

# Transactions Letters

## Multicarrier On-Off Keying for Fast Frequency Hopping Multiple Access Systems in Rayleigh Fading Channels

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**Abstract**—We study multicarrier on-off keying (MCOOK) for fast frequency-hopping (FFH) multiple-access systems. The performance of a sub-optimum receiver is analyzed for Rayleigh fading channels and a closed form expression for bit error rate (BER) has been derived for the binary case. We find that the number of chips per symbol required to optimize the performance of the system varies from system to system. Comparison with FFH M-ary frequency shift keying (MFSK) shows that in the interference-limited region (large MAI), MCOOK performs better than MFSK and further, the gain in the performance increases with increase in the value of modulation index  $M$ .

**Index Terms**—Fast frequency hopping, multicarrier on-off keying, multiple access systems, Rayleigh fading channel.

### I. INTRODUCTION

**D**UE to the growing demand for wireless communications, the problem of increasing the number of users in the same bandwidth has received quite a lot of attention. In Frequency Hopping (FH) systems, the most commonly used modulation scheme has been  $M$ -ary frequency shift keying (MFSK) which is somewhat wasteful of bandwidth [1]. To improve the performance, [1] introduced a new modulation scheme for Slow Frequency Hopping (SFH) systems based on Multicarrier On-Off Keying (MCOOK) and [2] proposed an extension of the same by combining  $M$ -ary amplitude-shift keying and orthogonal frequency-division multiplexing. Both the above SFH schemes have been shown to result in higher spectral efficiency than SFH schemes employing MFSK. However MCOOK is yet to be studied for Fast Frequency Hopping (FFH) systems. In this letter we study Fast Frequency Hopping with MCOOK modulation and analyze the performance of a sub-optimum receiver for the same.

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The rest of the paper is organized as follows. In Section II we describe the system model. In Section III we consider a sub-optimum receiver. In Section IV we derive the bit error rate (BER) expressions for the sub-optimum receiver in a Rayleigh fading channel and derive a closed form expression for the binary case. Finally we analyze the performance of the system in Section V using numerical and simulation results and conclude the paper in Section VI.

### II. SYSTEM MODEL

We consider a system with  $K$  active users transmitting over a common channel. The bits of  $k$ th user are modulated by  $L$  parallel On-Off Keying (OOK) modulators (Fig. 1(a)) forming an  $M$ -ary symbol with the binary representation,  $\mathbf{d}_l^{(k)} = (d_{0,l}^{(k)}, d_{1,l}^{(k)}, \dots, d_{L-1,l}^{(k)})$ ;  $d_{i,l}^{(k)} = 0$  or  $1$ ,  $0 \leq i \leq L-1$ ,  $0 \leq l \leq M-1$ . The spread bandwidth is divided into  $Q$  hopping slots (Fig. 1(b)) and each slot into  $L$  rows corresponding to  $L$  tone frequencies of the OOK modulator. Unlike [1] the symbol period  $T_s$  (duration of  $L$  bits) is divided into  $H$  chips, each of duration  $T_c = T_s/H$ . Each user is assigned a length  $H$  address, the elements of which can take values over  $0$  to  $Q-1$ . These addresses determine the hopping frequencies of the users in respective chips. With this model, the signal transmitted by user  $k$  for one chip duration ( $0 \leq t \leq T_c$ ) can be written as:

$$s^{(k)}(t) = \sqrt{\frac{2E_c}{T_c}} \sum_{i=0}^{L-1} d_{i,l}^{(k)} \cos[2\pi(f_{h_j}^{(k)} + f_i)t + \theta_{i,j}^{(k)}] \quad (1)$$

where  $E_c = E_s/H$  is the chip energy of each tone,  $E_s$  being the energy of the  $M$ -ary symbol,  $f_i$  is the tone frequency corresponding to the  $i$ th bit of a symbol and  $f_{h_j}^{(k)}$  is the hopping frequency of  $k$ th user in  $j$ th chip ( $1 \leq j \leq H$ ). The phase of  $k$ th user in  $(i, j)$ th slot is  $\theta_{i,j}^{(k)}$  which is assumed to be i.i.d. and uniformly distributed over  $0$  to  $2\pi$ .

### III. RECEIVER STRUCTURE

We consider a synchronous system in which signals from all the users are time aligned at the receiver. We assume that within a hopping slot the separation between the  $L$  tone frequencies is smaller than the coherence bandwidth and hence

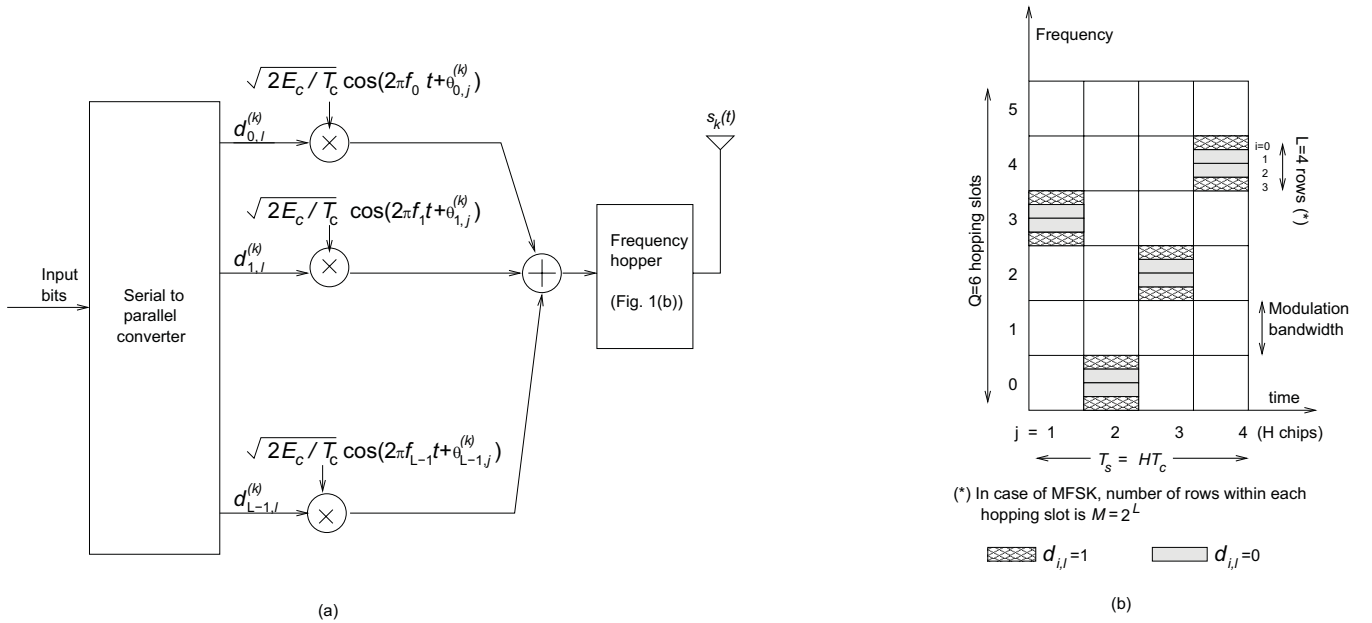


Fig. 1. (a) Transmitter for FFH/MCOOK system; (b) Frequency-hopping system  $M = 16$ ,  $L = \log_2 M = 4$ ,  $Q = 6$ ,  $H = 4$ .

fading is identical for all the tones. Thus the received signal for  $K$  users is,

$$r(t) = \sum_{k=0}^{K-1} \left[ \sqrt{\frac{2E_c}{T_c}} \beta_j^{(k)} \sum_{i=0}^{L-1} d_{i,l}^{(k)} \cos[2\pi(f_{h_j}^{(k)} + f_i)t + \theta_{i,j}^{(k)}] + n(t) \right], \quad 0 \leq t \leq T_c \quad (2)$$

where  $n(t)$  is a sample of WGN with zero mean and two sided PSD  $N_0/2$ , and  $\beta_j^{(k)}$ 's are Rayleigh random variables. The random phase offset in Rayleigh fading channel can be taken into account by absorbing it into  $\theta_{i,j}^{(k)}$ . We use non-coherent detection and assume the zeroth user to be the desired user. Let  $r^{(0)}(t)$  denote the signal obtained after dehopping  $r(t)$  with respect to the 0th user's address. Then,

$$r^{(0)}(t) = \sum_{k=0}^{K-1} \left[ \sqrt{\frac{2E_c}{T_c}} \beta_j^{(k)} \sum_{i=0}^{L-1} d_{i,l}^{(k)} \cos[2\pi(f_{h_j}^{(k)} - f_{h_j}^{(0)} + f_i)t + \theta_{i,j}^{(k)}] + n(t) \right] \quad (3)$$

The separation between  $f_i$ 's is assumed to be an integral multiple of  $1/T_c$ . Thus the correlator outputs of the 0th user for  $i$ th tone and  $j$ th chip given that there are  $m_j$  hits<sup>1</sup> in  $j$ th chip, is,

$$\begin{aligned} r_{i,jI} &= \int_0^{T_c} r^{(0)}(t) \sqrt{\frac{2}{T_c}} \cos(2\pi f_i t) dt \\ &= d_{i,l}^{(0)} \sqrt{E_c} \beta_j^{(0)} \cos \theta_{i,j}^{(0)} + \sum_{k=1}^{m_j} d_{i,l}^{(k)} \sqrt{E_c} \beta_j^{(k)} \cos \theta_{i,j}^{(k)} \\ &\quad + n_{i,jI} \\ r_{i,jQ} &= \int_0^{T_c} r^{(0)}(t) \sqrt{\frac{2}{T_c}} \sin(2\pi f_i t) dt \end{aligned} \quad (4)$$

<sup>1</sup>The number of hits in a given chip is equal to the number of interfering users in that chip i.e. the number of users whose address element in the given chip is same as that of the desired user.

$$= - \left[ d_{i,l}^{(0)} \sqrt{E_c} \beta_j^{(0)} \sin \theta_{i,j}^{(0)} + \sum_{k=1}^{m_j} d_{i,l}^{(k)} \sqrt{E_c} \beta_j^{(k)} \sin \theta_{i,j}^{(k)} + n_{i,jQ} \right] \quad (5)$$

With random address assignment, the probability of  $m_j$  hits in a chip,  $P(m_j)$ , is given by (3) of [1] with  $m$  and  $P_h$  replaced by  $m_j$  and  $1/Q$  respectively. For  $f_i = 1/T_c$ ,  $Q$  is related to the total spread bandwidth  $B$  as  $Q = B/(\log_2 M/T_c)$ . We define the number of interfering signals<sup>2</sup> for 0th user in  $i$ th tone and  $j$ th chip corresponding to  $m_j$  hits as  $h_{i,j}(m_j) = \sum_{k=1}^{m_j} d_{i,l}^{(k)}$  with  $P(h_{i,j}(m_j)) = \frac{1}{2^{m_j}} \frac{m_j!}{(h_{i,j}(m_j))! (m_j - h_{i,j}(m_j))!}$  (see [1]).

#### A. Sub-Optimum Receiver

Analogous to that derived in [1], the optimum receiver for the FFH/MCOOK system considered here turns out to be complicated and is of little practical interest. In this section we discuss a sub-optimum receiver motivated by the sub-optimum receivers discussed in [1], [3], [4]. As discussed in [1] the outputs of the energy detectors<sup>3</sup>,  $r_{i,j} = r_{i,jI}^2 + r_{i,jQ}^2$  for  $1 \leq j \leq H$ ,  $0 \leq i \leq L-1$ , provide sufficient statistics for the detection. The received signal is non-coherently detected and for each of the  $L$  rows of correlator pairs, the outputs of the energy detectors corresponding to  $H$  successive chips are added. The  $L$  different values thus obtained are compared with a threshold  $\gamma$  to detect  $L$  bits of the desired user. The detection rule is stated as<sup>4</sup>:

$$\sum_{j=1}^H r_{i,j} \begin{cases} > \gamma \Rightarrow \hat{d}_i = 1 \\ < \gamma \Rightarrow \hat{d}_i = 0 \end{cases} \quad (6)$$

<sup>2</sup>The number of interfering signals is equal to the number of interfering users in  $j$ th chip whose transmitted bit is 1 corresponding to the  $i$ th frequency.

<sup>3</sup>The block of two correlators, squarers and the adder is called energy detector.

<sup>4</sup> $\hat{\mathbf{d}} = (\hat{d}_0, \hat{d}_1, \dots, \hat{d}_{L-1})$  is the decoded symbol.

#### IV. PERFORMANCE ANALYSIS OF THE SUB-OPTIMUM RECEIVER

Since  $\theta_{i,j}^{(k)}$  are i.i.d. uniform random variables, therefore for a given symbol  $\mathbf{d}_l$  of the desired user and a given hit pattern  $\mathbf{h}_j(m_j)$ ,  $r_{i,j_I}$  and  $r_{i,j_Q}$  in (4) and (5) are independent Gaussian random variables with zero mean. The variance of  $r_{i,j_I}$  and  $r_{i,j_Q}$  is,

$$\sigma_{r_{i,j_I}}^2 = \sigma_{r_{i,j_Q}}^2 = E[r_{i,j_I}^2] = \frac{1}{2}(d_{i,l}\bar{E}_c + h_{i,j}(m_j)\bar{E}_c + N_0)$$

where  $\bar{E}_c = E[\beta_j^2] E_c$  (7)

and the density of  $r_{i,j}$  given  $h_{i,j}(m_j)$  and  $\mathbf{d}_l$  is,

$$f(r_{i,j} | h_{i,j}(m_j), \mathbf{d}_l) = \frac{1}{\sigma_{r_{i,j}}^2} \exp\left(-\frac{r_{i,j}^2}{\sigma_{r_{i,j}}^2}\right) \quad (8)$$

$$\text{where } \sigma_{r_{i,j}}^2 = E[r_{i,j}^2] = \sigma_{r_{i,j_I}}^2 + \sigma_{r_{i,j_Q}}^2 \quad (9)$$

The density of  $r_{i,j}$  given  $m_j$  hits and  $\mathbf{d}_l$ , can be obtained as:

$$f(r_{i,j} | m_j, \mathbf{d}_l) = \sum_{h_{i,j}(m_j)} f(r_{i,j} | h_{i,j}(m_j), \mathbf{d}_l) P(h_{i,j}(m_j)) \quad (10)$$

We define a hit vector  $\mathbf{m} = (m_1, m_2, \dots, m_H)$ , whose elements are the number of hits from the first to the last chip. Let  $\mathbf{r}_i = (r_{i,1}, r_{i,2}, \dots, r_{i,H})$  be a vector consisting of the outputs of energy detectors of all the chips corresponding to  $i$ th correlator pair. Since  $r_{i,j_I}$  and  $r_{i,j_Q}$  and hence  $r_{i,j}$  are independent,  $f(\mathbf{r}_i | \mathbf{m}, \mathbf{d}_l) = \prod_{j=1}^H f(r_{i,j} | m_j, \mathbf{d}_l)$ . Therefore the probability of choosing the symbol  $\mathbf{d}_k$  when the transmitted symbol is  $\mathbf{d}_l$ <sup>5</sup> and the hit vector is  $\mathbf{m}$ , is,

$$P_M(\mathbf{d}_l \rightarrow \mathbf{d}_k | \mathbf{m}, \mathbf{d}_l) = \prod_{i=0}^{L-1} \iint \dots \int_{a_i < \sum_{j=1}^H r_{i,j} < b_i} f(\mathbf{r}_i | \mathbf{m}, \mathbf{d}_l) d\mathbf{r}_i \quad (11)$$

where,  $a_i = 0, b_i = \gamma$  for  $d_{i,k} = 0$  and  $a_i = \gamma, b_i = \infty$  for  $d_{i,k} = 1$ . Hence the probability of detecting  $\mathbf{d}_l$  as  $\mathbf{d}_k$  is,

$$P_M(\mathbf{d}_l \rightarrow \mathbf{d}_k | \mathbf{d}_l) = \sum_{m_1=0}^{K-1} \dots \sum_{m_H=0}^{K-1} P_M(\mathbf{d}_l \rightarrow \mathbf{d}_k | \mathbf{m}, \mathbf{d}_l) P(\mathbf{m}) \quad (12)$$

where  $P(\mathbf{m})$  is the probability of  $\mathbf{m}$  given by  $P(\mathbf{m}) = \prod_{j=1}^H P(m_j)$ . Since all the symbols are equally likely, the probability of symbol error is,

$$P_M = \frac{1}{M} \sum_{l=0}^{M-1} \sum_{\substack{k=0 \\ k \neq l}}^{M-1} P_M(\mathbf{d}_l \rightarrow \mathbf{d}_k | \mathbf{d}_l) \quad (13)$$

The probability of bit error can then be written as [5],

$$P_b = \frac{1}{M \log_2 M} \sum_{l=0}^{M-1} \sum_{\substack{k=0 \\ k \neq l}}^{M-1} d_h(k, l) P_M(\mathbf{d}_l \rightarrow \mathbf{d}_k | \mathbf{d}_l) \quad (14)$$

<sup>5</sup>It should be noted that the symbol error probabilities are not symmetric in MCOOK.

#### A. Performance of the Sub-Optimum Receiver for $M=2$

For  $M = 2$  i.e. binary MCOOK, the joint density of  $r_1$  and  $r_H$  when  $d_l = 0$ , can be written in closed form as:<sup>6</sup>

$$f(r_1, r_2, \dots, r_H | d_l = 0) = \prod_{j=1}^H f(r_j | d_l = 0) = \sum_{n_1=0}^{K-1} \dots \sum_{n_H=0}^{K-1} C_{n_1} C_{n_2} \dots C_{n_H} \times \frac{1}{\prod_{j=1}^H (N_0 + n_j \bar{E}_c)} \exp\left(-\sum_{j=1}^H \frac{r_j^2}{(N_0 + n_j \bar{E}_c)}\right) \quad (15)$$

$$\text{where } C_{n_j} = \sum_{x=n_j}^{K-1} P(m_j = x) P(h_j(m_j = x) = n_j),$$

$$1 \leq j \leq H \text{ and } 0 \leq n_j \leq K-1 \quad (16)$$

In order to distinguish the values taken by  $m_j$  and  $h_j(m_j)$ , we denote the probability of  $m_j$  taking the value  $x$  by  $P(m_j = x)$  and the probability of  $h_j(m_j)$  taking the value  $n_j$  when the value of  $m_j$  is  $x$ , by  $P(h_j(m_j = x) = n_j)$ . When the desired user's symbol is '1', the variance of  $r_j$  contains an additional  $\bar{E}_c$  term. Therefore the conditional density  $f(r_1, r_1, \dots, r_H | d_l = 1)$  is obtained by replacing all the  $n_j$ 's in (15) by  $n_j + 1$ . The average bit error probability is  $P_b = 0.5[P_b(d_l = 0) + P_b(d_l = 1)]$  where,

$$P_b(d_l = 0) = 1 - \int \dots \int_{0 < \sum_{j=1}^H r_j < \gamma} f(r_1, \dots, r_H | d_l = 0) dr_1 \dots dr_H \quad (17)$$

$$P_b(d_l = 1) = \int \dots \int_{0 < \sum_{j=1}^H r_j < \gamma} f(r_1, \dots, r_H | d_l = 1) dr_1 \dots dr_H \quad (18)$$

To obtain the final expression for  $P_b$  we need to integrate the RHS of (15). The integral of each of the terms in the sum in (15) can be obtained as follows, with  $A_j = N_0 + n_j \bar{E}_c$ ,

$$\int \dots \int_{0 < \sum_{j=1}^H r_j < \gamma} \frac{1}{\prod_{j=1}^H A_j} \exp\left(-\sum_{j=1}^H \frac{r_j^2}{A_j}\right) dr_1 dr_2 \dots dr_H = 1 - \sum_{j=1}^H \frac{A_j^{H-1}}{\prod_{\substack{b=1 \\ b \neq j}}^H (A_j - A_b)} \exp\left(-\frac{\gamma^2}{A_j}\right) \quad (19)$$

If  $A_j = A_b$  for any  $(j, b) \in \{1, 2, \dots, H\}$ , the integral is found as the limit  $A_j \rightarrow A_b$  of (19).

#### V. RESULTS AND DISCUSSION

As in [1], the average bit error probability  $P_b$  of FFH/MCOOK system for the sub-optimum receiver discussed

<sup>6</sup>As  $L = 1$  in this case, we do not write the subscript  $i$  of  $r_{i,j}$  and use  $d_l$  to denote the only element of  $\mathbf{d}_l$ .

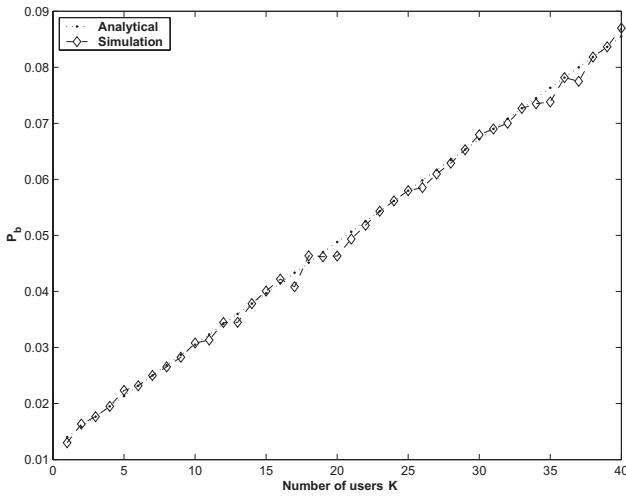


Fig. 2. Average bit error probability  $P_b$  vs.  $K$ , analytical and simulation results; MCOOK,  $M = 2$ , spread bandwidth = 0.8 MHz,  $T_s = 5 \times 10^{-4}$  s,  $H = 2$  and  $\frac{E_b}{N_0} = 15$  dB; optimum values of  $\gamma$  are in the range  $\gamma = 0.3408$  to  $\gamma = 0.3933$  for  $K = 1$  to  $K = 40$ .

is a function of the threshold  $\gamma$ , the optimum value of which depends on the system parameters. Since the signal and the interference in every row is independent, the  $\gamma$  that optimizes the performance in the case of a single row ( $L = 1$  or  $M = 2$ ) is same as the  $\gamma$  that is optimal for multiple rows ( $L > 1$  or  $M > 2$ ) if the number of hopping slots  $Q$  and other system parameters are fixed. Thus optimal  $\gamma$  for a given system can be obtained by considering an equivalent<sup>7</sup> system with  $M = 2$  and finding an optimum  $\gamma$  for it. This can be done by using the analytical expression for  $P_b$  from Section IV-A and equating the derivative of  $P_b$  with respect to  $\gamma$ , to 0. All the results that follow have been obtained using the optimum  $\gamma$  for the system under consideration.

In Fig. 2 we plot  $P_b$  against  $K$  as obtained from (14) and from simulations. Each simulation point is obtained by running the simulation for 200,000 bits. As is clear, the simulation result closely matches the theoretical result which proves the validity of the analytical result derived in Section IV-A.

In Fig. 3 we evaluate the performance of binary MCOOK ( $M = 2$ ) system for varying number of chips when the total spread bandwidth is fixed. The probability of error calculated as described in Section IV-A, is plotted against the number of users. We observe that the curves corresponding to different values of  $H$  intersect each other. Here  $H = 1$  corresponds to the SFH case discussed in [1]. It can be seen that for a given range of users, there exists an optimum value of  $H$  for which  $P_b$  is minimum. The point of intersection between any two values of  $H$  varies from system to system. The two factors that pull the location of the intersection point in opposing directions as  $H$  is increased, are: reduction in the number of hopping slots that increases the chances of hits and the increased diversity obtained by increased number of hops per symbol. In particular, it can be verified that as the spread bandwidth is increased, the intersection point between any two given values of  $H$  shifts to higher values because large

<sup>7</sup>A system with  $M = 2$  in which  $Q$  and other system parameters are same as those of the given system.

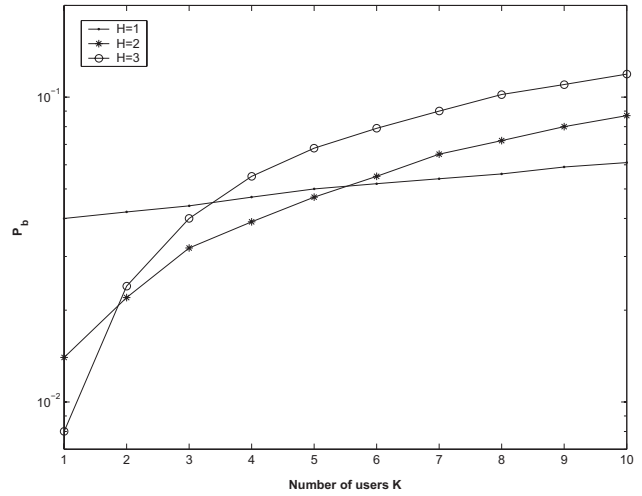


Fig. 3. Average bit error probability  $P_b$  vs.  $K$  for varying  $H$ ; MCOOK,  $M = 2$ , spread bandwidth=0.18 MHz,  $T_s = 5 \times 10^{-4}$  s and  $\frac{E_b}{N_0} = 15$  dB; optimum values of  $\gamma$  are in the range (i)  $H = 1$ ,  $\gamma = 0.200$  to  $\gamma = 0.202$  (ii)  $H = 2$ ,  $\gamma = 0.34$  to  $\gamma = 0.40$  (iii)  $H = 3$ ,  $\gamma = 0.46$  to  $\gamma = 1.00$  for  $K = 1$  to  $K = 10$ .

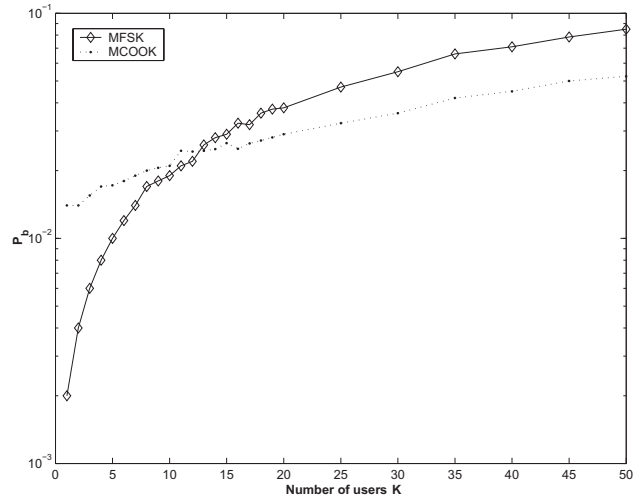


Fig. 4. Average bit error probability  $P_b$  vs.  $K$  for MCOOK and MFSK;  $M = 16$ , spread bandwidth = 8 MHz,  $T_s = 5 \times 10^{-4}$  s,  $H = 2$ , and  $\frac{E_b}{N_0} = 15$  dB; optimum values of  $\gamma$  are in the range  $\gamma = 0.3408$  to  $\gamma = 0.3623$  for  $K = 1$  to  $K = 50$ .

bandwidth suppresses the first factor and allows the second factor to dominate for larger range of  $K$ . Also, as the spread bandwidth increases, the performance for all values of  $H$  improves as expected. By evaluating the expressions in section IV-A it can be verified that, for  $K = 1$ ,  $P_b$  decreases as  $H$  increases because there are no hits and the only factor that affects  $P_b$  is the diversity gain from increasing hops. In general, because of the complexity of the expressions involved, it is difficult to obtain a pattern for the intersection points as a function of system parameters and hence, depending on the application, the system needs to be optimized for optimal  $H$ .

For  $M > 2$ , we present only simulation results since the expressions for  $P_b$  become complicated for analytical evaluation.

In Figs. 4 and 5, we compare the performance of FFH/MCOOK system with that of FFH/MFSK system. For

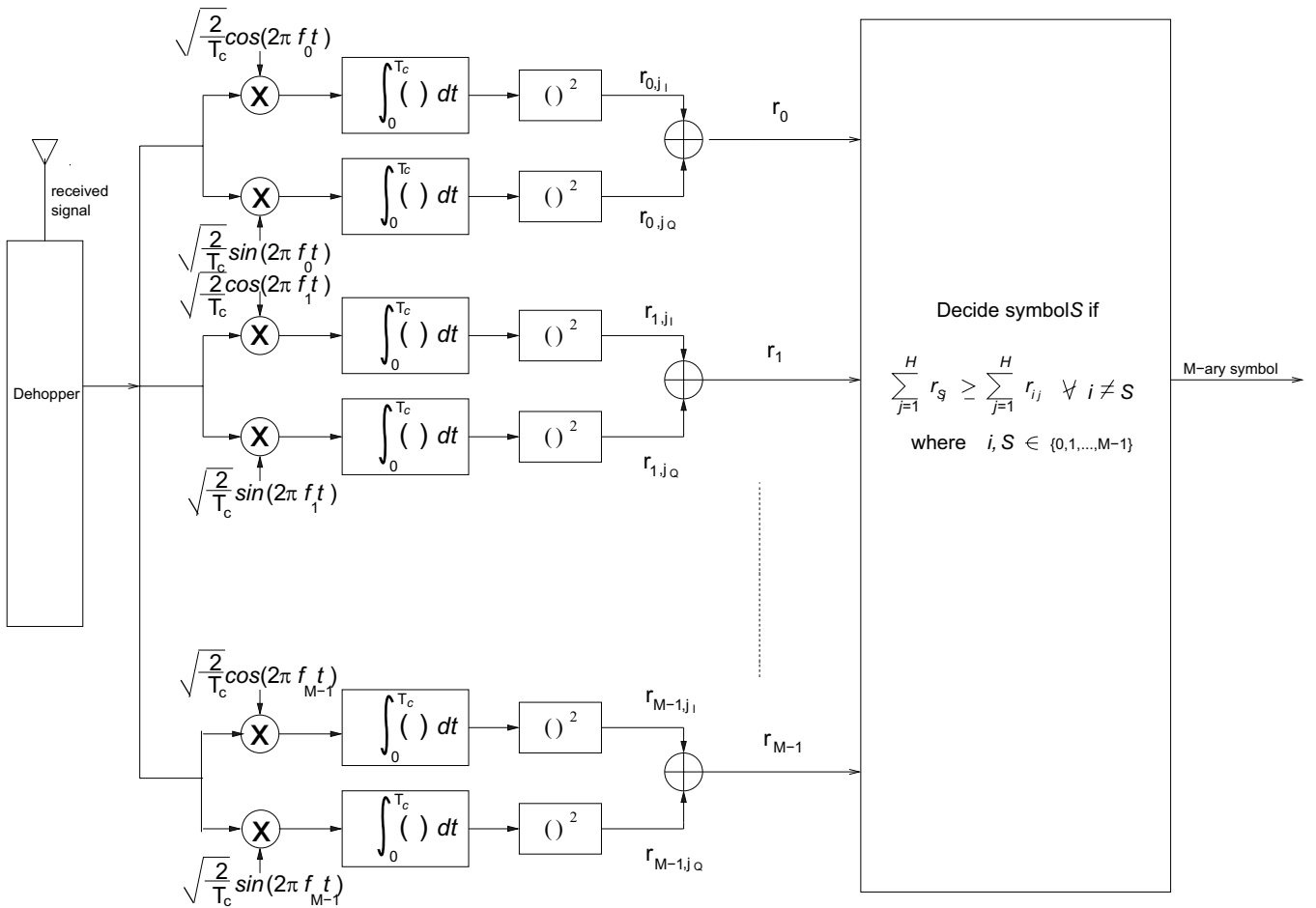


Fig. 6. Receiver for FFH/MFSK.

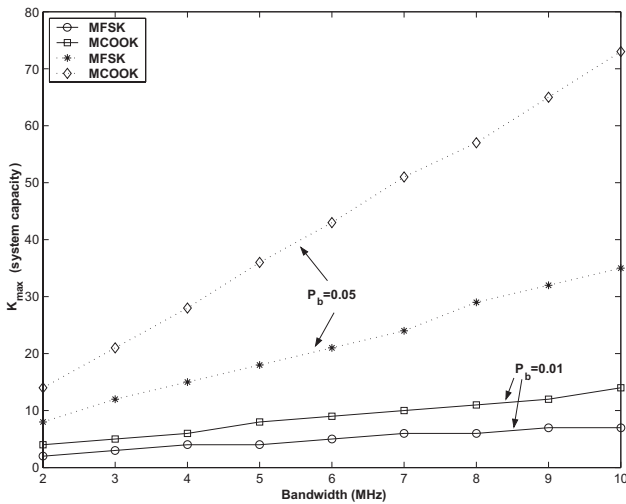


Fig. 5. Maximum user capacity  $K_{max}$  vs spread bandwidth under the constraints of  $P_b = 0.01$  and  $P_b = 0.05$ ;  $M = 16$ ,  $T_s = 5 \times 10^{-4} s$ ,  $H = 2$ ,  $\frac{E_b}{N_0} = \infty$ .

the purpose of comparison, we consider the MFSK receiver shown in Fig. 6.

Fig. 4 shows the plot of  $P_b$  vs  $K$  for MCOOK and MFSK systems, both using  $M = 16$ , having same spread bandwidth

and operating at same  $\bar{E}_b/N_0$ .<sup>8</sup> It is observed that the two curves intersect. In low MAI region, MFSK performs better while in high MAI region, MCOOK performs better. The smaller slope of MCOOK curve is due to the fact that in MCOOK, only half of the bits (the bits which are one) carry energy, so when the number of users increase, the interference does not increase as much as in MFSK in which all the symbols carry equal energy. In the low MAI region, it is unlikely in MFSK that the sum of energy of chips in any other row exceeds that of the correct row whereas in MCOOK, whenever there is a hit, all the rows are affected and the decision for one or the other row may go wrong resulting in a symbol error. The relative increase in the number of hopping slots in MCOOK does not dominate the above factor because the interference is already low and hence, in the low MAI region, it is MFSK which performs better. It can be verified that the point above which MCOOK performs better than MFSK, shifts towards lower number of users as  $M$  is increased. This is because of the increased gain factor,  $M/\log_2 M$ , of the number of hopping slots in MCOOK over MFSK as  $M$  is increased.

For a given bandwidth and a given number of users, if we plot  $P_b$  vs  $\bar{E}_b/N_0$  for the two systems, a plot similar to that in

<sup>8</sup> $E_b = E_s/2$  for MCOOK system. For MFSK system,  $E_b$  is  $(1/\log_2 M)$  times the symbol energy of MFSK system.  $\bar{E}_b = E[\beta^2] E_b$ .

Fig. 9 of [1] is obtained<sup>9</sup> which shows that in FFH case too, MCOOK performs better than MFSK once  $\overline{E}_b/N_0$  is above a particular threshold. Further, the value of  $\overline{E}_b/N_0$  above which MCOOK performs better, decreases as the number of users increase because of comparative increase in MAI.

In Fig. 5 we plot the maximum number of users supported by MCOOK and MFSK systems in the interference limited region against the spread bandwidth for a given bit error probability. Both the systems considered use two chips per symbol. As can be seen, in this case MCOOK provides capacity gain over MFSK by a factor of 2 approximately. It should be noted that this gain is lesser than the gain obtained for  $H = 1$  in [1]<sup>10</sup>. This agrees with the result obtained above (Fig. 3) that in the interference limited region, the performance of MCOOK system improves as the number of chips is decreased.

## VI. CONCLUSION

We analyzed Multicarrier On-Off Keying as a modulation scheme for FFHMA systems. The BER was analyzed for a sub-optimum receiver in a Rayleigh fading channel and a closed form expression was derived for the binary case. It was observed that the number of chips per symbol required

to optimize the performance varies from system to system. The performance of FFH/MCOOK system was compared with FFH/MFSK and it was found that in the interference-limited region (large MAI), MCOOK performs better than MFSK. The performance gain over MFSK increases with the modulation index  $M$ .

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<sup>9</sup>Due to lack of space we are not providing the plot here.

<sup>10</sup>[1] compares the two systems for  $M = 8$ . The corresponding gain for  $M = 16$  will be even higher.