A Low Complexity Symbol Timing Estimator for MIMO Systems Using Two Samples Per Symbol

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***Abstract*— A new algorithm for data-aided symbol timing estimation in multiple antenna systems is proposed. By utilizing the information about the transmitted pulse, the proposed method achieves better performance and lower computational complexity than the recently proposed Discrete Fourier Transform (DFT) based interpolation method. Further, the method needs only two samples per symbol which is half of the oversampling factor required by the DFT based interpolation method.**

***Index Terms*— MIMO, symbol timing, pulse shape, oversampling factor, feedforward.**

I. Introduction

We consider the problem of data-aided feedforward symbol timing estimation in multi-antenna systems. The optimum sample selection algorithm was the first work to address this problem [1]. However, in order to obtain a reasonable performance, the algorithm required a large oversampling factor. Recently a Discrete Fourier Transform (DFT) based interpolation method [2] which works well even with an oversampling factor of four has been proposed. The method has better performance and requires lesser computational complexity than the optimum sample selection algorithm.

In this paper we propose a modification of [2] that achieves improved performance by utilizing the information about the transmitted pulse. The proposed method has significantly lower computational complexity than the DFT method and requires an oversampling factor of only two.

The organization of the paper is as follows: Section II presents the system model used. The proposed estimator is given in Section III. Simulation results and discussions are presented in Section IV and finally Section V concludes the paper.

II. System Model

We consider the space time code based modem of [1] with N transmit and M receive antennas. The baseband equivalent model and the notations are the same as in [2]. Hence the matched filtered signal at jth receive antenna is given by,

$$\mathbf{r_j}(m) = \sqrt{\frac{E_s}{N}} \sum_{i=1}^{N} h_{ij} \sum_{n} c_i(n) p(\frac{mT}{Q} - nT - \epsilon T) + \eta_j(m), \quad 1 \le j \le M \quad (1)$$

where E_s/N is the symbol energy; T is the symbol period; h_{ij}s are independent complex channel gains corresponding to the channel between ith transmit and jth receive antenna; Q is the oversampling factor; ϵ is the timing error and $p(t)$ is a Nyquist pulse with bandwidth $(1+\alpha)/2T$, α being the excess bandwidth factor; $\eta_j(m) = n_j(t) \otimes g_r(t)|_{t=mT/Q}$, $g_r(t)$ being the receive filter and $n_j(t)$ is the complex-valued circularly distributed Gaussian white noise at the jth receive antenna, with power density N_0 ($\otimes$ denotes convolution); $c_i(n)$ are training sequences of length L_t with cyclic prefix and suffix of identical lengths. Substituting $m = lQ + k$ (where l and k are integers) and $\epsilon' = \epsilon - k_0/Q$ (where $k_0 = -\lfloor (1/2 - \epsilon)Q \rfloor$) in (1) we get [2, eq. 3],

$$\mathbf{r_j}(lQ+k) = \sqrt{\frac{E_s}{N}} \sum_{i=1}^{N} h_{ij} \sum_{n} c_i(n) p(\frac{kT}{Q} + (l-n)T - \epsilon' T) + \eta_j(lQ+k) \quad (2)$$

where $k = 0, 1, \ldots Q-1$. Defining

$$\mathbf{r_j}(k) = [r_j(k)\ r_j(Q+k) \ldots r_j((L_t-1)Q+k)]^T$$
$$\mathbf{c_i} = [c_i(1)\ c_i(2) \ldots c_i(L_t)]^T$$

where the superscript T denotes transpose, the likelihood function for timing estimation is given by

$$\Lambda(k) = \sum_{i=1}^{N} \sum_{j=1}^{M} \left| \mathbf{c_i}^H \mathbf{r_j}(k) \right|^2 \quad (3)$$

where superscript H denotes conjugate transpose.

III. Symbol Timing Estimation using Pulse Shape Information

Our aim is to estimate ϵ' from the likelihood function $\Lambda(k)$ given in (3). Since $\Lambda(0), \Lambda(1) \ldots \Lambda(Q-1)$ correspond to samples of continuous time likelihood function $\Lambda(\epsilon')$, the problem of estimating ϵ' can be expressed as,

$$\hat{\epsilon}' \triangleq arg \max_{\epsilon'} \Lambda(\epsilon') \quad (4)$$

Two algorithms have been proposed in the literature to address this problem. The optimum sample selection algorithm [1] suggests selecting $\hat{\epsilon}' = \hat{k}/Q$, where

$$\hat{k} = arg \max_{k} \Lambda(k) \quad (5)$$

As shown in [2] this algorithm exhibits an error floor of $1/12Q^2$ in the Mean Square Error (MSE) of ϵ'. Therefore a large value of Q is required which results in high computational complexity. The DFT based technique suggested in [2]

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uses interpolation for estimation of ϵ'. After simplification, it reduces to computing

$$\hat{\epsilon}' = -\frac{1}{2\pi} arg\left\{ \sum_{k=1}^{Q} \Lambda(k) e^{-j2\pi k/Q} \right\} \quad (6)$$

This method needs only four samples per symbol and in terms of MSE performs better as compared to the optimum sample selection algorithm of [1]. However both the above schemes do not exploit the fact that the receiver is aware of the pulse shape used by the transmitter.

We now propose a new algorithm which attempts to utilize this information. Taking $c_i(n)$ to be perfect sequences as proposed in [2] and neglecting the noise term in [2, eq. 33], we get,

$$\Lambda(k) = W^2 h p^2(kT/Q - \epsilon' T) \quad (7)$$

where $W = \sqrt{E_s/N} L_t$ and $h = \sum_i \sum_j |h_{ij}|^2$. The information about the pulse shape can be utilized provided h is eliminated from (7). This can be done by taking $Q = 2$ and defining the ratio,

$$r \triangleq \frac{\Lambda(0)}{\Lambda(1)} = \frac{p^2(-\epsilon' T)}{p^2(0.5T - \epsilon' T)} = f(\epsilon') \quad (8)$$

In practice, it is sufficient to estimate ϵ' only as it represents the time difference between the first sample of the training sequence and the next nearest optimum sampling instance [2]. For the two samples per symbol case, ϵ' lies in the interval $(0, 0.5)$. A plot of r as a function of ϵ' over this range shows that it is one to one for raised cosine as well as the Nyquist pulses in [4], [5]. Therefore we propose $\hat{\epsilon}' = f^{-1}(r)$ as the new estimator for ϵ'.

Finding an analytical expression for $f^{-1}(r)$ is intractable because the above Nyquist pulses are either ratios involving trigonometric functions or are expressed as transcendental series. Hence we propose to approximate $f^{-1}(r)$ by a polynomial of appropriate degree. For a system using a particular pulse shape, we will need to determine the coefficients of the polynomial only once. Such a polynomial can be found by using standard least squares approach [6, Chap. 11]. For example using 20 points and a fifth degree polynomial approximation for $\epsilon' \in (0, 0.5)$, we get

$$\hat{\epsilon}'_{RC} = -0.0114r^5 + 0.1040r^4 - 0.3897r^3 + 0.8008r^2 - 1.0677r + 0.8139 \quad (9)$$

for a raised cosine pulse with roll off factor 0.3. Similarly for a "Better Than" raised cosine pulse [4] with same roll off factor, we get

$$\hat{\epsilon}'_{BTRC} = -0.0109r^5 + 0.1012r^4 - 0.3842r^3 + 0.7904r^2 - 1.0410r + 0.7944 \quad (10)$$

where r is defined in (8). The evaluation of these polynomials requires only four multiplications if implemented using Horner's rule [7].

The computational complexity of the proposed method can be compared with the method of [2] by estimating the number of multiplications in each case. In both the schemes the computationally most demanding part is calculation of $\mathbf{c_i}^H \mathbf{r_j}(k)$ requiring MNL_tQ complex multiplications. The

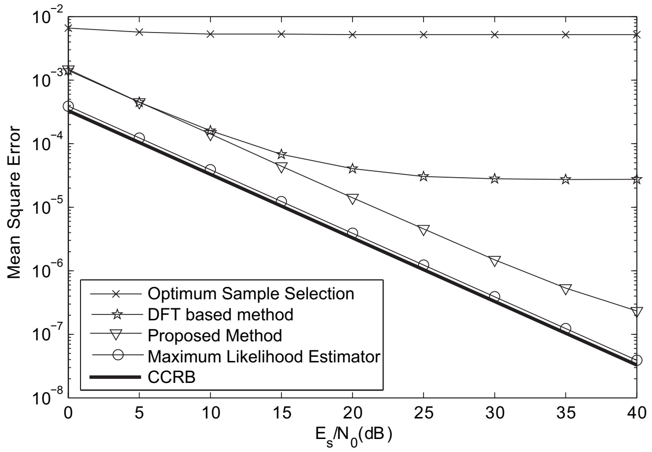

Fig. 1. Comparison of various schemes for $N = 2$, $M = 4$, $L_t = 32$ and $Q = 2$ ($Q = 4$ for DFT based and optimum sample selection methods).

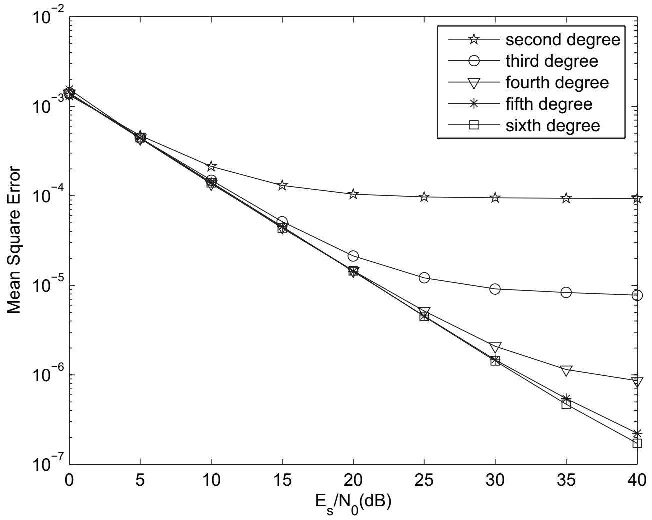

Fig. 2. Variation in MSE with the degree of polynomial used for approximating $f^{-1}(r)$ for $L_t = 32$, $N = 2$, $M = 4$.

DFT method requires Q to be at least four while the proposed method works with only two samples per symbol i.e. $Q = 2$. Thus for typical values of $M = 2$, $N = 4$ and $L_t = 32$ the proposed algorithm requires about 500 less multiplications. In the above comparison, we have ignored the fact that the DFT method further requires an arg operation and the proposed method further requires a division and four multiplications (for a fifth degree polynomial approximation).

IV. Simulation Results and Discussion

The performance of the estimators (9) and (6) has been compared using Monte Carlo simulations. The MSE is calculated by averaging over 10^5 estimates. We have taken ϵ to be uniformly distributed in $[-0.5, 0.5]$ and h_{ij}s as independent complex Gaussian distributed random variables with zero mean and variance 0.5 each for the real as well as the imaginary parts. The pulse shape is assumed to be raised cosine with excess bandwidth $\alpha = 0.3$. Training sequence as given in [2, sec. 4] with cyclic prefix and suffix of length 4 is used. The oversampling factor Q is assumed to be 2 for the proposed and 4 for the DFT based interpolation method [2]

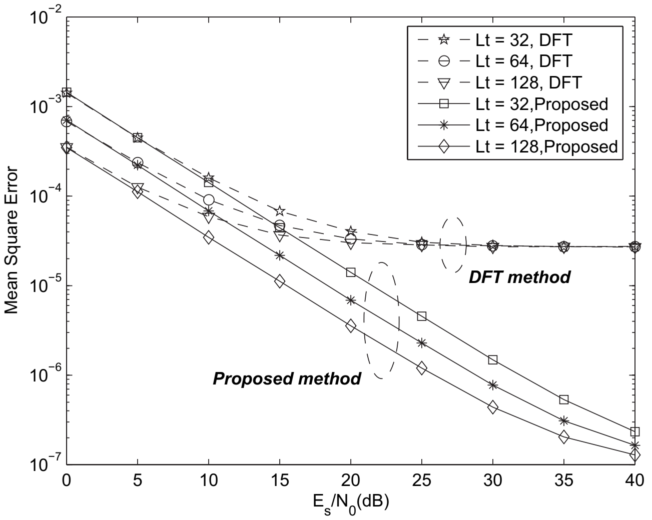

Fig. 3. Variation in MSE with the length of training sequences for $N = 2$, $M = 4$.

and the optimum sample selection algorithm [1]. The Signal-to-Noise Ratio (SNR) is defined as E_s/N_0.

Fig. 1 shows the performance of the proposed, DFT and optimum sample selection methods for perfect sequences. Clearly, the proposed method outperforms the DFT method of [2] at all SNRs. It may be noted that the proposed method is valid only when perfect sequences are used. The Maximum Likelihood (ML) estimate [3] and the Conditional Cramer-Rao Bound (CCRB) [3, eq. 32], each using optimal sequences and $Q = 2$, are also plotted in Fig. 1. These curves serve as benchmarks for comparing the performance with any sequence. Since perfect sequences are not optimal, it is not surprising to note that the ML algorithm and CCRB are far away from the performance of the proposed algorithm.

As shown in Fig. 2, performance of the proposed method improves if a higher degree polynomial is used to approximate $f^{-1}(r)$. In Fig. 1, fifth degree polynomial has been used. Comparing Fig. 1 and Fig. 2, we can see that third and fourth degree polynomials also perform better than the DFT method.

Fig. 3 shows that, as expected, the performance improves with increase in the length of the training sequences. Here also the proposed method outperforms the DFT method at all SNRs.

V. Conclusion

A data-aided symbol timing estimator for multi-antenna systems has been proposed. Unlike the existing methods in the literature, the proposed estimator makes use of the information about the pulse shape. In terms of MSE, it shows improved performance compared to the DFT based interpolation method and the optimum sample selection algorithm. Furthermore the proposed method uses only two samples per symbol resulting in reduced computational complexity compared to the DFT method.

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