www.ietdl.org

Published in IET Communications
Received on 8th October 2009
Revised on 22nd September 2010
doi: 10.1049/iet-com.2009.0645

ISSN 1751-8628

Performance analysis of multiple input single output systems using transmit beamforming and antenna selection with delayed channel state information at the transmitter

Y.N. Trivedi A.K. Chaturvedi

Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur, India
E-mail: tyogesh@iitk.ac.in

Abstract: The authors consider a multiple input single output (MISO) system in which both transmit beamforming and antenna selection (AS) are implemented using delayed channel state information (CSI) at the transmitter (CSIT). The performance of the system has been analysed for three different AS schemes wherein two out of N antennas are selected at the transmitter. The authors have derived closed-form expressions for the probability density function of the received signal-to-noise, bit error rate and outage probability for each AS scheme considered. The expressions have been obtained as a function of the correlation between perfect CSI at the receiver (CSIR) and delayed CSIT. The authors also discuss some special cases and compare them with the results available in the literature.

1 Introduction

Multiple antennas can significantly improve the performance of wireless communication systems using transmit beamforming (TB) [1] or space time coding (STC) [2]. In particular, beamforming provides spatial diversity, whereas STC provides spatial diversity along with temporal diversity [3].

However, to realise the full potential of TB systems, we need channel state information at the transmitter (CSIT) and a radio frequency (RF) chain corresponding to each transmit antenna. There are two ways to obtain CSIT [4]. One is through channel reciprocity (in case of time division duplexing (TDD)). The other is through feedback (in case of frequency division duplexing (FDD)), in which the channel is estimated at the receiver and fed back to the transmitter via a dedicated (feedback) channel. To partially alleviate the requirement of multiple RF chains, without sacrificing the potential gains, antenna selection (AS) [5, 6] has been proposed in the literature. Using AS, the available antennas can be used with a reduced number of RF chains and still provide diversity [7]. Therefore performance of STC systems [8–12] and TB systems [13] have been analysed for different AS schemes.

In case of TB systems with transmit AS, the feedback information (i.e. CSIT) consists of the indices of the selected transmit antennas and the corresponding beamforming vector. However, even if we assume a perfectly estimated CSI at the receiver (CSIR) and a noiseless feedback link from the receiver to the transmitter, it is difficult to obtain perfect CSIT because of the time-varying nature of the wireless channel and non-zero delay in the feedback link. A couple of papers have dealt with systems using TB based on delayed CSIT [14–17]. However, the effect of delayed CSIT, when both TB and AS are used, has not been considered in the literature.

In this paper, we analyse the effect of delayed CSIT in a system with N transmit antennas and one receive antenna, which uses both TB and AS. This system model is suitable for downlink applications, where the base station (BS) can have multiple antennas but the mobile station can accommodate only one antenna. Assuming perfect CSIR, only two out of the N transmit antennas are selected by the receiver and the indices and beamforming vector for the same are fed back to the transmitter over a noiseless link with some finite delay. Thus TB and AS are effectively implemented with delayed CSIT. For this system, we consider three different AS schemes available in the literature. One of them is optimum [16] while the other two are sub optimum [11] with reduced complexity.

Our major contribution in this paper is the derivation of the closed-form expressions for the probability density function (pdf) of the received signal-to-noise ratio (SNR) for three AS schemes. Using this we obtain the expressions for bit error rate (BER) and outage probability for binary phase shift keying (BPSK) constellation. We also discuss some special cases for example, $N = 2$ (i.e. without antenna selection), perfect CSIT and no CSIT.

The rest of the paper is organised as follows. Section 2 describes the system model and in Section 3 we present a detailed performance analysis of all the schemes considered. In Section 4, we discuss some special cases while in Section 5 we present the results. The paper is concluded in Section 6.

Notations: Bold italics letters denote column vectors. The transpose, Hermitian, absolute value, norm and real part are denoted by $(.)^T$, $(.)^*$, $|.|$, $\|.\|$ and Re{.}, respectively. We use $\mathcal{Q}(.)$ and $\mathcal{J}_0(.)$ to denote the Gaussian Q-function and the zeroth order Bessel's function of the first kind, respectively.

2 System model

We consider a MISO system with N $(N \geq 2)$ transmit antennas and one receive antenna. At any time, only two of the N transmit antennas are selected for transmission and hence the system is denoted as $(N, 2; 1)$. The low pass equivalent received signal r is represented by

$$r = \boldsymbol{h}^* \boldsymbol{w} x + n \tag{1}$$

where $n \sim \mathcal{CN}(0, N_0)$, x is a transmitted BPSK symbol with average power E_s and channel $\boldsymbol{h} = [h_U \; h_V]^T$ in which U and V denote the indices of the two selected antennas. The channel coefficients corresponding to a transmit antenna are correlated in time and circularly symmetric, complex Gaussian random variables with zero mean and unit variance. However, the channel coefficients corresponding to two different transmit antennas are independent.

We assume that perfect CSI for all the N channels is available at the receiver (perfect CSIR). The receiver feeds back $\boldsymbol{h}$ and $\{U, V\}$ to the transmitter via a noiseless link with some delay. Let us denote the delayed CSIT as $\widehat{\boldsymbol{h}}$ and the correlation $E[\boldsymbol{h}\widehat{\boldsymbol{h}}^*] = \rho \boldsymbol{I}_2$, where $0 \leq \rho \leq 1$ and $\boldsymbol{I}_2$ is an identity matrix of order 2×2. In (1), $\boldsymbol{w}$ is the unit beamforming vector given by $\boldsymbol{w} = \widehat{\boldsymbol{h}}/\|\widehat{\boldsymbol{h}}\|$. The block diagram of the system in shown in Fig. 1.

We further assume that the perfect information of delay in the feedback link is also available at the receiver. Having the knowledge of delay and perfect CSI, the receiver can easily determine the transmit beamforming vector as has also been done in [14–17]. Therefore using the perfect knowledge of the the effective channel $\boldsymbol{h}^* \boldsymbol{w}$ at the receiver, the decision of the transmitted symbol is given by the sign of the decision variable

$$z = \text{Re}\{(\boldsymbol{h}^* \boldsymbol{w})^* r\} \tag{2}$$

Now we briefly describe the three transmit antenna selection schemes to be considered at the receiver.

Scheme 1 [18]: In this scheme, the two antennas with the highest and second highest channel power gains are selected, and their indices are fed back to the transmitter. Since the channels are time-varying and the feedback link is assumed to have a delay, at any given time the indices of the selected antennas are given by

$$\{U, V\} = \arg \max_{\substack{1 \leq u, v \leq N \\ u \neq v}} \{|\widehat{h}_u|^2 + |\widehat{h}_v|^2\} \tag{3}$$

Scheme 2: In this case, the N antennas are divided into two groups with N_1 and N_2 antennas such that $N_1 + N_2 = N$. From each group, the antenna with the highest channel power gain is selected. This is a generalisation of the scheme given in [11]. The indices of the selected antennas are determined by

$$\begin{aligned} U &= \arg \max_{1 \leq u \leq N_1} \{|\widehat{h}_u|^2\} \\ V &= \arg \max_{1 \leq v \leq N_2} \{|\widehat{h}_v|^2\} \end{aligned} \tag{4}$$

Scheme 3 [11]: Here the N antennas are divided into $N/2$ sets such that each set has two adjacent antennas. For convenience, we assume that N is an even integer. The set for which the summation of the channel power gains is highest, is selected. Let K denote the index of the selected set. Then

$$K = \arg \max_{1 \leq k \leq N/2} \{|\widehat{h}_{2k-1}|^2 + |\widehat{h}_{2k}|^2\}$$

and the indices of the two selected antennas are

$$U = 2K - 1, \quad V = 2K$$

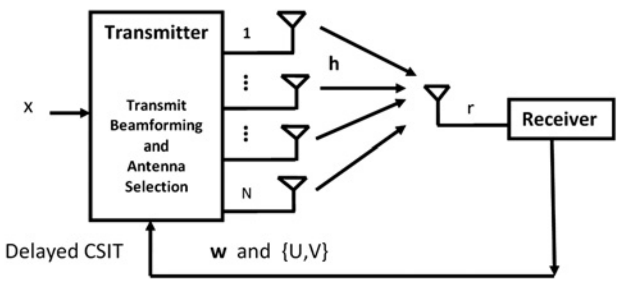

Fig. 1 *Block diagram*

3 Performance analysis

Expanding (2), by substituting the value of r from (1), we obtain

$$z = \text{Re}\{|\boldsymbol{h}^* \boldsymbol{w}|^2 x + (\boldsymbol{h}^* \boldsymbol{w})^* n\} \tag{5}$$

The instantaneous (with respect to fading) SNR γ can be represented as

$$\gamma = |\boldsymbol{h}^* \boldsymbol{w}|^2 \frac{E_s}{N_0} \tag{6}$$

Now it is difficult to derive the pdf of γ in some simple form. Therefore we use an indirect approach in which we represent γ as a function of $|\widehat{\boldsymbol{h}}|^2$. To do this, we introduce correlation (ρ) between $\boldsymbol{h}$ and $\widehat{\boldsymbol{h}}$, and represent $\boldsymbol{h}$ as a function of $\widehat{\boldsymbol{h}}$ and an error term by using a Gauss–Markov process model as done in [19]

$$\boldsymbol{h} = \rho \widehat{\boldsymbol{h}} + \sqrt{1 - \rho^2} \boldsymbol{\delta} \tag{7}$$

where $\boldsymbol{\delta} = [\delta_1 \; \delta_2]^T$. Since $E[\boldsymbol{h}\widehat{\boldsymbol{h}}^*] = \rho \boldsymbol{I}_2$ and $\boldsymbol{h} \sim \mathcal{CN}(0, \boldsymbol{I}_2)$, it easily follows that $\boldsymbol{\delta} \sim \mathcal{CN}(0, \boldsymbol{I}_2)$. Furthermore $\boldsymbol{\delta}$ is also independent of $\widehat{\boldsymbol{h}}$. It can be seen that $\rho = 0$ represents no CSIT whereas $\rho = 1$ corresponds to perfect CSIT. Now substituting the value of $\boldsymbol{h}$ from (7) in (6), we obtain

$$\gamma = \left| \rho \|\widehat{\boldsymbol{h}}\| + \sqrt{1 - \rho^2} \boldsymbol{\delta}^* \boldsymbol{w} \right|^2 \frac{E_s}{N_0} \tag{8}$$

Let us denote the inner product $\boldsymbol{\delta}^* \boldsymbol{w}$ as $\eta_1 + j\eta_2$. It can be easily shown that the distribution of $\eta_1 + j\eta_2$ reduces to

IET Commun., 2011, Vol. 5, Iss. 6, pp. 827–834
doi: 10.1049/iet-com.2009.0645

$\mathcal{CN}(0, 1)$ and is independent of $\hat{\boldsymbol{h}}$. Using this, we can write (8) as

$$\gamma = (A + X_1)^2 + X_2^2 \tag{9}$$

where

$$A = \sqrt{\rho^2 \|\hat{\boldsymbol{h}}\|^2 \frac{E_s}{N_0}}$$
$$X_1 = \sqrt{(1-\rho^2)\frac{E_s}{N_0}}\eta_1 \tag{10}$$
$$X_2 = \sqrt{(1-\rho^2)\frac{E_s}{N_0}}\eta_2$$

Here X_1 and X_2 are independent Gaussian variables with mean zero and identical variance $(1-\rho^2)E_s/2N_0$. As $\eta_1 + j\eta_2$ is independent of $\hat{\boldsymbol{h}}$, X_1 and X_2 are also independent of A. Therefore for a given A, γ has non-central chi-squared distribution with two degrees of freedom with the pdf [20]

$$p_{\gamma/A}(\gamma/a) = \frac{\mathrm{e}^{-(a^2+\gamma/((1-\rho^2)E_s/N_0))}}{(1-\rho^2)E_s/N_0}\mathcal{J}_0\left(\frac{2\mathrm{i}\sqrt{\gamma}a}{(1-\rho^2)E_s/N_0}\right) \tag{11}$$

where $\mathrm{i} = \sqrt{-1}$.

Now we require the pdf of A, which can be derived from the pdf of $\|\hat{\boldsymbol{h}}\|^2$ and (10). The pdf of $\|\hat{\boldsymbol{h}}\|^2$ will be different for each antenna selection scheme and we have derived them in Appendix 1.

After getting the pdf $p_A(a)$, $p_\gamma(\gamma)$ can be determined by

$$p_\gamma(\gamma) = \int_0^\infty p_{\gamma/A}(\gamma/a)p_A(a)\mathrm{d}a \tag{12}$$

Now BER P_e can be derived by averaging $\mathcal{Q}(\sqrt{2\gamma})$ over the distribution $p_\gamma(\gamma)$ as [20]

$$P_e = \int_0^\infty \mathcal{Q}(\sqrt{2\gamma})p_\gamma(\gamma)\mathrm{d}\gamma \tag{13}$$

where $Q(x)$ represents the area under the tail of the Gaussian pdf. This derivation can be easily extended to the symbol error rate (SER) for M-ary modulation schemes like M-PSK or M-QAM [21].

The probability of outage, P_{out}, is defined as

$$P_{\text{out}} = P(\gamma < \gamma_{\text{th}}) = \int_0^{\gamma_{\text{th}}} p_\gamma(\gamma)\mathrm{d}\gamma \tag{14}$$

where γ_{th} is the threshold (minimum required SNR below which the system is in outage).

3.1 Scheme 1

We have derived $p_\gamma(\gamma)$ and using which we have obtained P_e and P_{out} in Section 8.1 of Appendix 1, which can be represented as follows

$$P_e = N(N-1)\left[\sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k}\left\{\frac{1-\sigma_0}{2} - \frac{1-\sigma_k}{k+2}\right\} + \frac{1-\sigma_0}{4} - \frac{\rho^2\sigma_0^3}{8E_s/N_0}\right] \tag{15}$$

where

$$\sigma_k^2 = \frac{(E_s/N_0)(k+2-k\rho^2)}{k+2+(E_s/N_0)(k+2-k\rho^2)}, \quad k = 0, 1, \ldots$$

$$P_{\text{out}} = \frac{N(N-1)}{2}\left[1 - \mathrm{e}^{-\gamma_{\text{th}}/(E_s/N_0)}\left\{1 + \rho^2\frac{\gamma_{\text{th}}}{E_s/N_0}\right\} + 2\sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k}\left\{\frac{k}{k+2} - \mathrm{e}^{-\gamma_{\text{th}}/(E_s/N_0)} + \frac{2}{k+2}\mathrm{e}^{-(\gamma_{\text{th}}/(E_s/N_0))((k+2)/(k+2-k\rho^2))}\right\}\right] \tag{16}$$

3.2 Scheme 2

We have derived $p_\gamma(\gamma)$ and using which we have obtained P_e and P_{out} in Section 8.2 of Appendix 1, which can be represented as follows

$$P_e = \frac{N_1N_2}{2}\sum_{p=0}^{P-1}\frac{\binom{N_1-1}{p}\binom{N_2-1}{p}}{1+p}\left\{\frac{1-\sigma_p}{1+p} - \frac{\sigma_p\rho^2}{2(1+p-p\rho^2)(1+p+E_s/N_0(1+p-p\rho^2))}\right\} + \frac{N_1N_2}{2}\sum_{n=0}^{N_1-1}\sum_{\substack{m=0\\m\neq n}}^{N_2-1}\binom{N_1-1}{n}\binom{N_2-1}{m}\frac{(-1)^{m+n}}{n-m} \times\left\{\frac{1-\sigma_m}{1+m} - \frac{1-\sigma_n}{1+n}\right\} \tag{17}$$

where

$$\sigma_i^2 = \frac{E_s/N_0(1+i-i\rho^2)}{1+i+E_s/N_0(1+i-i\rho^2)}, \quad i = 0, 1, \ldots$$

$$P_{\text{out}} = N_1N_2\sum_{p=0}^{P-1}\frac{\binom{N_1-1}{p}\binom{N_2-1}{p}}{(1+p-p\rho^2)(1+p)} \times\left[(1-\rho^2)\left\{1-\mathrm{e}^{-\frac{\gamma_{\text{th}}}{\mu_p}}\right\} + \frac{\rho^2}{1+p} \times\left\{1 - \frac{\gamma_{\text{th}}+\mu_p}{\mu_p}\mathrm{e}^{-\gamma_{\text{th}}/\mu_p}\right\}\right] + N_1N_2\sum_{n=0}^{N_1-1}\sum_{\substack{m=0\\m\neq n}}^{N_2-1}\frac{\binom{N_1-1}{n}\binom{N_2-1}{m}(-1)^{m+n}}{n-m} \times\left\{\frac{1-\mathrm{e}^{-\gamma_{\text{th}}/\mu_m}}{1+m} - \frac{1-\mathrm{e}^{-\gamma_{\text{th}}/\mu_n}}{1+n}\right\} \tag{18}$$

where

$$\mu_i = \frac{E_s/N_0(1+i-i\rho^2)}{1+i}, \quad i = 0, 1, \ldots$$

3.3 Scheme 3

We have derived $p_\gamma(\gamma)$ and using which we have obtained P_e and P_{out} in Section 8.3 of Appendix 1, which can be represented as follows

$$P_e = \frac{N}{4}\sum_{m=0}^{N/2-1}\sum_{k=0}^{m}\sum_{n=0}^{k+1} \times \frac{\binom{N/2-1}{m}\binom{m}{k}((k+1)!)^2(\rho^2)^{k-n+1}(-1)^m(1-\rho^2)^n}{n!(k-n+1)!(1+m-m\rho^2)^{k+1}(1+m)^{k-n+2}} \times \left[1-\sigma_m-\sum_{t=1}^{k-n+1}\{1.3..(2t-1)\} \times \left\{\frac{1+m}{2(1+m-m\rho^2)E_s/N_0}\right\}^t \frac{\sigma_m^{2t+1}}{t!}\right] \quad (19)$$

where

$$\sigma_m^2 = \frac{(E_s/N_0)(1+m-m\rho^2)}{1+m+(E_s/N_0)(1+m-m\rho^2)}, \quad m=0,1,\ldots$$

(see (20))

$$P_{\text{out}} = \frac{\frac{N}{2}\sum_{m=0}^{N/2-1}\sum_{k=0}^{m}\sum_{n=0}^{k+1}\binom{N/2-1}{m}\binom{m}{k}(-1)^m((k+1)!)^2(\rho^2)^{k-n+1}(1-\rho^2)^n}{n!(k-n+1)!(1+m)^{k-n+2}(1+m-m\rho^2)^{k+1}} \times \left[1-\mathrm{e}^{-(\gamma_{\text{th}}/(E_s/N_0))((1+m)/(1+m-m\rho^2))}\sum_{t=0}^{k-n+1}\frac{1}{t!}\left\{\frac{\gamma_{\text{th}}}{E_s/N_0}\frac{1+m}{1+m-m\rho^2}\right\}^t\right] \quad (20)$$

4 Special cases

In this section, we reduce the expressions for the BER and the outage probability for some special cases like $N=2$, perfect CSIT and no CSIT and compare them with the existing cases in the literature. In all cases μ denotes $\sqrt{E_s/(E_s+N_0)}$.

4.1 N = 2

Since $N=2$, both the antennas will always be selected and hence there will be no antenna selection problem here. In this case, P_{out} for all the schemes reduces to

$$P_{\text{out}} = (1-\rho^2)[1-\mathrm{e}^{-\gamma_{\text{th}}N_0/E_s}] + \rho^2\left[1-\left\{1+\frac{\gamma_{\text{th}}N_0}{E_s}\right\}\mathrm{e}^{-\gamma_{\text{th}}N_0/E_s}\right] \quad (21)$$

and P_e for all the schemes reduces to

$$P_e = (1-\rho^2)\left[\frac{1}{2}(1-\mu)\right] + \rho^2\left[\frac{1}{4}(2-3\mu+\mu^3)\right] \quad (22)$$

In both these equations, the first term in square brackets corresponds to a single input single output (SISO) system and the second term in square brackets corresponds to an MRC [20] system with one transmit and two receive antennas. Hence the outage as well as the BER performance is the weighted average of the performances of the SISO and the MRC systems. Recently (12) in [17] has drawn the same conclusion for outage probability.

It may also be pointed out that by plugging the moment generating function from (11) of [14] in (5.3) of [21], we can easily obtain the BER expression for the case of N_t transmit antennas as shown in Appendix 3. As expected, for $N_t = 2$, the obtained BER expression reduces to (22).

4.2 Perfect CSIT ($\rho = 1$)

In this case, we would like to compare the performance of our Scheme 1 with (1; N, 2) hybrid selection/maximum ratio combining (HS/MRC) [18] in which there is one transmit antenna and the output of the two antennas with the largest SNRs are combined at the receiver. We know that the performance of MRC (with perfect CSIR) is identical to beamforming (with perfect CSIT). For a (1; N, 2) HS/MRC system, (9) of [18] expresses the BER and, as expected, it exactly matches with the BER expression of our Scheme 1 as given in (15), when $\rho = 1$.

4.3 No CSIT ($\rho = 0$)

In this case, using (37) from Appendix 2, BER for Scheme 1 reduces to

$$P_e = \frac{1-\mu}{2} \quad (23)$$

which represents the BER for a SISO system [20]. Thus we can say that both diversity gain as well as antenna selection gain cannot be achieved when no CSIT is available. This can be proved for the remaining two schemes also.

5 Results

For the purpose of simulations, we generated the channel using the recently proposed simulation model [22] for Jakes correlation. In all the cases, the simulation results were found to be closely matching with their analytical counter parts. In the figures below, we present the results obtained by evaluating the analytical expressions. In all the figures average SNR is E_s/N_0.

Fig. 2 shows the BER against SNR curves for different values of ρ for a (3, 2; 1) system using Scheme 1. It shows the degradation in BER when ρ decreases from 1 to 0. As we have seen in the special cases, the performance for $\rho = 0$ is same as the performance of a SISO system, where as for $\rho = 1$, full antenna selection gain is obtained. For the intermediate values of ρ, the loss in the performance is because of the combined effect of not implementing perfect CSIT based beamforming and not selecting the best antennas at all times.

doi: 10.1049/iet-com.2009.0645

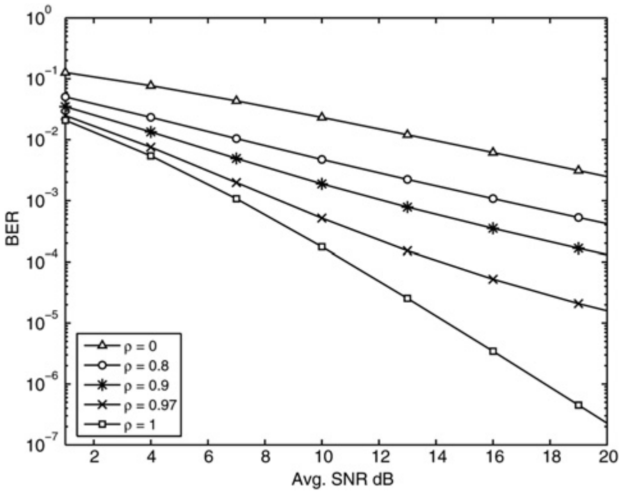

Fig. 2 *BER against average SNR for (3, 2; 1) system for scheme 1*

Figs. 3 and 4 show the BER and the outage probability ($\gamma_{th} = 4$ dB) respectively, for all the three schemes for (4, 2; 1) and (6, 2; 1) systems at $\rho = 0.9$. As we increase the number of antennas from 4 to 6, there is an enhancement in the performance because of increased diversity gain.

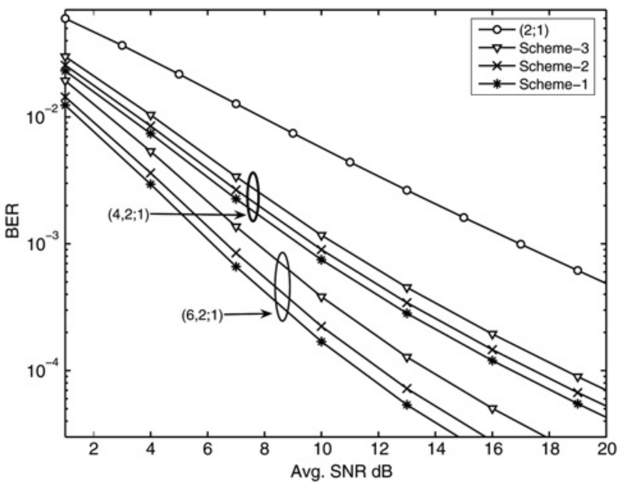

Fig. 3 *BER against average SNR for $\rho = 0.9$*

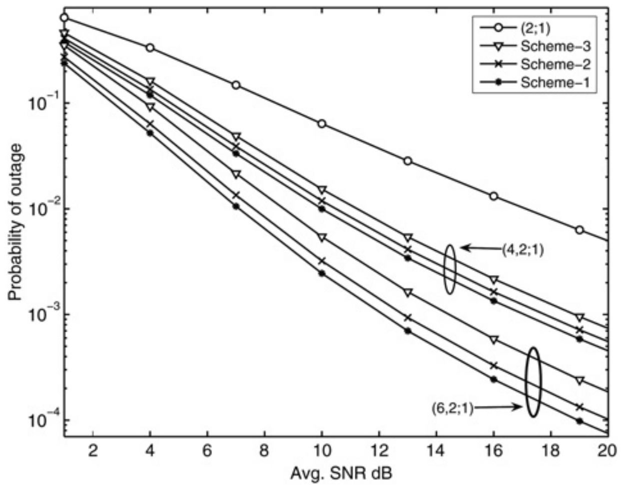

Fig. 4 *Probability of outage against average SNR, $\rho = 0.9$ and $\gamma_{th} = 4$ dB*

Comparing the performance of Schemes 2 and 3, Scheme 1 has a better performance because of better selection of antennas. Further, this difference in the performance increases when N increases from 4 to 6. The performance of a scheme depends on the number of feedback bits allocated for antenna selection. Like for $N = 8$, schemes 1, 2 and 3 require 6, 4 and 2 feedback bits, respectively, to represent the indices of the two selected antennas. In both the figures, we have also presented the performance of the (2; 1) system using (21) and (22). The improvement in the performance of the three schemes over the (2; 1) system can be seen even when antenna selection is done with delayed CSIT.

6 Conclusion

We have considered an MISO system with N transmit antennas and one receive antenna. The system implements transmit beamforming after selecting only two of the N transmit antennas. Both the transmit beamforming vector as well as the choice of the two transmit antennas are done on the basis of delayed CSIT. Three different AS schemes, wherein one is optimum and the remaining two are sub-optimum with less complexity, have been considered. Exact closed-form expressions for the pdf of the received SNR, BER and outage probability have been obtained as a function of the correlation (ρ) between perfect CSIR and delayed CSIT. The performance of the system is found to depend critically on this ρ. Further, some special cases have been compared with the results available in the literature.

7 References

1 Lo, T.K.Y.: 'Maximum ratio transmission', *IEEE Trans. Commun.*, 1999, **47**, (10), pp. 1458–1461

2 Tarokh, V., Seshadri, N., Calderbank, A.R.: 'Space time codes for high data rate wireless communications: performance criterion and code construction', *IEEE Trans. Inf. Theory*, 1998, **44**, pp. 744–765

3 Arora, D., Agathoklis, P.: 'Performance evaluation of uplink transmission using space time block coding and beamforming'. IEEE Int. Symp. on SPIT 2004, December 2004, pp. 405–408

4 Paulraj, A., Nabar, R., Gore, D.: 'Introduction to space-time wireless communications' (Cambidge University Press, 2003)

5 Molisch, A.F., Win, M.Z.: 'MIMO systems with antenna selection', *IEEE Microw. Mag.*, 2004, **5**, (1), pp. 46–56

6 Ghrayeb, A.: 'A survey on antenna selection for MIMO communication systems'. Information and Communication Technologies, ICTTA'06, 2006, vol. 2, pp. 2104–2109

7 Gore, D.A., Paulraj, A.: 'MIMO antenna subset selection with space-time coding', *IEEE Trans. Signal Process.*, 2002, **50**, (10), pp. 2580–2588

8 Chen, Z., Yuan, J., Vucetic, B., Zhou, Z.: 'Performance of Alamouti scheme with transmit antenna selection', *Electron. Lett.*, 2003, **39**, (23), pp. 1666–1668

9 Chen, Z., Vucetic, B., Yuan, J.: 'Space-time trellis codes with transmit antenna selection', *Electron. Lett.*, 2003, **39**, (11), pp. 854–855

10 Badic, B., Fuxjaeger, P., Weinrichter, H.: 'Performance of quasi-orthogonal space-time code with antenna selection', *Electron. Lett.*, 2004, **40**, (20), pp. 1282–1284

11 Chen, Z., Collings, I.B., Zhuo, Z., Vucetic, B.: 'Transmit antenna selection schemes with reduced feedback rate', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (2), pp. 1006–1016

12 Fan, B., Sun, W., Shen, D.: 'Performance analysis of orthogonal space-time block coding with reduced-complexity antenna selection'. IET Int. Conf. on Wireless, Mobile and Multimedia Networks, November 2006, pp. 1–4

13 You, C., Hwang, I., Kim, Y., Tarokh, V.: 'Dual antenna selection algorithms and feedback strategies with reduced complexity for multiple-input multiple-output systems', *IET Microw. Antennas Propag.*, 2009, **3**, (6), pp. 906–916

14 Ma, Y., Zhang, D., Leith, A., Wang, Z.: 'Error performance of transmit beamforming with delayed and limited feedback', *IEEE Trans. Wirel. Comm.*, 2009, **8**, (3), pp. 1164–1170

15 Onggosansui, E.N., Gatherer, A., Dabak, A., Hosur, S.: 'Performance analysis of closed loop transmit diversity in the presence of feedback delay', *IEEE Trans. Commun.*, 2001, **49**, (9), pp. 1618–1630
16 Zhou, S., Giannakis, G.B.: 'Optimal transmitter eigen beam forming and space time block coding based on channel mean feedback', *IEEE Trans. Signal Process.*, 2002, **50**, (10), pp. 2599–2613
17 Annapureddy, V.S., Marathe, D.V., Ramya, T.R., Bhashyam, S.: 'Outage probability of multiple-input single-output (MISO) systems with delayed feedback', *IEEE Trans. Commun.*, 2009, **57**, (2), pp. 319–326
18 Eng, T., Kong, N., Milstein, L.B.: 'Comparision of diversity combining techniques for Rayleigh-fading channels', *IEEE Trans. Commun.*, 1996, **44**, (9), pp. 1117–1129
19 Chen, C., Sezgin, A., Cioffi, J.M., Paulraj, A.: 'Antenna selection in space-time block coded systems: performance analysis and low-complexity algorithm', *IEEE Trans. Signal Process.*, 2008, **56**, (7), pp. 3303–3314
20 Proakis, J.G.: 'Digital communication' (MH, 2001, 4th edn.)
21 Simon, M.K., Alouini, M.S.: 'Digital communication over fading channels – a unified approach to performance analysis' (John Wiley and Sons, 2000)
22 Zheng, Y.R., Xiao, C.: 'Simulation models with correct statistical properties for rayleigh fading channels', *IEEE Trans. Commun.*, 2003, **51**, (6), pp. 920–928
23 Papoulis, A., Pillai, S.U.: 'Probability, random variables and stochastic processes' (MH, 4th edn.)
24 Gradshteyn, I.S., Ryzhik, I.M.: 'Table of integrals, series and products' (Academic Press, 2000, 6th edn.)
25 David, H.A.: 'Order statistics' (Wiley, New York, 1970)

8 Appendix 1

The pdf of $\|\widehat{\boldsymbol{h}}\|^2$ will be different for each of the schemes considered. However, we will denote $\|\widehat{\boldsymbol{h}}\|^2$ by Z for all the three schemes. In Section 8.1 below, we derive $p_Z(z)$, $p_A(a)$, $p_\gamma(\gamma)$, P_e and P_{out} for Scheme 1. This exercise is repeated for Schemes 2 and 3 in Sections 8.2 and 8.3 respectively.

8.1 Scheme 1

In this case, $p_Z(z)$ is expressed by (6) from [18] as

$$p_Z(z) = N(N-1)\mathrm{e}^{-z}\left[\frac{z}{2} + \sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k} \times (1-\mathrm{e}^{-kz/2})\right], \quad z \geq 0 \tag{24}$$

Now using (10), $p_A(a)$ can be represented [23] as

$$p_A(a) = \frac{2N(N-1)}{E_s/N_0\rho^2}\mathrm{e}^{-(a^2/(E_s/N_0\rho^2))}\left[\frac{a^3}{2(E_s/N_0)\rho^2} + a\sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k}\{1-\mathrm{e}^{-ka^2/(2E_s/N_0\rho^2)}\}\right], \quad a \geq 0 \tag{25}$$

Plugging (11) and (25) in (12), we obtain

$$p_\gamma(\gamma) = \frac{2N(N-1)}{(1-\rho^2)\rho^2(E_s/N_0)^2}\mathrm{e}^{-(\gamma/(1-\rho^2)(E_s/N_0))} \times \int_{a=0}^{\infty}\mathrm{e}^{-(a^2/\rho^2(1-\rho^2)(E_s/N_0))}\left[\frac{a^3}{2\rho^2(E_s/N_0)} + a\sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k} \times (1-\mathrm{e}^{-(ka^2/2\rho^2E_s/N_0)})\mathcal{J}_0\left(\frac{2\mathrm{i}\sqrt{\gamma}a}{(1-\rho^2)E_s/N_0}\right)\right]\mathrm{d}a \tag{26}$$

Simplifying (26) by using (6.631.1), (9.212.2) and (9.212.5) from [24], we obtain

$$p_\gamma(\gamma) = \frac{N(N-1)}{2E_s/N_0}\left[2\sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k} \times\left\{\mathrm{e}^{-\gamma/(E_s/N_0)} - \frac{2}{2+k-k\rho^2}\mathrm{e}^{-(\gamma/(E_s/N_0))(k+2)/(k+2-k\rho^2)}\right\} + \left(1-\rho^2+\frac{\rho^2\gamma}{E_s/N_0}\right)\mathrm{e}^{-\gamma/(E_s/N_0)}\right], \quad \gamma \geq 0 \tag{27}$$

Plugging (27) in (13), we obtain P_e as shown in (15). Similarly plugging (27) in (14), we obtain P_{out} as shown in (16).

8.2 Scheme 2

Let us denote the maximum channel power gains for groups 1 and 2 by X and Y, respectively. The pdf of X can be expressed [25] as

$$p_X(t) = N_1\sum_{n=0}^{N_1-1}\binom{N_1-1}{n}(-1)^n\mathrm{e}^{-(n+1)t}, \quad t \geq 0 \tag{28}$$

The pdf of Y is same as (28) with N_1 replaced by N_2. The pdf of $Z = X + Y$ can be determined as [23]

$$\begin{aligned} p_Z(z) &= \int_0^z p_X(t)p_Y(z-t)\,\mathrm{d}t \\ &= N_1N_2\left[\sum_{p=0}^{P-1}\binom{N_1-1}{p}\binom{N_2-1}{p}z\mathrm{e}^{-(p+1)z} + \sum_{n=0}^{N_1-1}\sum_{\substack{m=0\\m\neq n}}^{N_2-1}\binom{N_1-1}{n}\binom{N_2-1}{m}(-1)^{m+n} \times\left\{\frac{\mathrm{e}^{-(m+1)z}-\mathrm{e}^{-(n+1)z}}{n-m}\right\}\right], \quad z \geq 0 \end{aligned} \tag{29}$$

where $P = \min\{N_1, N_2\}$. Now using (10), $p_A(a)$ can be represented [23] as

$$p_A(a) = \frac{2N_1N_2a^3}{\rho^4(E_s/N_0)^2}\sum_{p=0}^{P-1}\binom{N_1-1}{p}\binom{N_2-1}{p}\mathrm{e}^{-((p+1)a^2/\rho^2(E_s/N_0))} + \frac{2N_1N_2a}{\rho^2(E_s/N_0)}\sum_{n=0}^{N_1-1}\sum_{\substack{m=0\\m\neq n}}^{N_2-1}\binom{N_1-1}{n}\binom{N_2-1}{m}(-1)^{m+n} \times\left\{\frac{\mathrm{e}^{-(m+1)(a^2/\rho^2(E_s/N_0))}-\mathrm{e}^{-(n+1)(a^2/\rho^2(E_s/N_0))}}{n-m}\right\}, \quad a \geq 0 \tag{30}$$

Plugging (11) and (30) in (12) and simplifying it using

doi: 10.1049/iet-com.2009.0645

(6.631.1) and (6.631.4) from [24], we obtian

$$p_\gamma(\gamma) = \frac{N_1N_2}{E_s/N_0}\sum_{p=0}^{P-1}\frac{\binom{N_1-1}{p}\binom{N_2-1}{p}}{(1+p-p\rho^2)^2} \times\left\{1-\rho^2+\frac{\gamma\rho^2}{(1+p-p\rho^2)E_s/N_0}\right\}e^{-\gamma(1+p)/(E_s/N_0(1+p-p\rho^2))} + \frac{N_1N_2}{E_s/N_0}\sum_{n=0}^{N_1-1}\sum_{\substack{m=0\\m\neq n}}^{N_2-1}\frac{\binom{N_1-1}{n}\binom{N_2-1}{m}(-1)^{m+n}}{n-m} \times\left\{\frac{e^{-\gamma(1+m)/(E_s/N_0)(1+m-m\rho^2)}}{1+m-m\rho^2} - \frac{e^{-\gamma(1+n)/(E_s/N_0)(1+n-n\rho^2)}}{1+n-n\rho^2}\right\},\ \gamma\geq 0 \tag{31}$$

Plugging (31) in (13), we obtain P_e as shown in (17). Similarly plugging (31) in (14), we obtain P_{out} as shown in (18).

8.3 Scheme 3

The channel power gain (X) corresponding to any one transmitter–receiver antenna pair is chi-squared distributed with four degrees of freedom. The pdf and CDF of X can be expressed [20] as $p_X(x) = xe^{-x}$ and $F_X(x) = 1 - e^{-x}(1+x)$, respectively, where $x \geq 0$. For this scheme, the pdf of Z can be expressed as [25]

$$p_Z(z) = \frac{N}{2}[F_X(z)]^{(N/2)-1}p_X(z) = \frac{N}{2}[1-e^{-z}(1+z)]^{(N/2)-1}ze^{-z}, \quad z\geq 0 \tag{32}$$

Now using (10), $p_A(a)$ can be represented [23] as

$$p_A(a) = N\sum_{m=0}^{N/2-1}\sum_{k=0}^{m}\binom{N/2-1}{m}\binom{m}{k}(-1)^m\frac{a^{2k+3}}{(\rho^2E_s/N_0)^{k+2}} \times e^{-(a^2/(\rho^2E_s/N_0))(1+m)}, \quad a\geq 0 \tag{33}$$

Plugging (11) and (33) in (12) and using (6.631.1) from [24], we simplify (12) as

$$p_\gamma(\gamma) = \frac{N}{2(1-\rho^2)E_s/N_0}\sum_{m=0}^{N/2-1}\sum_{k=0}^{m}\binom{N/2-1}{m}\binom{m}{k} \times\frac{(-1)^m\Gamma(k+2)(1-\rho^2)^{k+2}}{(1+m-m\rho^2)^{k+2}}e^{-\gamma/((1-\rho^2)E_s/N_0)} \times\boldsymbol{\Phi}\left(k+2,1;\frac{\gamma\rho^2}{(E_s/N_0)(1-\rho^2)(1+m-m\rho^2)}\right), \quad \gamma\geq 0 \tag{34}$$

where $\boldsymbol{\Phi}(.)$ is a degenerate hypergeometric function [24]. Now using (9.215.1) and (9.212.4) from [24], we can represent $\boldsymbol{\Phi}(L, 1; z)$, where L is an integer, as

$$\boldsymbol{\Phi}(L,1;z) = \sum_{k=0}^{L-1}\frac{\binom{L-1}{k}}{(L-k-1)!}z^{L-k-1}e^z \tag{35}$$

Using (35) into (34), we obtain

$$p_\gamma(\gamma) = \frac{N}{2}\sum_{m=0}^{N/2-1}\sum_{k=0}^{m}\sum_{n=0}^{k+1}\frac{\binom{N/2-1}{m}\binom{m}{k}(-1)^m\gamma^{k-n+1}}{n!(E_s/N_0)^{k-n+2}} \times\left\{\frac{(k+1)!}{(k-n+1)!}\right\}^2\frac{(\rho^2)^{k+1-n}(1-\rho^2)^n}{(1+m-m\rho^2)^{2k-n+3}} \times e^{-((1+m)/(1+m-m\rho^2))(\gamma/(E_s/N_0))}, \quad \gamma\geq 0 \tag{36}$$

Plugging (36) in (13) and simplifying it using (3.351.1) from [24] and (2.1.100) from [20], we get P_e as shown in (19). Similarly plugging (36) in (14) and using (3.351.1) from [24], we get P_{out} as shown in (20).

9 Appendix 2

For $N \geq 2$, we show that

$$\frac{N(N-1)}{2}\left[1+2\sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k+2}\right] = 1 \tag{37}$$

Proof: Let us consider

$$X = -\int_{x=-1}^{0}(x(1+x)^{N-2}-x)\,dx = \frac{1}{N(N-1)} - \frac{1}{2} \tag{38}$$

Now expanding the series $(1+x)^{N-2}$ in X, we obtain

$$X = -\int_{x=-1}^{0}\left\{x\sum_{k=0}^{N-2}\binom{N-2}{k}x^k - x\right\}dx = -\int_{x=-1}^{0}\left\{\sum_{k=0}^{N-2}\binom{N-2}{k}x^{k+1} - x\right\}dx = -\int_{x=-1}^{0}\sum_{k=1}^{N-2}\binom{N-2}{k}x^{k+1}dx = \sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^{k+2}}{k+2} = \sum_{k=1}^{N-2}\binom{N-2}{k}\frac{(-1)^k}{k+2} \tag{39}$$

Equating (38) and (39), we obtain the desired result. □

10 Appendix 3: P_e for $N_t \times 1$ system

$$P_e = \frac{1}{2}\sum_{m=0}^{N_t-1}\binom{N_t-1}{m}(\rho^2)^{N_t-m-1}(1-\rho^2)^m \times \left[1-\mu-\sum_{k=1}^{N_t-m-1}\frac{1\times 3\times\cdots\times(2k-1)\mu^{2k+1}}{k!2^k\gamma_c^k}\right] \quad (40)$$

IET Commun., 2011, Vol. 5, Iss. 6, pp. 827–834
doi: 10.1049/iet-com.2009.0645