A New Adaptive Serial Search PN Code Acquisition Scheme for DS-CDMA Systems

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***Abstract*— In this work we have derived the pdf of the decision variable for an adaptive serial search PN code acquisition scheme in Nakagami-m fading environment which is better suited for modeling urban multipath mobile radio communication channel. The detection and false alarm probabilities have also been derived. These can be used for the computation of mean and variance of acquisition time.**

I. INTRODUCTION

EXTENSIVE work has been carried out in the past for pseudo-noise (PN) code acquisition for unfaded and Rayleigh faded received signal, in additive white gaussian noise (AWGN) [1], [2]. In [3] acquisition of unfaded and in [4] Rayleigh faded received signal in AWGN has been considered using different adaptive threshold methods.

In this work, we have investigated the probability density function (pdf) of the decision variable when the received signal for code acquisition in a direct sequence spread spectrum (DS-SS) system is subjected to Nakagami-m fading. The Nakagami-m model for fading channels was investigated in [6] where it was pointed out that more accurate statistical description of urban radio multipath fading channels is provided by Nakagami-m distribution when compared with Rayleigh distribution. The detection and false alarm probability expressions for code acquisition have also been derived using the adaptive threshold detection method. For adapting the threshold according to channel conditions cell averaging constant false alarm rate (CA-CFAR) technique of [4], [5] is incorporated in the hypothesis testing device and will be discussed in sequel. The paper is organized as follows: In section II a description of model used is presented. In section III pdf of the decision variable along with detection probabilities is derived. Numerical results are presented in section IV.

II. SYSTEM MODEL

The initial code acquisition for DS-SS systems is modeled as a hypothesis testing problem in [1], [2]. The band pass representation of serial search hypothesis testing device of [1] is shown in Fig.1(a). The decision processor block of [1] contains a threshold device. The acquisition scheme of [1] is modified for adaptive threshold operation by incorporating the CA-CFAR scheme of [5], shown in Fig.1(b), in the decision processor block of Fig.1(a). When non-coherent reception of unfaded received signal in AWGN is considered, the pdf of normalized decision variable Z^* as given by [1] are

$$p_0(Z^*) = \frac{(Z^*)^{N_B-1}}{(N_B-1)!}\exp(-Z^*) \tag{1}$$

and

$$p_1(Z^*) = \left(\frac{Z^*}{N_B\gamma}\right)^{\frac{N_B-1}{2}} \exp(-Z^* - N_B\gamma) \cdot I_{N_B-1}(2\sqrt{N_B\gamma Z^*}) \tag{2}$$

where $p_i(Z^*)(i = 0, 1)$ denotes the pdf of normalized decision variable under the binary hypotheses: signal samples absent or present on PN code correlation curve respectively. While

$$Z^* \triangleq \frac{ZN_B}{2\sigma^2}$$
$$N_B \triangleq B\tau_d = \frac{\tau_d}{T} \tag{3}$$

where $\frac{1}{T}$ is sampling rate, B is the bandwidth of band pass filter and $B = \frac{1}{T}$. σ^2 is variance of AWGN, τ_d is integration (dwell) time and $I_n(x)$ is modified Bessel function of first kind, n^{th} order. Predetection signal to noise ratio (SNR) $\gamma = \frac{A^2}{2\sigma^2}$ where A is received signal amplitude. For adaptive operation of the decision processor, the detection threshold is continuously updated according to the fading conditions on the channel to provide desired false alarm probability. Approximately independent samples Z_i of Z^* [1] are serially shifted into a shift register of size $M + 1$. Each of the M reference cells is assumed to contain Gaussian noise, while only one cell corresponding to Z_o contains signal plus noise. Non coherent integration of N_B samples after the square law envelope detection is used for comparison with the adaptive threshold. Noise estimation is done by processing the contents of neighboring M cells on either side of the test cell. The integrated noise power estimate from $L = MN_B$ samples is obtained by

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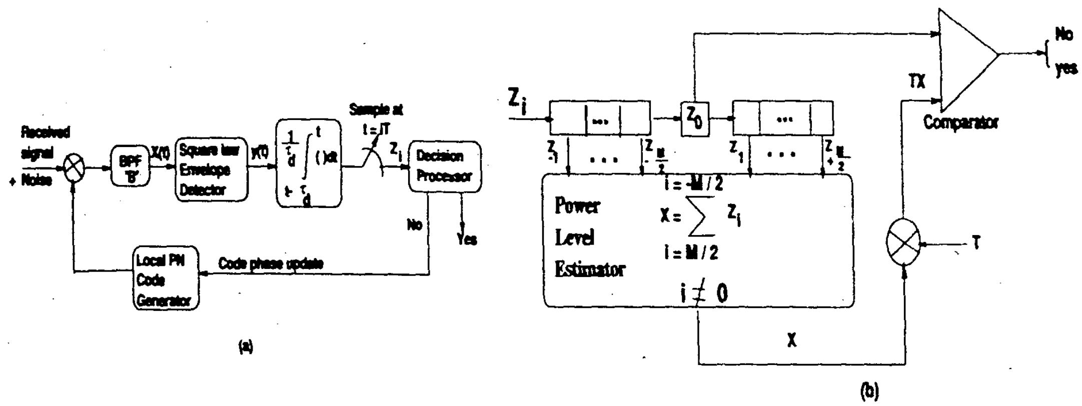

Fig. 1. **(a)** Serial search hypothesis testing device [1] **(b)** Cell average CFAR decision processor [5]

$X = \sum_{i=-\frac{M}{2}}^{i=+\frac{M}{2}} Z_i$. The threshold TX, where the threshold coefficient T is used as a scale factor, varies according to the fading channel conditions, to achieve the desired false alarm probability for a given shift register of window size M. The pdf of X as obtained in [5] is given by

$$p(X) = \frac{X^{L-1}}{(L-1)!}\exp(-X) \quad (4)$$

In Nakagami-m fading environment, the received SNR will be a random variable [7] with distribution

$$p(\gamma) = \frac{\gamma^{m-1}}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \exp\left(-\frac{m\gamma}{\Omega}\right) \quad (5)$$

where $\Gamma(m)$ is the gamma function, m is the fading parameter that ranges from $\frac{1}{2}$ to ∞ and $\Omega = E(\gamma)$.

III. PROBABILITY DENSITY FUNCTION For Z^* AND DETECTION PROBABILITIES

Even in severe fading environments, $p_0(Z^*)$ given by (1) will remain unchanged as it does not depend upon received SNR. However, $p_1(Z^*)$ as given by (2) will now become conditional pdf i.e., $p_1(Z^*/\gamma)$ as it does depend upon received SNR. Under the assumption that Nakagami-m fading is slow enough that the amplitude as well as phase of received signal remains constant over one integration duration (τ_d) but fast enough that successive τ_d segments are essentially independent, the pdf of decision variable Z^* under Nakagami-m fading will then be obtained by averaging the pdf (2) over (5) i.e.,

$$p_1(Z^*) = \int_0^\infty p_1(Z^*/\gamma)p(\gamma)d\gamma \quad (6)$$

$$p_1(Z^*) = \int_0^\infty \frac{\left(\frac{m}{\Omega}\right)^m \gamma^{m-1}\exp\left(-\frac{m\gamma}{\Omega}\right)}{\Gamma(m)}\left(\frac{Z^*}{N_B\gamma}\right)^{\frac{N_B-1}{2}} \cdot \exp^{-(Z^*+\gamma N_B)} I_{N_B-1}(2\sqrt{N_B Z^*\gamma})d\gamma \quad (7)$$

using the series expansion for I_{N_B-1} [9] and collecting all the γ terms inside the integral we have

$$p_1(Z^*) = \frac{\left(\frac{m}{\Omega}\right)^m}{\Gamma(m)}\left(\frac{Z^*}{N_B}\right)^{\frac{N_B-1}{2}} \cdot \exp(-Z^*)\sum_{r=0}^{\infty}\frac{(N_B Z^*)^{\frac{N_B+2r-1}{2}}}{r!(N_B+r-1)!} \cdot \underbrace{\int_0^\infty \gamma^{m+r-1}\exp\left[-\left(\frac{m}{\Omega}+N_B\right)\gamma\right]d\gamma}_{T_1} \quad (8)$$

Letting $\gamma\left(\frac{m}{\Omega}+N_B\right) = \theta$, the integral T_1 can be evaluated using the definition of gamma function [9] i.e., $T_1 = \frac{\Gamma(m+r)}{\left(\frac{m}{\Omega}+N_B\right)^{m+r}}$. Substituting T_1 in (8) and denoting $u \triangleq \frac{\Omega N_B}{m}$,

$$p_1(Z^*) = \frac{(Z^*)^{N_B-1}\exp(-Z^*)}{\Gamma(m)(1+u)^m} \cdot \sum_{r=0}^{\infty}\frac{\Gamma(m+r)}{r!\Gamma(N_B+r)}\left(\frac{uZ^*}{1+u}\right)^r \quad (9)$$

The resulting pdf after some algebra can be expressed as

$$p_1(Z^*) = \frac{(Z^*)^{N_B-1}\exp(-Z^*)}{\Gamma(m)(1+u)^m(N_B-1)!}\,{}_1F_1\left(m, N_B; \frac{uZ^*}{1+u}\right) \quad (10)$$

where ${}_1F_1(a,b;x)$ is the confluent hypergeometric function given as ${}_1F_1(a,b;x) = \sum_{r=0}^{\infty}\frac{\Gamma(b)\Gamma(r+a)x^r}{\Gamma(a)\Gamma(r+b)r!}$, $b \neq 0, -1, -2, ...$[9].

The detection probability (P_D) for a given value of T is then obtained by

$$P_D = \int_0^\infty p(X)\int_{TX}^\infty p_1(Z^*)dZ^*dX \quad (11)$$

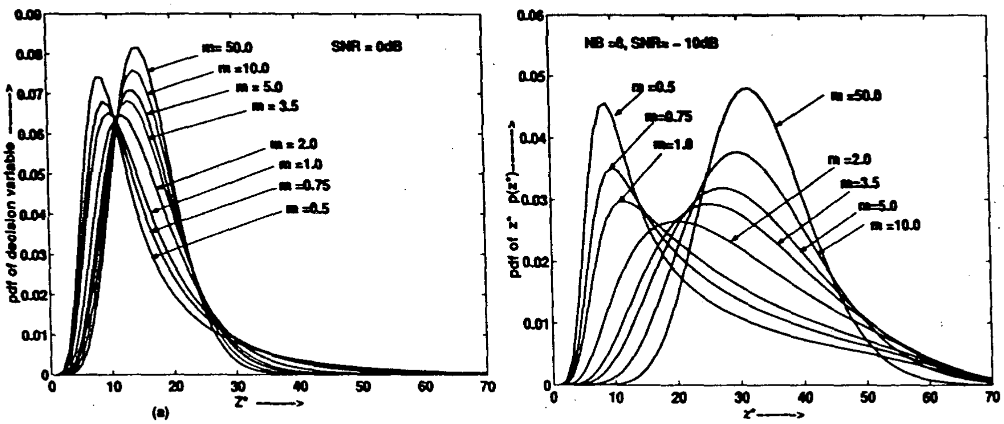

Fig. 2. pdf of Z^* using (9) for different m

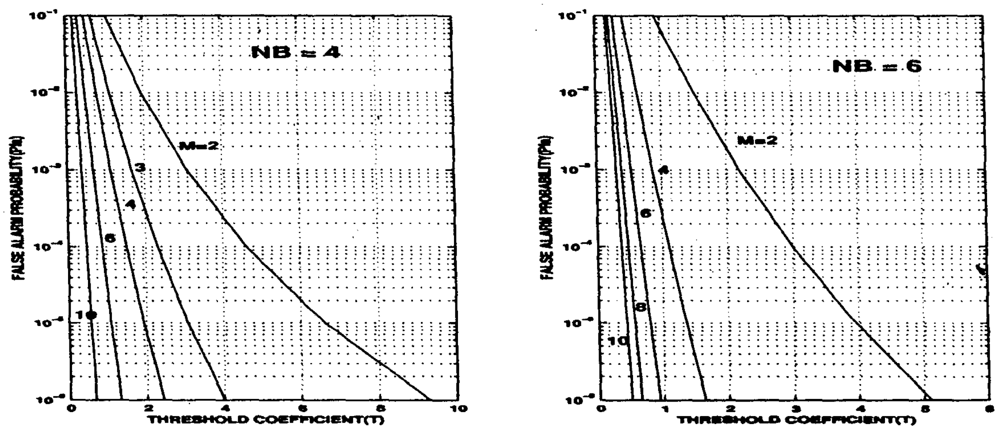

Fig. 3. P_{FA} vs. threshold coefficient (T) using (14) for different M and N_B

while threshold coefficient T is determined from a designated false alarm probability (P_{FA}) [5] by

$$P_{FA} = \int_0^\infty p(X) \int_{TX}^\infty p_0(Z^*) dZ^* dX \tag{12}$$

Substituting (10) and (4) in (11), (1) and (4) in (12) and performing the required integrations the final expressions for probabilities are obtained as

$$P_D = \frac{1}{(1+u)^m (1+T)^L} \sum_{r=0}^{\infty} \begin{pmatrix} m+r-1 \\ r \end{pmatrix} \left(\frac{u}{1+u}\right)^r \cdot \sum_{p=0}^{N_B+r-1} \begin{pmatrix} L+p-1 \\ p \end{pmatrix} \left(\frac{T}{1+T}\right)^p \tag{13}$$

and

$$P_{FA} = \sum_{p=0}^{N_B-1} \begin{pmatrix} L+p-1 \\ L-1 \end{pmatrix} \frac{T^p}{(1+T)^{L+p}} \tag{14}$$

where notation $\begin{pmatrix} a \\ b \end{pmatrix}$ is used to denote $\frac{\Gamma(a-1)}{\Gamma(b-1)\Gamma(a-b-1)}$

IV. NUMERICAL RESULTS

Numerical evaluation of (10) is shown in Fig.2 for fading more severe ($m < 1$) and less severe ($m > 1$) than Rayleigh ($m = 1$) at an average SNR $\Omega = 0dB$ and -10dB. It can be observed that as m is increased the symmetry of curves increases and they become more and more peaked. For low SNR (-10 dB) peaking occurs at a larger value of m, while for moderate SNR (0 dB) the peaking occurs at comparatively lower values of m. The false alarm probability (14) against threshold coefficient (T) is plotted in Fig.3. It can be observed for specific N_B and P_{FA}, T decreases with increasing M. Moreover, for wide dynamic range of P_{FA} from 10^{-6} to 10^{-1}, the variation in threshold coefficient decreases with increasing M. This is due to the

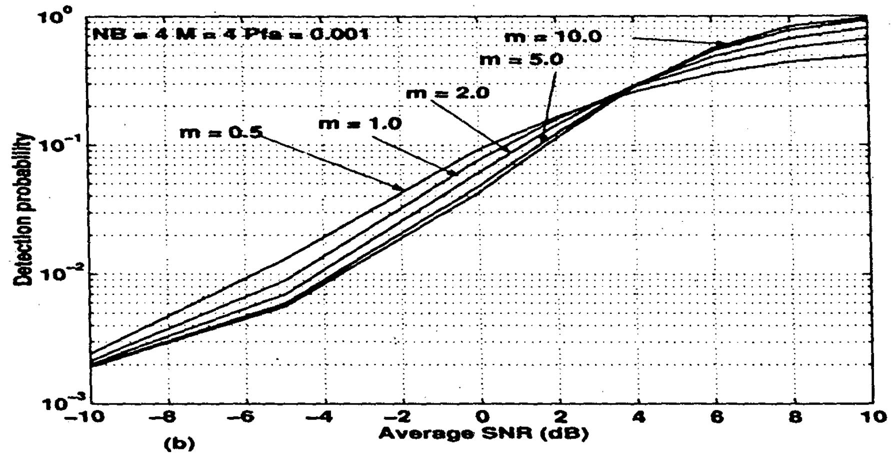

Fig. 4. **Detection probability, P_D vs average SNR, Ω using (13) for different m**

fact that more samples in decision processor will make the channel estimate better which will lead to better threshold settings to achieve the desired false alarm probability. Further, it can be observed that by increasing N_B, the whole family of curves shifts to left indicating reduction in T. The detection probability (13) was also numerically evaluated for $N_B = 4$, $M = 4$, and $P_{FA} = 0.001$ and is shown in Fig.4. With increasing m (reducing fading severity), improved detection performance is obtained when P_D is greater than 0.2. However, for P_D less than 0.2, better detection performance is obtained with reducing m i.e., increasing fading severity. Similar phenomenon has also been observed in [5], [8] and implies that there exists an optimum N_B for given m and average SNR Ω, for which P_D is maximum.

V. CONCLUSION

In this work we have attempted to bridge the gap that was existing for detection performance characterization of code acquisition problem under fading conditions more severe than Rayleigh. These fading conditions are often encountered on urban mobile radio environment. A closed form expression for P_{FA} is also provided. Though, a closed form expression for P_D has eluded us, nevertheless, (13) is more general as compared to those reported for Rayleigh fading case [4] and for only integer values of signal strength fluctuation parameter, $K \geq 1$ [5][1]. Depending on the fading severity, (13) can be evaluated numerically for desired accuracy. The P_D and P_{FA} as obtained from (13) and (14) can be further used in the mean acquisition time $(\overline{T}_{acq})$ and variance of acquisition time (σ^2_{acq}) expressions for serial search approach to obtain the code acquisition time performance under different fading conditions.

[1] K (an integer and $1 \leq K \leq \infty$) in radar literature represents the degree of signal strength fluctuation i.e., smaller values of K, corresponds to the deeper signal fluctuations and vice-versa. In that sense K of [5] is same as fading parameter m used in this work

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