Evaluation of Error Probabilities in the Presence of Timing Errors and Fading

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***Abstract*— Using the approximate Fourier series technique we obtain expressions for the probability of error for bandlimited BPSK signalling in the presence of timing errors and fading. The derived results can be used to compute the error probabilities to any desired accuracy for Nakagami-m and Weibull fading channels. The effect of timing error on the performance of the Raised Cosine pulse has been evaluated for several fading parameters. We also compare the performance of some useful Nyquist pulses known.**

***Index Terms*— Timing error, fading, pulse analysis.**

I. Introduction

For bandlimited communications in the Additive White Gaussian Noise (AWGN) channel, it was shown in [1] that in the presence of timing errors the probability of error can be considerable even when the error probability due to noise alone or timing error alone is negligible. An expression for error probability in the presence of Intersymbol Interference (ISI) was derived in [2] for the AWGN channel. This expression proved to be extremely useful in the context of evaluating the sensitivity of various Nyquist pulses to timing errors in the AWGN channel [4], [5], [6]. In this paper, we derive the corresponding expressions for Nakagami-m and Weibull fading channels. We use these results to analyse the impact of timing errors on the performance of some useful Nyquist pulses in these channels.

II. Probability of Error in the Presence of Timing Errors and Fading

The system model of [3] can be modified to analyse the effect of timing error in fading channels. We assume a single user scenario with no cochannel interference. A flat, slow fading channel and bandlimited BPSK signalling is considered. Let b_ks be the transmitted data bits that are independent and take the values $\{-1, +1\}$ with equal probabilities. If $g_T(t)$ is the transmitted baseband pulse shape such that $\int_{-\infty}^{\infty} g_T^2(t)dt = 1$, then the received signal is given by

$$y(t) = \sqrt{2P_sT} \cdot r \cdot s(t) \cdot \cos(w_ct + \theta) + n(t) \quad (1)$$

where $s(t) = \sum_{k=-\infty}^{\infty} b_k g_T(t - kT)$, T is the bit interval, r and θ are the gain and phase introduced by fading, w_c is the carrier frequency and $n(t)$ is AWGN. The phase θ is assumed to be uniformly distributed between $[0, 2\pi)$ and without loss of generality $E[r^2] = 1$, where $E[\cdot]$ denotes expectation. Hence, P_s is the received signal power. Let $g(t)$ be a Nyquist pulse denoting the overall impulse response of the cascade of the transmit and receive filters. At the receiver, the decision statistic Z for the zeroth data symbol b_o after coherent detection, matched filtering and sampling at $t = \epsilon$ can be expressed as

$$Z_0 = r(b_0g_0 + z_0) + n_0 \quad (2)$$

where $z_0 = \sum_{k\neq 0} b_kg_k$, n_0 is zero mean Gaussian and $g_k = \sqrt{\frac{P_sT}{2}} g(\epsilon - kT)$. Thus, z_0 denotes the interference caused by neighboring symbols due to a timing error of ϵ at the sampling instant. Without loss of generality, we assume $E[n_0^2] = 1$. Then as in [3], SNR can be expressed as $(\frac{P_sT}{2})$.

When $b_0 = -1$, error occurs if $Z_0 > 0$. Thus from (2), the probability of error conditioned on z_0 and r can be written as

$$P_{e|\{z_0,r\}} = P(n_0 > r(g_0 - z_0)) = Q(r(g_0 - z_0)) \quad (3)$$

where $P(\cdot)$ denotes probability and $Q(\cdot)$ represents the complementary distribution function (cdf) of a zero mean Gaussian random variable with unit variance. Probability of error P_e can be found by averaging $P_{e|\{z_0,r\}}$ over all values of z_0 and r. Since z_0 and r are mutually independent, P_e conditioned only on z_0 can be expressed as

$$P_{e|z_0}(g_0 - z_0) = \int_0^{\infty} Q(r(g_0 - z_0))f_r dr \quad (4)$$

where f_r is the probability density function (pdf) of the fading gain r.

Let f_V denote the pdf of the distribution with $P_{e|z_0}$ as cdf. Then $f_V = -\frac{\partial P_{e|z_0}(g_0-z_0)}{\partial(g_0-z_0)}$. For an even f_V, the probability of error, P_e, can be found using (16) and (26) of [2] as

$$P_e = \frac{1}{2} - \frac{2}{\pi} \sum_{l=1,odd}^{L_f} \left\{ \frac{A_l \sin(l\omega g_0)}{l} \prod_{k=-\infty,k\neq 0}^{\infty} \cos(l\omega g_k) \right\} + \Delta \quad (5)$$

where $T_f = 2\pi/\omega$ is the period of the approximate Fourier series of $P_{e|z_0}$ used in the derivation of (5), L_f is the largest index of l and A_l is given by

$$A_l = \int_{-\infty}^{\infty} f_V(x) \cos(l\omega x)dx \quad (6)$$

The expression of P_e given in (5) has limited utility unless the error Δ can be made arbitrarily small. It was shown in [2] that $|\Delta|$ is upper bounded as

$$|\Delta| < P_{e|z_0} \left(\frac{T_f}{2} - \sum_{k=-\infty}^{\infty} |g_k| \right) + \frac{2}{\pi} \int_{L_f}^{\infty} \frac{A_l}{l} dl \quad (7)$$

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The first term in the R.H.S. of (7), say Δ_1, is due to the use of the approximate Fourier series while the second term, say Δ_2, is due to truncation of the series. By choosing the values of T_f and L_f to be large enough, Δ can be made arbitrarily small. Hence the expression in (5) can be used to compute the error probabilities due to timing error to any desired accuracy. We evaluate $A_l, \Delta_1,$ and Δ_2 for Nakagami-m and Weibull fading channels in the subsequent sections.

III. Nakagami-m Fading Channel

When f_r follows Nakagami-m distribution with parameters (m, Ω), the conditional probability with respect to z_0, $P_{e|z_0}(g_0 - z_0)$, can be obtained for the case of coherent BPSK as [11]

$$P_{e|z_0}(g_0 - z_0) = \frac{B_{X_o}(m, \frac{1}{2})}{2B(m, \frac{1}{2})} \tag{8}$$

where $X_o = 2m/(2m + \Omega(g_0 - z)^2)$ and $B_{X_o}(a,b) = \int_0^{X_o} u^{a-1}(1-u)^{b-1}du$ is known as the incomplete beta function and $B(a,b) = B_1(a,b)$ [8]. It can be verified that $f_V = -\frac{\partial P_{e|z_0}(g_0 - z_0)}{\partial(g_0 - z_0)}$ is even. Hence, A_l can be found using (8.432.5) of [7] in (6) as

$$A_l = \frac{2}{\Gamma(m)}\left(\frac{ml^2\omega^2}{2\Omega}\right)^{m/2} \mathrm{K}(m, \sqrt{\frac{2m}{\Omega}}l\omega) \tag{9}$$

where $\mathrm{K}(\cdot,\cdot)$ is the modified Bessel function of the second kind [7]. Substituting (9) in (5) and rearranging, we get the error probability for Nakagami-m channel in the presence of timing errors as

$$P_e = \frac{1}{2} - \frac{4}{\pi\Gamma(m)}\left(\frac{m\omega^2}{2\Omega}\right)^{m/2} \sum_{l=1,odd}^{L_f} l^{m-1}\mathrm{K}(m, \sqrt{\frac{2m}{\Omega}}l\omega) \cdot \sin(l\omega g_0) \prod_{k=-\infty, k\neq 0}^{\infty} \cos(l\omega g_k) + \Delta_1 + \Delta_2 \tag{10}$$

where Δ_1 and Δ_2 have been defined in the preceding section. Using (7) and (8)

$$\Delta_1 = \frac{B_{X_o}(m, \frac{1}{2})}{2B(m, \frac{1}{2})} \tag{11}$$

where $X_o = 2m/(2m + \Omega(T_f/2 - \sum_{k=-\infty}^{\infty}|g_k|)^2)$. $B_{X_o}(\cdot,\cdot)$ is an increasing function of X_o and hence in this case, a decreasing function of T_f. So Δ_1 can be made infinitesimally smaller by choosing a large enough T_f. Using the second term on the R.H.S. of (7), the error bound Δ_2 can be found as

$$\Delta_2 = \frac{4}{\pi\Gamma(m)}\left(\frac{m}{2\Omega}\right)^{m/2} \int_{L_f}^{\infty} \frac{(l\omega)^{m+1}\mathrm{K}(m, \sqrt{\frac{2m}{\Omega}}l\omega)}{(l\omega)l} dl \tag{12}$$

As $l > L_f$ in the integrand of (12), Δ_2 is upper bounded by

$$\Delta_2 < \frac{4}{\pi\Gamma(m)\omega L_f^2}\left(\frac{m}{2\Omega}\right)^{m/2} \int_{L_f}^{\infty} (l\omega)^{m+1}\mathrm{K}(m, \sqrt{\frac{2m}{\Omega}}l\omega)dl \tag{13}$$

Substituting $y = \sqrt{\frac{2m}{\Omega}}l\omega$ in (13) and using (3) of (7.14.1) of [9] we get

$$\Delta_2 < \frac{1}{\pi 2^{m-2}\Gamma(m)}\left(\sqrt{\frac{2m}{\Omega}}L_f\omega\right)^{m-1} \mathrm{K}(m+1, \sqrt{\frac{2m}{\Omega}}L_f\omega) \tag{14}$$

We know that when $\Omega = 1$ and $m \to \infty$, Nakagami-m channel reduces to an AWGN channel. In Appendix A we show that when we let $m \to \infty$ in (10), as expected, the obtained expression turns out to be identical to that in [2] for an AWGN channel.

IV. Weibull Fading Channel

Experimental data have shown that the Weibull fading channel model exhibits an excellent fit both for indoor and outdoor environments [12]. We now consider the case when pdf f_r in (4) follows Weibull distribution with parameters (c, β) [12]. Substituting f_r in (4) we get

$$P_{e|z_0}(g_0 - z_0) = \frac{c}{\beta}\int_0^{\infty} Q(r(g_0 - z_0))r^{c-1}e^{-\frac{r^c}{\beta}}dr \tag{15}$$

Clearly, $f_V = -\frac{\partial P_{e|z_0}(g_0 - z_0)}{\partial(g_0 - z_0)}$ is even. So, using (6), A_l for this case is given by

$$A_l = \frac{c}{\beta\sqrt{2\pi}}\int_{-\infty}^{\infty}\left(\int_0^{\infty} e^{\frac{-r^2x^2}{2}} r^c e^{-\frac{r^c}{\beta}}\cos(lwx)dr\right)dx \tag{16}$$

As the inner integral converges, the order of the two integrals can be interchanged. Interchanging the integrals and then using (3.896) of [7], A_l becomes

$$A_l = \frac{c}{\beta}\int_0^{\infty} e^{-\frac{l^2\omega^2}{2r^2}} r^{c-1} e^{-\frac{r^c}{\beta}}dr \tag{17}$$

For all integer values of c, A_l can be expressed in terms of Meijer's G function [7].

As shown in Appendix B, A_l can be expressed as

$$A_l = \frac{\sqrt{2c}}{(2\pi)^{c/2}} G_{0,c+2}^{c+2,0}\left(\frac{(\omega l)^{2c}}{\beta^2 2^{c+2}c^c}\,\middle|\, \begin{matrix}\{\}\\ S\end{matrix}\right) \tag{18}$$

where $G_{p,q}^{m,n}(\cdot)$ is Meijer's G function defined in (9.301) of [7] and $S = \{0, \frac{1}{c}, \frac{2}{c}, \cdots, 1, \frac{1}{2}\}$. The error probability for the Weibull fading channel can now be written by substituting A_l from (18) in (5) as

$$P_e = \frac{1}{2} - \frac{2\sqrt{2c}}{(2\pi)^{c/2}\pi}\sum_{l=1,odd}^{L_f} G_{0,c+2}^{c+2,0}\left(\frac{(\omega l)^{2c}}{\beta^2 2^{c+2}c^c}\,\middle|\, \begin{matrix}\{\}\\ S\end{matrix}\right) \cdot \frac{\sin(l\omega g_0)}{l}\prod_{k=-\infty,k\neq 0}^{\infty}\cos(l\omega g_k) + \Delta_1 + \Delta_2 \tag{19}$$

The error bound Δ_1 for the Weibull case is calculated as

$$\Delta_1 = \frac{c}{\beta}\int_0^{\infty} Q(rT_{eff})r^{c-1}e^{\frac{-r^c}{\beta}}dr$$

where $T_{eff} = (T_f/2 - \sum_{k=-\infty}^{\infty}|g_k|)$. Using $Q(x) = 0.5(1 - \mathrm{erf}(x/\sqrt{2}))$ with $\mathrm{erf}(\cdot)$ denoting the error function given by $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ and the fact that $e^{\frac{-r^c}{\beta}} < 1$ we get

$$\Delta_1 < \frac{c}{2\beta(T_{eff}/\sqrt{2})^c}\int_0^{\infty}(1 - \mathrm{erf}(u))\cdot u^{c-1}du$$

TABLE I

VALUES OF T_f, N_L, P_e AND $\Delta = (\Delta_1 + \Delta_2)$ FOR NAKAGAMI-m AND WEIBULL FADING CHANNELS FOR $\epsilon = 0.15T$

Channel	Parameter	(T_f, N_L) SNR = 15dB	$P_e \pm (\Delta_1 + \Delta_2)$ SNR = 15dB	(T_f, N_L) SNR=20dB	$P_e \pm (\Delta_1 + \Delta_2)$ SNR = 20dB
Nakagami-m	$m = 1$	$(377, 242)$	$(1.8483 \pm 0.0018)e-2$	$(650, 458)$	$(6.1202 \pm 0.0061)e-3$
	$m = 3$	$(55, 31)$	$(1.6101 \pm 0.0010)e-3$	$(85, 56)$	$(8.6939 \pm 0.0081)e-5$
	$m = 4$	$(42, 23)$	$(8.5106 \pm 0.0073)e-4$	$(69, 45)$	$(2.2539 \pm 0.0022)e-5$
	$m = 5$	$(37, 20)$	$(5.3637 \pm 0.0053)e-4$	$(62, 40)$	$(7.7091 \pm 0.0075)e-6$
	$m = 10$	$(31, 16)$	$(1.6523 \pm 0.0012)e-4$	$(50, 33)$	$(2.8868 \pm 0.0021)e-7$
Weibull	$c = 2$	$(675, 460)$	$(1.8482 \pm 0.0018)e-2$	$(1175, 865)$	$(6.1197 \pm 0.0061)e-3$
	$c = 4$	$(64, 30)$	$(2.1521 \pm 0.0020)e-3$	$(175, 105)$	$(2.2429 \pm 0.0022)e-4$
	$c = 6$	$(58, 28)$	$(5.4938 \pm 0.0051)e-4$	$(100, 56)$	$(1.8586 \pm 0.0018)e-5$
	$c = 8$	$(43, 21)$	$(2.2568 \pm 0.0022)e-4$	$(80, 44)$	$(2.5706 \pm 0.0023)e-6$

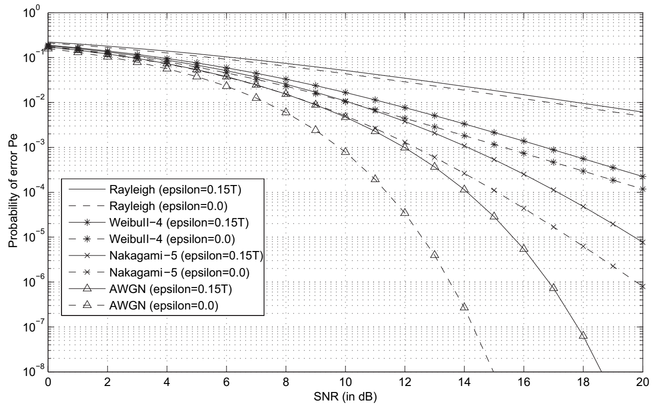

Fig. 1. Probability of error for different fading distributions for RC pulse ($\alpha = 0.35$).

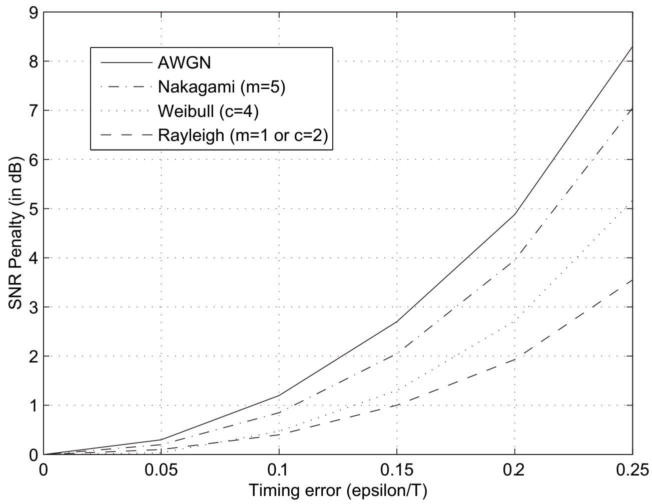

Fig. 2. SNR Penalty for different fading distributions for RC pulse ($\alpha = 0.35$).

where $u = rT_{eff}/\sqrt{2}$. Using 6.281 of [7]

$$\Delta_1 < \frac{1}{2\sqrt{\pi}\beta(T_{eff}/\sqrt{2})^c} \cdot \Gamma(\frac{c+1}{2}) \qquad (20)$$

where $\Gamma(\cdot)$ represents the Gamma function given by $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ [7]. The error bound Δ_2 can be calculated by substituting A_l from (18) in the second term on the R.H.S. of (7). The obtained integral can be evaluated by using the change of variable $l' = (l/L_f)^{2c}$ in the integral (20.5.2) of [10], and is given by

$$\Delta_2 = \frac{\sqrt{2}}{\pi\sqrt{c}(2\pi)^{c/2}} G^{c+3,0}_{1,c+3}\left(\frac{(\omega L_f)^{2c}}{\beta^2 2^{c+2} c^c} \middle| \begin{array}{c} 1 \\ 0,\, S \end{array} \right) \qquad (21)$$

V. NUMERICAL RESULTS

Error probabilities have been evaluated using a total of 2^9 interfering symbols. Without loss of generality, we assume $E(r^2) = 1$ which implies $\Omega = 1$ for the Nakagami-m case and $\beta^{2/c}\Gamma(1+2/c) = 1$ for the Weibull case. All error probabilities have been calculated with a relative accuracy better than 0.1%, i.e $\frac{\Delta_1+\Delta_2}{P_e} < 0.001$. As Δ_1 depends only on T_f, it can be made sufficiently small by taking a large enough T_f. The error bound Δ_2, depends on both T_f and L_f and hence after T_f has been fixed, L_f can be increased till the required accuracy is obtained.

In Table 1 and Figs. 1 and 2, Raised Cosine (RC) pulse with roll-off factor $\alpha = 0.35$ has been used. Table 1 lists the values of T_f and $N_L = (L_f + 1)/2$ required for evaluating the error probabilities within a relative accuracy of 0.1%, i.e $\frac{\Delta_1+\Delta_2}{P_e} < 0.001$, for a timing error $\epsilon = 0.15T$ for two different SNRs. It can be noted that for maintaining the same relative accuracy, the number of terms N_L, required in (10), decreases as the value of m increases for a given SNR. Further, as the value of SNR increases for a given m, the number of terms needed also increases. Similar observations can be made for c for the case of Weibull fading channel.

Fig. 1 shows the error probabilities for zero timing error and when the timing error $\epsilon = 0.15T$ for different fading channels. P_e for the AWGN case has been calculated using the P_e, Δ_1 and Δ_2 expressions of [2]. We have verified that the curves obtained for $m = 1$ (Nakagami-m) and $c = 2$ (Weibull) match since both correspond to the Rayleigh distribution [12]. It is clear that in the case of Nakagami-m distribution, as m increases from 1 to ∞, the impact of timing errors on P_e gets more pronounced. Its impact is maximum when the channel is AWGN ($m \to \infty$). Similarly, in the case of Weibull fading, as c increases from 2 (Rayleigh) to 4, the sensitivity of P_e to timing errors increases.

Let us define SNR penalty as the additional SNR required for maintaining a given error probability when timing errors

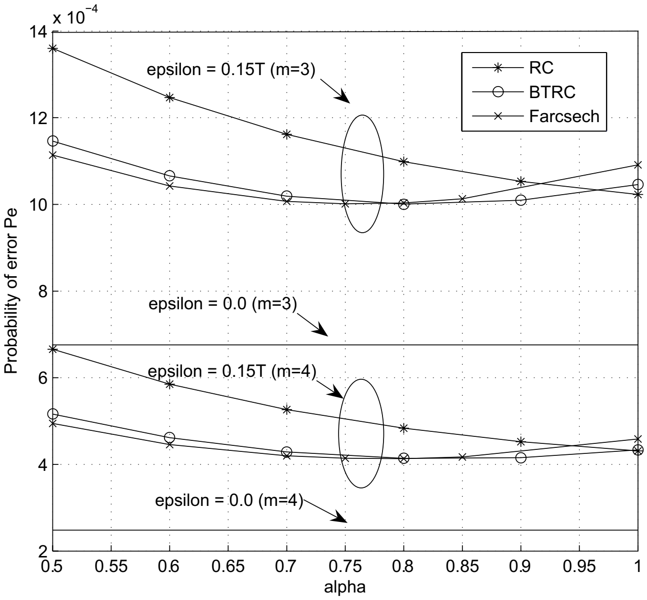

Fig. 3. Relative performance of pulses in a Nakagami-m fading channel.

are introduced. Fig. 2 shows the SNR penalty as a function of timing error when the error probability is fixed at 10^{-4}. It can be observed that the SNR penalty increases as m or c increases. Also, as expected, for a given value of m or c, SNR penalty increases as timing error increases.

Fig. 3 shows the performance of RC, Better Than Raised Cosine (BTRC) [4] and Farcsech [5] pulses in Nakagami-m fading channel at SNR=15 dB for a timing error $\epsilon = 0.15T$. Interestingly, none of the three pulses is consistently superior for either of the two values of m considered. At low values of α, Farcsech is the best pulse but as α tends to unity RC becomes better than the other two. In fact, which of these three pulses is the best depends on the values of α and m. For example, in a Nakagami-m fading channel with $m = 3$, SNR = 15dB and $\epsilon = 0.15T$, Farcsech performs the best for $0 \leq \alpha < 0.78$, BTRC is the best for $0.78 \leq \alpha \leq 0.96$ and RC is the best for $0.96 < \alpha \leq 1$.

VI. Conclusions

We have obtained expressions for the probability of error for bandlimited BPSK signalling in the presence of timing errors and fading. The derived results can be used to compute the error probabilities to any desired accuracy for Nakagami-m and Weibull fading channels. The results derived are also expected to help in the performance evaluation of systems in fading channels with ISI even if the ISI is not caused by a timing error. The effect of timing error on the performance of the Raised Cosine pulse has been evaluated for several fading parameters. It has also been observed that SNR penalty increases with m (Nakagami-m) or c (Weibull). Hence, in general, fading channels are less sensitive to timing errors than the AWGN channel. We also compare the performance of some useful Nyquist pulses known. The results obtained suggest that amongst RC, BTRC and Farcsech pulses, for a given SNR the best pulse depends on α, timing error and fading parameter.

Appendix A

In this appendix we show that for $\Omega = 1$ and by letting $m \to \infty$ in (10), it can indeed be verified that the obtained expression is identical to that obtained in [2] for an AWGN channel. We evaluate A_l as $m \to \infty$ using the asymptotic series expansion of $\mathrm{K}(\cdot,\cdot)$ given in (25) of 7.4.2 of [9]. Using the fact that $\sinh^{-1}(x) = \log_e(x + \sqrt{1+x^2})$ and the fact that $O((\sqrt{2ml\omega})^{-1})$ is very small compared to $\Gamma(1/2) = \sqrt{\pi}$ for large m, we get

$$A_l = \lim_{m\to\infty} \left[\frac{\sqrt{2\pi} m^{m-1/2}}{\Gamma(m)} \cdot \frac{e^{-m\sqrt{1+2l^2\omega^2/m}}}{(1+2l^2\omega^2/m)^{1/4}} \cdot \left(\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2l^2\omega^2}{m}} \right)^m \right] \quad (22)$$

Multiplying and dividing by e^m and using the fact that $\lim_{m\to\infty} \frac{m^{m-1/2}}{\Gamma(m)e^m} = 1/\sqrt{2\pi}$ from (1.182) of [8], (22) can be simplified to

$$A_l = \lim_{m\to\infty} \frac{e^m e^{-m\sqrt{1+2l^2\omega^2/m}}}{(1+2l^2\omega^2/m)^{1/4}} \left(\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2l^2\omega^2}{m}} \right)^m \quad (23)$$

Substituting $\sqrt{1+\frac{2l^2\omega^2}{m}} = p$ in (23), we get

$$A_l = \lim_{p\to 1} \frac{e^{-(\frac{2l^2\omega^2}{p+1})}}{\sqrt{p}} \cdot \left(\frac{1}{2} + \frac{p}{2} \right)^{\frac{2l^2\omega^2}{p^2-1}} \quad (24)$$

Breaking $1/(p^2-1)$ into partial fractions, we get

$$A_l = e^{-l^2\omega^2} \cdot \lim_{p\to 1} \left[\left(1+\frac{p-1}{2}\right)^{\frac{l^2\omega^2}{p-1}} \cdot \left(1+\frac{p-1}{2}\right)^{-\frac{l^2\omega^2}{p+1}} \right] \quad (25)$$

Using $\lim_{q\to 0}(1+q)^{a/q} = e^a$ in (25), A_l for AWGN channel can be expressed as

$$A_l = e^{-l^2\omega^2/2} \quad (26)$$

As expected, the expression obtained after substituting (26) in (5) agrees with the P_e expression obtained for the AWGN case in [2].

Appendix B

In this appendix we use the Mellin transform approach to express A_l in (17) in terms of Meijer's G function. We use the Mellin transform of e^{-ax^2} given in [12] to find the Mellin transform of A_l in (17). The Mellin transform of A_l with respect to l can be expressed as

$$M(s) = \frac{c}{2\beta} \left(\frac{\omega^2}{2} \right)^{-s/2} \Gamma(\frac{s}{2}) \int_0^\infty r^{s+c-1} e^{-\frac{r^c}{\beta}} dr \quad (27)$$

where s is a complex parameter. Expressing the integral in terms of $\Gamma(.)$, (27) can be written as

$$M(s) = \frac{\beta^{s/c}}{2} \left(\frac{\omega^2}{2} \right)^{-s/2} \Gamma\left(\frac{s}{2}\right) \Gamma\left(1+\frac{s}{c}\right) \quad (28)$$

Now we take the inverse Mellin transform and use change of variable, $s = 2cu$. Then applying Gauss Multiplication theorem as given in (1.2.11) of [8], on the $\Gamma(.)$ functions, we have

$$A_l = \frac{\sqrt{2c}}{(2\pi)^{c/2}} \cdot \frac{1}{2\pi j} \cdot \int_{\gamma - j\infty}^{\gamma + j\infty} \left(\frac{(\omega l)^{2c}}{\beta^2 2^{c+2} c^c} \right)^{-u} \cdot \prod_{k=0}^{c} \Gamma\left(u + \frac{1}{2}\right) \Gamma\left(u + \frac{k}{c}\right) du \quad (29)$$

From the definition of Meijer's G function given in (9.301) of [7], we get

$$A_l = \frac{\sqrt{2c}}{(2\pi)^{c/2}} G_{0,c+2}^{c+2,0}\left(\frac{(\omega l)^{2c}}{\beta^2 2^{c+2} c^c} \middle| \begin{array}{c} \{\} \\ S \end{array} \right) \quad (30)$$

where $G_{p,q}^{m,n}(\cdot)$ is Meijer's G function and $S = \{0, \frac{1}{c}, \frac{2}{c}, \cdots, 1, \frac{1}{2}\}$.

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