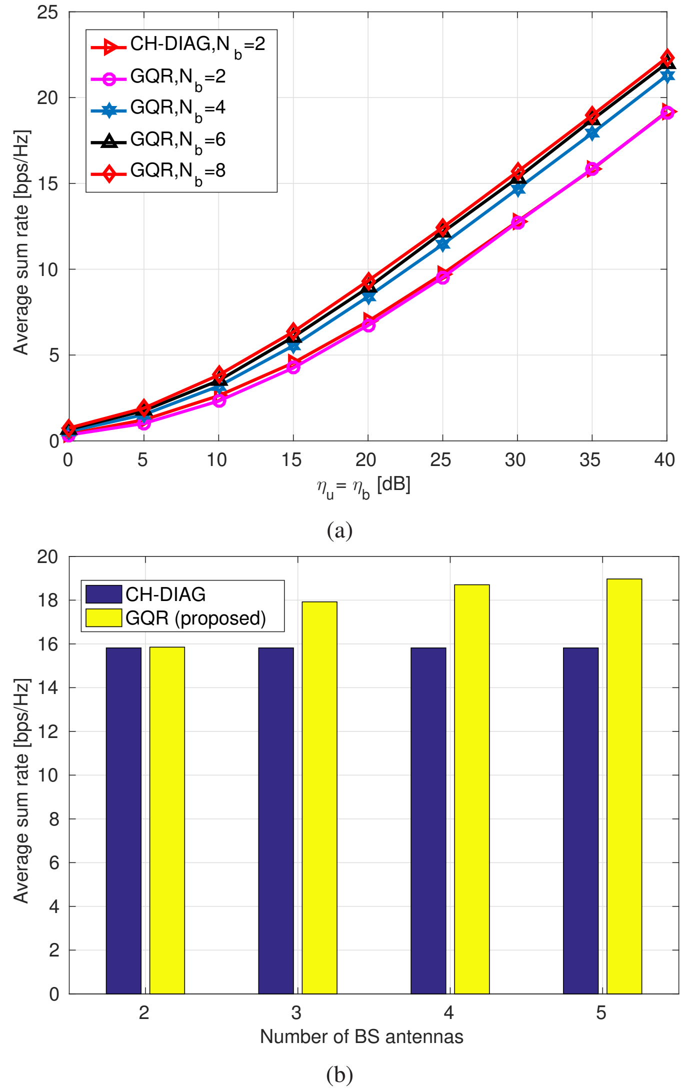

Fig. 7: (a) Average sum-rate comparison of the proposed GQRD design and the CH-DIAG design for different N_b values; and (b) Average sum-rate of the GQRD and the CH-DIAG design for different N_b at $\eta_1 = \eta_2 = 25$ dB. For both these figures, $N_u = 2$ user antennas and $N_r = 2$ relay antennas.

Appendix A
Proof of Lemma 4.2

To prove the lemma, we reproduce and simplify the relay power expression from (5).

$$\text{Tr}\left(\mathbf{W}(\mathbf{H}_u\boldsymbol{\Sigma}_u\mathbf{H}_u^H + \mathbf{H}_b\boldsymbol{\Sigma}_b\mathbf{H}_b^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{W}^H\right) \tag{A.1}$$

$$\overset{(a)}{=} \sum_{j=1}^{N_u} \rho^2\|\mathbf{W}\mathbf{h}_u^j\|^2 + \lambda_j\|\mathbf{W}\mathbf{h}_b^j\|^2 + \sigma_r^2\ \text{Tr}(\mathbf{W}\mathbf{W}^H)$$

$$\overset{(b)}{=} \sum_{j=1}^{N_u} \rho^2\|\boldsymbol{\Delta}\mathbf{Q}_h^H\mathbf{h}_u^j\|^2 + \lambda_j\|\boldsymbol{\Delta}\mathbf{Q}_h^H\mathbf{h}_b^j\|^2 + \sigma_r^2\text{Tr}(\mathbf{W}\mathbf{W}^H)$$

$$\overset{(c)}{=} \sum_{j=1}^{N_u} \rho^2\|\boldsymbol{\Delta}\bar{\mathbf{h}}_u^j\|^2 + \lambda_j\|\boldsymbol{\Delta}\bar{\mathbf{h}}_b^j\|^2 + \sigma_r^2\text{Tr}(\mathbf{W}\mathbf{W}^H)$$

$$\overset{(d)}{=} \sum_{j=1}^{N_u} \rho^2\|\boldsymbol{\Delta}\bar{\mathbf{h}}_u^j\|^2 + \lambda_j\|\boldsymbol{\Delta}\bar{\mathbf{h}}_b^j\|^2 + \sigma_r^2\text{Tr}(\boldsymbol{\Delta}\boldsymbol{\Delta}^H)$$

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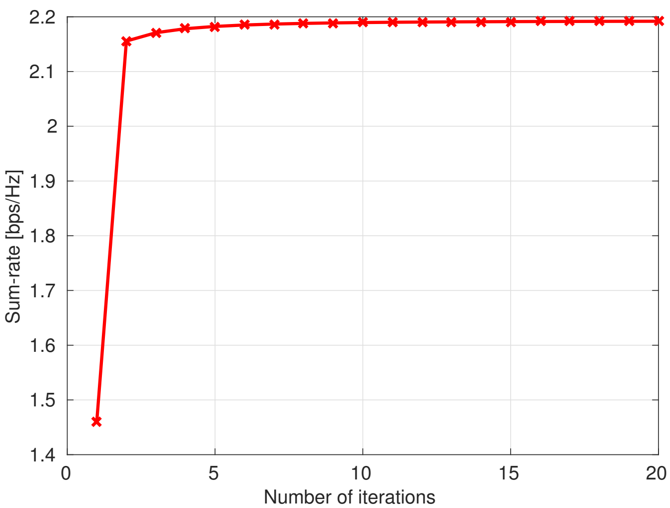


Fig. 8: Convergence of sum-rate at $\eta_u = \eta_b = 10$ dB

$$= \sum_{j=1}^{N_u}\sum_{n=1}^{N_r}\{\rho^2|\bar{h}^j_{u,\widehat{n}}|^2 + \lambda_j|\bar{h}^j_{b,\widehat{n}}|^2 + \sigma_r^2\}\delta_n \qquad \text{(A.2)}$$

In (a) we use: i) $\mathbf{\Sigma}_u = \mathbf{B}_u\mathbf{B}_u^H = \rho^2$; ii) $\mathbf{\Sigma}_b = \mathbf{B}_b\mathbf{B}_b^H = [\overline{\mathbf{U}}_h]\mathbf{\Lambda}\mathbf{\Lambda}^H[\overline{\mathbf{U}}_h]^H$; and iii) $\mathbf{h}_u^j$ and $\mathbf{h}_b^j$ to denote the jth column of $\mathbf{H}_u$ and $\mathbf{H}_b[\overline{\mathbf{U}}_h]^H$, respectively. Equality in (b) is because $\mathbf{Q}_g$ has orthonormal columns. In (c), we denote $\bar{\mathbf{h}}_u^j = \mathbf{Q}_h^H\mathbf{h}_u^j = [\bar{h}^j_{u,1}, \cdots, \bar{h}^j_{u,N_r}]^T$, $\bar{\mathbf{h}}_b^j = \mathbf{Q}_h^H\mathbf{h}_b^j = [\bar{h}^j_{b,1}, \cdots, \bar{h}^j_{b,N_r}]^T$ and $\widehat{n} = N_r - n + 1$. Equality in (d) is because Tr($\mathbf{AB}$) = Tr($\mathbf{BA}$); and 2) $\mathbf{Q}_h^H$ has orthonormal rows and $\mathbf{Q}_g$ has orthonormal columns. The relay power is a posynomial as the coefficients of λ_j and δ_n for all j, n in (A.2) are non-negative.

References

[1] A. Papadogiannis, M. Farber, A. Saadani, M. D. Nisar, P. Weitkemper, T. M. de Moraes, J. Gora, N. Cassiau, D. Ktenas, J. Vihriala, M. Khanfouci, and T. Svensson, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Veh. Technol. Mag.*, vol. 9, no. 9, pp. 29–37, Jun. 2014.

[2] M. Xia and S. Aissa, "Underlay cooperative af relaying in cellular networks: Performance and challenges," *IEEE Commun. Mag.*, vol. 51, no. 12, pp. 170–176, Dec. 2013.

[3] S. Won and L. Hanzo, "Synchronization issues in relay-aided cooperative MIMO networks," *IEEE Wireless Commun.*, vol. 21, no. 5, pp. 41–51, Oct. 2014.

[4] C. Xing, Y. Ma, Y. Zhou, and F. Gao, "Transceiver optimization for multi-hop communications with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 64, no. 6, pp. 1519–1534, Mar. 2016.

[5] C. Xing, F. Gao, and Y. Zhou, "A framework for transceiver designs for multi-hop communications with covariance shaping constraints," *IEEE Trans. Signal Process.*, vol. 63, no. 15, pp. 3930–3945, Aug. 2015.

[6] C. Xing, S. Ma, and Y. Zhou, "Matrix-monotonic optimization for MIMO systems," *IEEE Trans. Signal Process.*, vol. 63, no. 2, pp. 334–348, Jan. 2015.

[7] V. T. Muralidharan and B. Rajan, "Wireless network coding for MIMO two-way relaying," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3566–3577, Jul. 2013.

[8] ——, "Performance analysis of adaptive physical layer network coding for wireless two-way relaying," *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1328–1339, Mar. 2013.

[9] T. Unger and A. Klein, "Duplex schemes in multiple antenna two-hop relaying," *EURASIP J. Adv. Signal Process.*, vol. 2008, pp. 1–14, 2008.

[10] A. Dutta, K. Hari, and L. Hanzo, "Linear transceiver design for an amplify-and-forward relay based on the MBER criterion," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 3765–3777, Nov. 2014.

[11] M. Molu and N. Goertz, "Optimal precoding in the relay and the optimality of largest eigenmode relaying with statistical channel state information," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 2113–2123, Apr. 2014.

[12] H.-B. Kong, C. Song, H. Park, and I. Lee, "A new beamforming design for MIMO AF relaying systems with direct link," *IEEE Trans. Commun.*, vol. 62, no. 7, pp. 2286–2295, Jul. 2014.

[13] C. Li, J. Wang, F. C. Zheng, J. M. Cioffi, and L. Yang, "Overhearing-based co-operation for two-cell network with asymmetric uplink-downlink traffics," *IEEE Trans. Signal Inf. Process. Netw.*, vol. 2, no. 3, pp. 350–361, Sep. 2016.

[14] C. Li, H. J. Yang, F. Sun, J. M. Cioffi, and L. Yang, "Multiuser overhearing for cooperative two-way multiantenna relays," *IEEE Trans. Veh. Technol.*, vol. 65, no. 5, pp. 3796–3802, May 2016.

[15] R. Budhiraja and B. Ramamurthi, "Transceiver design for non-concurrent two-way MIMO AF relaying with QoS guarantees," *IEEE Trans. Veh. Technol.*, vol. 65, no. 12, pp. 9651–9661, Dec. 2016.

[16] R. Budhiraja, Karthik KS, and B. Ramamurthi, "Linear precoders for non-regenerative asymmetric two-way relaying in cellular systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 9, pp. 5002–5014, Sep. 2014.

[17] G. Amarasuriya, C. Tellambura, and M. Ardakani, "Sum rate analysis of two-way MIMO AF relay networks with zero-forcing," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4456–4469, Sep. 2013.

[18] K.-J. Lee, H. Sung, E. Park, and I. Lee, "Joint optimization for one and two-way MIMO AF multiple-relay systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3671–3681, Dec. 2010.

[19] R. Wang and M. Tao, "Joint source and relay precoding designs for MIMO two-way relaying based on MSE criterion," *IEEE Trans. Signal Process.*, vol. 60, no. 3, pp. 1352–1365, Mar. 2012.

[20] S. Xu and Y. Hua, "Optimal design of spatial source-and-relay matrices for a non-regenerative two-way MIMO relay system," *IEEE Trans. Wireless Commun.*, vol. 10, no. 5, pp. 1645–1655, May 2011.

[21] Y. Rong, "Joint source and relay optimization for two-way linear non-regenerative MIMO relay communications," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6533–6546, Dec. 2012.

[22] W.-C. Choi, S. Choi, A. Heo, and D.-J. Park, "Joint optimization of source and relay for MIMO two-way relay networks using MSE duality," *IEEE Commun. Lett.*, vol. 18, no. 7, pp. 1246–1249, Jul. 2014.

[23] A. Heo, W. Choi, J. Park, and D. Park, "Sum rate maximization of an MIMO two-way relay system using MSE duality," *IEEE Commun. Lett.*, vol. 19, no. 5, pp. 863–866, May 2015.

[24] F. Roemer and M. Haardt, "A low-complexity relay transmit strategy for two-way relaying with MIMO amplify and forward relays," in *Proc. Int. Conf. Acoustics, Speech and Signal Process. (ICASSP)*, Dallas, Texas, USA, Mar. 2010, pp. 3254–3257.

[25] J. Li, M. Matthaiou, and T. Svensson, "I/Q imbalance in two-way AF relaying," *IEEE Trans. Commun.*, vol. 62, no. 7, pp. 2271–2285, Jul. 2014.

[26] H. Yan, H. Nguyen, and J. Su, "Distributed precoding for OFDM in two-way relaying communications," *IEEE Trans. Veh. Technol.*, vol. 64, no. 5, pp. 1930–1941, May 2015.

[27] H. Park, H. J. Yang, J. Chun, and R. Adve, "A closed-form power allocation and signal alignment for a diagonalized MIMO two-way relay channel with linear receivers," *IEEE Trans. Signal Process.*, vol. 60, no. 11, pp. 5948–5962, Nov. 2012.

[28] Y. Zhang, L. Ping, and Z. Zhang, "Low cost pre-coder design for MIMO AF two-way relay channel," *IEEE Signal Process. Lett.*, vol. 22, no. 9, pp. 1369–1372, Sep. 2015.

[29] E. Chiu and V. Lau, "Cellular multiuser two-way MIMO AF relaying via signal space alignment: Minimum weighted SINR maximization," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4864–4873, Sep. 2012.

[30] C. Sun, C. Yang, Y. Li, and B. Vucetic, "Transceiver design for multi-user multi-antenna two-way relay cellular systems," *IEEE Trans. Commun.*, vol. 60, no. 10, pp. 2893–2903, Oct. 2012.

[31] J. Zhang, F. Roemer, and M. Haardt, "Beamforming design for multi-user two-way relaying with MIMO amplify and forward relays," in *Proc. Int. Conf. Acoustics, Speech and Signal Process. (ICASSP)*, Prague, May 2011, pp. 2824–2827.

[32] R. Budhiraja and B. Ramamurthi, "Joint precoder and receiver design for AF non-simultaneous two-way MIMO relaying," *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 2942–2955, Jun. 2015.

[33] E. Anderson, Z. Bai, and J. Dongarra, "Generalized QR factorization and its applications," *Linear Algebra and its Applications*, vol. 162, pp. 243 – 271, 1992.

[34] T. Yang, X. Yuan, L. Ping, I. Collings, and J. Yuan, "A new physical-layer network coding scheme with eigen-direction alignment precoding for MIMO two-way relaying," *IEEE Trans. Commun.*, vol. 61, no. 3, pp. 973–986, Mar. 2013.

[35] J. Gao, S. Vorobyov, H. Jiang, J. Zhang, and M. Haardt, "Sum-rate maximization with minimum power consumption for MIMO DF two-way relaying: Part I – relay optimization," *IEEE Trans. Signal Process.*, vol. 61, no. 14, pp. 3563–3577, Jul. 2013.

[36] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699–712, Jun. 2009.

[37] A. Khabbazibasmenj, F. Roemer, S. Vorobyov, and M. Haardt, "Sum-rate maximization in two-way AF MIMO relaying: Polynomial time solutions to a class of DC programming problems," *IEEE Trans. Signal Process.*, vol. 60, no. 10, pp. 5478–5493, Oct. 2012.

[38] G. Li, Y. Wang, and P. Zhang, "Optimal linear MMSE beamforming for two way multi-antenna relay systems," *IEEE Commun. Lett.*, vol. 15, no. 5, pp. 533–535, May 2011.

[39] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691–1706, Jul. 2003.

[40] C. Xing, S. Li, Z. Fei, and J. Kuang, "How to understand linear minimum mean-square-error transceiver design for multiple-input-multiple-output systems from quadratic matrix programming," *IET Communications*, vol. 7, no. 7, pp. 1231–1242, Aug. 2013.

[41] M. Chiang, C. W. Tan, D. Palomar, D. O'Neill, and D. Julian, "Power control by geometric programming," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2640–2651, Jul. 2007.

[42] S. Shi, M. Schubert, and H. Boche, "Rate optimization for multiuser MIMO systems with linear processing," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 4020–4030, Aug. 2008.

[43] C.-B. Chae, T. Tang, R. Heath, and S. Cho, "MIMO relaying with linear processing for multiuser transmission in fixed relay networks," *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 727–738, Feb. 2008.

[44] K. Phan, T. Le-Ngoc, S. Vorobyov, and C. Tellambura, "Power allocation in wireless multi-user relay networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2535–2545, May 2009.

[45] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge Univ. Press, 2004.

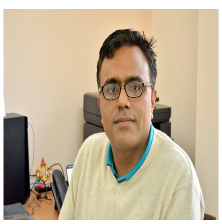
[46] N. J. Hingam, *Accuracy and Stability of Numerical Algorithms*. Philadelphia: Society for Industrial and Applied Mathematics (SIAM), 1961.

[47] G. H. Golub and C. F. V. Loan, *Matrix Computations*. Baltimore, MD: The Johns Hopkins University Press, 1996.

[48] R. Budhiraja and B. Ramamurthi, "Multiuser two-way non-regenerative MIMO relaying with non-concurrent traffic," *IEEE Trans. Veh. Technol.*, vol. 64, no. 7, pp. 3268–3273, Jul. 2014.

[49] ——, "Joint transceiver design for QoS-constrained MIMO two-way non-regenerative relaying using geometric programming," *IEEE Trans. Wireless Commun.*, vol. 15, no. 5, pp. 3453–3465, May 2016.

[50] C. Leow, Z. Ding, and K. Leung, "Joint beamforming and power management for nonregenerative MIMO two-way relaying channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 9, pp. 4374–4383, Nov. 2011.



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