# Generalization of Hybrid Time Divisioning for Power Allocation in DMT-Based DSL Systems

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***Abstract*— We propose an algorithm that seeks to generalize the recently proposed Hybrid Time Divisioning scheme. It tries to make optimal groupings amongst all the (User, Subchannel) pairs by deciding which users should transmit alone in a subchannel and which users should transmit along with each other in order to yield substantial data rate increments. This time divisioning based modification can be applied over any resource allocation scheme (e.g Multi User Discrete Bit Loading, Iterative Waterfilling, Optimal Spectrum Balancing) which involves simultaneous transmission of all the users.**

***Index Terms*— Discrete multi-tone, digital subscriber line, time divisioning.**

## I. Introduction

A HYBRID Time Divisioning (HTD) scheme was proposed in [1] in order to address the issue of unnecessary rate reductions due to simultaneous transmission of all users in a DSL system. It used the concept of time divisioning to divide the (User, Subchannel) pairs (denoted as $(u, s)$) into two groups namely pure-TD (now onwards referred to as Ungrouped or UG pairs) and non-TD (referred to as Grouped or G pairs). However, because of this restriction to only two groups, HTD cannot yield the maximum rate gains possible using time divisioning.

In this paper we propose a Generalized HTD (GHTD) scheme that can create several user groups in every subchannel. The algorithm selects the users which should transmit alone in a subchannel and the users which should group and transmit along with each other, in order to achieve substantial data rate increments. An optimal approach towards this problem would be to make an exhaustive evaluation of all the possible configurations and groupings of users in every subchannel and select the best (in terms of the aggregate data rate) out of these. However, for practical cases of 50-60 users and 256 subchannels this is clearly not feasible. GHTD though like HTD is a suboptimal algorithm, but it creates nearly optimal groupings amongst the $(u, s)$ pairs.

Rest of the paper is organized as follows. Section II reviews the DSL environment and models it as an interference channel. Section III describes the GHTD scheme and gives algorithms for its application over Iterative Waterfilling (IW) [3] and Multi User Discrete Bit Loading (MDBL) [2]. Simulation results are presented in Section IV. We conclude the paper in Section V.

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## II. System Model

When DMT technique is used, a DSL channel can be modelled as N independent frequency non-selective subchannels, each of which is an interference channel of M users. Without crosstalk cancellation, signal-to-interference-plus-noise ratio (SINR) of user $i$ in subchannel $j$ is given by

$$\gamma_j^i = \frac{P_j^i}{N_0 W + \sum_{k=1 k\neq i}^{M} \beta_{ki}^j P_j^k} \tag{1}$$

where $P_j^i$ denotes the power of user $i$ in the $j$th subchannel, $\beta_{ki}^j$ is the coupling coefficient value of user $k$ towards user $i$ in subchannel $j$, $W$ is the bandwidth allocated to every subchannel and $N_0$ is the noise power per Hz. We assume the direct channel coupling coefficient values to be equal to 1 i.e. $\beta_{ii}^j = 1$, $\forall (i = 1, \cdots M, j = 1, \cdots N)$. Further there is a maximum power constraint for every user. Thus for the $i$th user we have $P_1^i + P_2^i + \cdots + P_N^i \leq P_{max}^i$.

## III. Generalized Hybrid Time Divisioning

All the users of subchannel $j$, belonging to a common group (say group $A$) transmit simultaneously for a fraction $\upsilon_j^A = \frac{\alpha_j^A}{M}$ of the total time, where $\alpha_j^A$ gives the total number of users in subchannel $j$ belonging to group $A$. For such a G pair $(i, j)$, the maximum achievable data rate is given by

$$R_j^i = \upsilon_j^A W \log_2(1 + \gamma_{jA}^i) \tag{2}$$

where

$$\gamma_{jA}^i = \frac{P_j^i}{N_0 W + \text{Interference from other group A pairs of } j} \tag{3}$$

Similarly for the case of UG pairs i.e. those pairs $(k, l)$ which transmit alone in their respective subchannels, without any grouping with other pairs, the maximum achievable data rate is given by

$$R_l^k = \frac{W}{M} \log_2(1 + \gamma_{lU}^k) \tag{4}$$

where

$$\gamma_{lU}^k = \frac{P_l^k}{N_0 W} \tag{5}$$

Having specified the data rates for G and UG classes of $(u, s)$ pairs we need to come up with a scheme that can precisely calculate the number of groups in every subchannel as well as can allocate specific users to these groups such that the aggregate data rate ($R_{agg} = \sum_{i=1}^{M}\sum_{j=1}^{N} R_j^i$) is maximized. This is the main objective of Generalized Hybrid Time Divisioning. Since the problem is not amenable to a theoretical solution, we propose an algorithmic approach to seek its solution in the following sections.

### *A. GHTD Algorithm*

GHTD with the creation of many groups of users within each subchannel can be considered as a generalization of Hybrid Time Divisioning. Like [1] and [2], it also assumes the presence of a Spectrum Management Center at the central office end to administer the Dynamic Spectrum Management requirements of the algorithm.

It proceeds through several stages. Stage 1 starts by assigning the $(u, s)$ pair facing maximum interference in subchannel 1 as a UG pair which would transmit alone. The resulting changes in crosstalk interferences and SINR values due to this UG pair assignment are taken care by redoing power allocation using the underlying power allocation scheme (IW, MDBL etc.) on which GHTD is being applied. Aggregate data rate is now calculated and is compared with the rate before the modification was made. If an increment is observed we make the change permanent. Otherwise, we undo the modification. This procedure is repeated over all the $N$ subchannels. It is not necessary to get an $R_{agg}$ improvement by ungrouping the maximum interference facing pair of every subchannel. However we check for such a possibility in each of them.

Next we start with a second level of iteration that again traverses through all the $N$ subchannels repeating a similar procedure. In this iteration we try to ungroup and convert the next maximum interference facing G pair (left at the end of first iteration) of every subchannel as a UG pair. These iterations continue as long as the configuration modifications keep on giving $R_{agg}$ increments. Finally we terminate at a point where even after traversing through all the $N$ subchannels no further gains can be obtained. This is the completion of stage 1. The $(u, s)$ pairs which still remain grouped at the end of stage 1 can be termed as group 1 pairs within their respective subchannels.

Stage 2 aims at achieving further data rate improvements by carrying out groupings amongst the UG pairs left at the end of stage 1. It starts with an initialization that all the UG pairs of subchannel $j$ from stage 1 belong to a common group (say $C_j$) and hence transmit together, though separately from the group 1 pairs of subchannel $j$. This initialization is done for all the subchannels i.e $\forall(j = 1, \cdots N)$. A strategy similar to stage 1 is then followed in order to mark out UG pairs, from the $(u, s)$ pairs belonging to these newly created $C_j$ groups $\forall(j = 1, \cdots N)$. This might leave some pairs still in $C_j$. Thus we achieve a second level pairing amongst the $(u, s)$ pairs which were marked as UG pairs at the end of Stage 1. These newly created G pairs can now be termed as group 2 pairs within their respective subchannels. We continue this to further stages with each stage contributing an improvement in the aggregate data rate, by making efficient groupings amongst the UG pairs left out in the previous stage. The algorithm finally terminates at a stage where no further gains can be obtained.

Although theoretically GHTD is a suboptimal algorithm, it is expected that its performance will be close to optimal. This is because at each step in GHTD we only ungroup those users who face maximum interference within their respective subchannels. There is a high probability that such users can transmit at better data rates in their own disjointly allocated time slots, rather than transmitting all the time along with the other users in the presence of high crosstalk interference. Thus by continuously repeating this strategy over different stages we finally expect to end up with nearly optimal groupings.

### *B. GHTD Over Iterative Waterfilling*

We define an $M \times N$ matrix $\Lambda$ that denotes the group to which a $(u, s)$ pair belongs. For a G pair $(i, j)$, belonging to any general group $A$ in subchannel $j$, $\Lambda_{ij} = A$. For a UG pair $(k, l)$, $\Lambda_{kl} = 0$. We also use the variables - $Loop_{cntr}$, $R_{agg}^{temp1}$, $R_{agg}^{temp2}$ and arrays $\Delta$, $\Delta^{temp}$, $\Lambda^{temp}$, $\beta^{temp}$ to specify the algorithm. The complete operation of GHTD using IW as the underlying power allocation scheme is given below:

1) Initialize:
   a) $Loop_{cntr} = 0$.
   b) $\beta_{ki}^{j,temp} = \beta_{ki}^{j}$, $\forall(i, k = 1, \cdots M, j = 1, \cdots N)$.
   c) $\Lambda_{ij} = 1$, $\Lambda_{ij}^{temp} = 1$, $\forall(i = 1, \cdots M, j = 1, \cdots N)$ i.e. all the $(u, s)$ pairs of subchannel $j$, are G pairs belonging to group 1 of $j$ and transmit simultaneously in the beginning.
2) Allocate power to every $(u, s)$ pair using IW. Store the resulting $\gamma_j^i$ values in an another $M \times N$ matrix $\Delta_{ij}$ i.e. $\Delta_{ij} = \gamma_j^i$ $\forall$ $(i = 1, \cdots M, j = 1, \cdots N)$.
3) Initialize a temporary matrix $\Delta^{temp}$ with $\Delta_{ij}^{temp} = \Delta_{ij}$, $\forall(i, j)$. Calculate $R_{agg}$, and store it in $R_{agg}^{temp1}$.
4) *Stage 1:*
   a) Let $J$ = 1. i.e. considering subchannel 1.
   b) While (1)
      i) Find a user $i$ such that $\Lambda_{iJ}^{temp} = 1$ and $\Delta_{iJ}^{temp} = \min(\Delta_{kJ}^{temp})$, $\forall(k = 1, \cdots M)$ i.e. the group 1, G pair $(i, J)$ having the minimum SINR value in subchannel $J$.
      ii) Make $\Lambda_{iJ}^{temp} = 0$, $\beta_{ki}^{J,temp} = 0$ and $\beta_{ik}^{J,temp} = 0$, $\forall(k = 1, \cdots M)$ i.e. make $(i, J)$ a UG pair.
      iii) Allocate power to every $(u, s)$ pair using IW. Use $\beta^{temp}$ for the coupling coefficient values. Store the resulting SINR values in $\Delta^{temp}$. Calculate $R_{agg}$ and store it in $R_{agg}^{temp2}$.
      iv) If $R_{agg}^{temp2} \geq R_{agg}^{temp1}$ make:
         - $\Delta = \Delta^{temp}$, $\Lambda = \Lambda^{temp}$, $\beta = \beta^{temp}$ and $Loop_{cntr} = 0$.
         - $R_{agg}^{temp1} = R_{agg}^{temp2}$ and then $R_{agg}^{temp2} = 0$.

         i.e. Make the configuration resulting from step $4(b(ii))$ permanent.
      v) Else make:
         - $\Delta^{temp} = \Delta$, $\Lambda^{temp} = \Lambda$, $\beta^{temp} = \beta$ and $Loop_{cntr} = Loop_{cntr} + 1$.
         - If $Loop_{cntr} = N$, break (i.e. come out of the while loop).

         Thus we revert back to the previous best configuration.

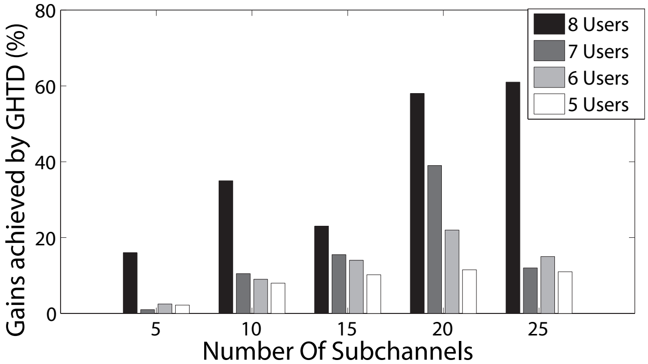


Fig. 1. GHTD over Multi User Discrete Bit Loading.

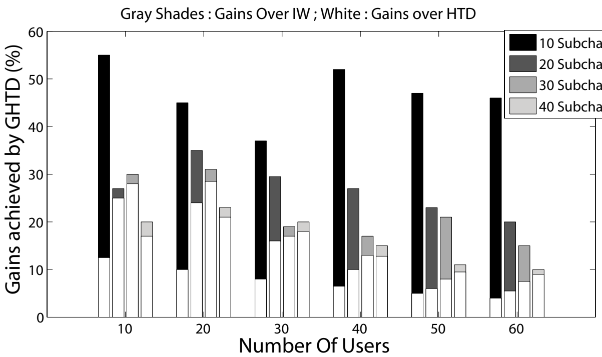


Fig. 2. GHTD over HTD and Iterative Waterfilling.

vi) $J = J + 1$. If $J = N + 1$ make $J = 1$.

5) Stage 2:
   a) $\forall(i = 1, \cdots M, j = 1, \cdots N)$, if $\Lambda_{ij} = 0$ make $\Lambda_{ij} = C_j$, $C_j \neq 1$ i.e. after stage 1 all the newly created UG pairs in subchannel $j$ are combined together into a single group $C_j$.
   b) With the above configuration, allocate power using IW. Calculate $R_{agg}$.
   c) Using a similar procedure as in stage 1 mark UG pairs, out of all the $(u, s)$ pairs belonging to these newly created groups $C_j$ $\forall(j = 1, \cdots N)$. This would leave some pairs still in $C_j$, thereby leading to a second level pairing amongst the $(u, s)$ which were marked as UG pairs at the end of Stage 1.

   Since Stage 2 operates over the configuration (CG) resulting from Stage 1, in the worst possible case of no further gain it would revert back to CG itself and thus cannot cause any $R_{agg}$ loss relative to Stage 1.

Repeat the above procedure to further stages. Finally terminate at a stage when no further gains can be obtained with respect to the previous stages. For practical cases of $50 - 60$ users in a DSL binder, the algorithm has been found to converge in 3 to 4 stages.

### *C. GHTD Over Multi User Discrete Bit Loading*

We use a modified form of the MDBL scheme that tries to achieve maximum aggregate data rates, given a total power constraint for every user. GHTD algorithm over MDBL is similar to that used for IW, with some minor modifications as given below:

1) At each iteration step use MDBL as the underlying resource allocation technique.
2) In step $4(b(ii))$, instead of marking the $(u, s)$ pairs with minimum SINR, mark those pairs as UG pairs which have been allocated the minimum number of bits in the previous permanent configuration.

## IV. Performance Evaluation

For simulation purposes, coupling coefficient ($\beta$) values are assumed to be uniformly distributed over 0 to 1. $N_0$ and $W$ values are taken as $0.01 \frac{W}{Hz}$ and $150\ Hz$ respectively. Further, a maximum power of $100\ W$ is distributed amongst all subchannels of a user. Figs. 1 and 2 show the gains (mean gain for 20 different sets of $\beta$ values) obtained by applying GHTD over MDBL, HTD and IW respectively.

We can see that GHTD results in significant performance improvements over all the three schemes, with rate gains as high as $56\%$ over IW and $62\%$ over MDBL for some cases. Note that the rate gains for GHTD over IW usually decrease with an increase in the number of subchannels. This is because a constant power has been allocated to each user and hence an increase in the number of subchannels results in a lower power per subchannel which in turn decreases the crosstalk interference amongst users. Lower are the interference values better is the performance of IW and hence the rate gains decrease. But we claim that the proposed scheme cannot perform worse than any underlying power allocation technique as under the extreme cases of very low coupling coefficients and high $\gamma_j^i$ values, GHTD will put all $(u, s)$ pairs in the same groups within their respective subchannels, thereby causing them to transmit simultaneously which would be similar to power allocation without any grouping applied. Thus we can always expect to achieve some rate benefits using GHTD, especially in high crosstalk interference scenarios.

## V. Conclusion

The proposed algorithm is a generalization of Hybrid Time Divisioning [1]. The scheme can be applied over any power allocation technique (e.g [2],[3],[4]) involving simultaneous transmission of all the users and results in significant performance improvements especially under cases of high crosstalk interference.

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