Optimization For Energy-Efficient OFDM Amplify and Forward Non-Concurrent Two-Way Relaying

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***Abstract*— We consider an orthogonal frequency division multiplexing based non-concurrent two-way relaying (ncTWR), where the base station (BS) serves a transmit user (TU) and a receive user (RU), whose receive signal is interfered by the TU transmit signal. The RU uses noisy side-information gathered by overhearing the TU transmit signal to neutralize the interference. We optimize the global energy efficiency (GEE) metric, which is a fractional function of the network sum-rate and its total energy consumption, by using a quadratic transformation-based algorithm. We numerically demonstrate that the proposed algorithm outperforms other state-of-the-art algorithms.**

***Index Terms*— Energy efficiency, quadratic transformation, sum-rate.**

I. Introduction

TWO-WAY relaying (TWR) enables two source nodes to exchange data in two channel uses [1]. In the first channel use of TWR, both source nodes simultaneously transmit their data signals. The amplify-and-forward (AF) relay receives a sum of these two signals, which it amplifies and retransmits in the second channel use. The two nodes, as they know their transmit signals, cancel the self-interference/back-propagating interference (BI) from their receive signals to make them BI-free. The inherent assumption in TWR is that the two source nodes always have data to exchange which they can transmit (resp. receive) in the first (resp. second) channel use.

The conventional TWR, based on the aforementioned assumption, can be readily incorporated in cellular systems, if a user always has data to exchange with the base station (BS). This assumption, typically, gets violated in cellular systems. For example, consider a user TU who is uploading a large file to a cloud, or a user RU, who is downloading a multimedia data from a server. We notice that TU only transmits data to the BS, and RU only receives data from the BS which, now, cannot use TWR to serve them as neither of them exchanges data with it. The BS will use one-way relaying to serve them and would consequently require four channel uses – two for each user. In the non-concurrent TWR (ncTWR) protocol considered in [1]–[5], the BS can serve both TU and RU in two channel uses as explained next.[1]

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[1]The first and the second channel uses of the ncTWR are commonly known as the multiple-access (MAC) and the broadcast (BC) phase, respectively.

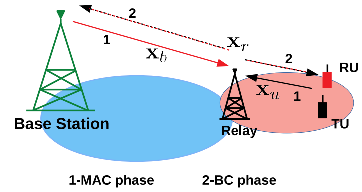


Fig. 1. *System Model*: In the MAC phase (1), both BS and TU transmit to the relay. In the BC phase, the relay amplifies the sum-signal received and transmits it to both BS and RU.

In the ncTWR MAC phase, as shown in Fig. 1, the BS and TU transmit their respective signals to the relay, which receives their sum. In the ncTWR BC phase, the AF relay first amplifies its receive sum signal, and then transmits it to both BS and RU. The BS, unlike the conventional TWR, now requires only two channel uses to send and receive data from TU and RU, respectively. The BI experienced by the i) BS is from its self-data which it can cancel; and ii) RU is due to the TU transmit data which, without any side information, it cannot cancel. This is unlike TWR wherein both the receive nodes can cancel the BI. References [1]–[4] designed precoders at the relay for a single-carrier ncTWR system to cancel the BI. In [4] and [5], which consider single-carrier and OFDM ncTWR systems respectively, the RU cancels its BI by overhearing the MAC-phase TU transmit signal. These works also maximized the system sum-rate. The aforementioned studies for single-carrier- and OFDM-based ncTWR are summarized in Table I.

The global energy efficiency (GEE) has become a key performance metric for wireless systems due to their high energy consumption [6]. We observe from ncTWR references in Table I that the GEE for ncTWR has not yet been optimized. The **contributions** of this work helps in doing that and are:

1) We propose a quadratic transformation (QT)-based approach to optimize GEE which is a coupled function of the i) rate and total power consumption in the GEE numerator and denominator respectively; and ii) TU, BS and relay power. 2) We show the GEE improvement of the proposed algorithm over geometric programming-based sum-rate maximization [5], OWR [7], equal and random power allocation [8].

II. System Model

We consider an OFDM ncTWR system, with K subbands, where the TU transmits uplink data to the BS and the RU receives downlink data from the BS via a half-duplex AF relay. We assume that all nodes are equipped with a single antenna. In the ncTWR MAC phase, both BS and TU transmit their respective OFDM signals to the relay, which for the kth subband, receives the following sum signal: $y_r[k] = h_b[k]x_b[k] + h_u[k]x_u[k] + n_r[k]$, for $k = 1, \ldots, K$. Here the scalar channels $h_b[k]$ and $h_u[k]$ are for the kth subband of the BS←Relay and the TU←Relay links, respectively. The transmit signal is $x_i[k] = \sqrt{p_i[k]}\tilde{x}_i[k]$ for $i = u, k$.

TABLE I

SUMMARY OF NCTWR LITERATURE FOCUSING ON SE AND GEE.

ncTWR	Main contribution	Metric optimized	Approach
[1]	Quantized precoder, single carrier	None	Diversity enhancing precoder
[2]	Power allocation, single carrier	Sum-rate	Geometric programming with approximated objective
[3]	Power allocation, single carrier	Sum-rate+QoS	Geometric programming
[5]	Power allocation, OFDM	Sum-rate	Geometric programming
[4]	Overhearing precoder, single carrier	Sum-rate	Semidefinte relaxation
Current work	Power allocation, OFDM	GEE	Iterative quadratic transformation

The information signal $\tilde{x}_i[k], \forall k$, follows circular symmetric complex Gaussian distribution, denoted as $\mathcal{CN}(0,1)$, and is independent and identically distributed (iid) across subbands which ensures that $\mathbb{E}[\sum_{k=1}^{K}|x_i[k]|^2] = \sum_{k=1}^{K} p_i[k] \leq P_i$. We assume that $n_r[k]$ is distributed as $\mathcal{CN}(0,1)$, and is iid across subbands. The RU overhears the MAC-phase TU signal and uses this noisy side information in the BC phase to cancel its BI. The overheard signal by the RU in the kth subband is

$$y_u^{(1)}[k] = h_o[k]x_u[k] + n_u^{(1)}[k], \quad (1)$$

where $h_o[k]$ is the kth subband overhearing-link channel for the TU←RU link. Here $n_u^{(1)}[k]$, distributed as $\mathcal{CN}(0,1)$, is RU noise and iid across subbands. In the BC phase of ncTWR, the relay amplifies its received signal and broadcasts it to both BS and RU. The relay transmit signal for the kth subband is $x_r[k] = \sqrt{p_r[k]}y_r[k] = \sqrt{p_r[k]}(h_b[k]x_b[k] + h_u[k]x_u[k] + n_r[k])$, where $p_r[k]$ is the amplification factor for the kth subband. The relay, with a maximum transmit power of P_r, imposes the following condition on its transmit signal:

$$\sum_{k=1}^{K} p_r[k]|h_b[k]|^2 p_b[k] + p_r[k]|h_u[k]|^2 p_u[k] + p_r[k] \leq P_r. \quad (2)$$

The relay transmit signal $x_r[k]$ is usually normalized to satisfy its transmit power constraint. The above transmit power constraint, similar to the existing literature [2], [3], [5], also ensures that $x_r[k]$ does not violate the relay power constraint.

The signals received by BS and RU on kth subband are

$$y_b[k] = \sqrt{p_r[k]}g_b[k]\Big(\underbrace{h_b[k]x_b[k]}_{\text{BI}} + h_u[k]x_u[k]\Big) + \bar{n}_b[k],$$
$$y_u^{(2)}[k] = \sqrt{p_r[k]}g_u[k]\Big(h_b[k]x_b[k] + \underbrace{h_u[k]x_u[k]}_{\text{BI}}\Big) + \bar{n}_u^{(2)}[k]. \quad (3)$$

The scalar channels $g_b[k]$ and $g_u[k]$ are for the kth subband of the Relay←BS and Relay←RU links, respectively. Also $\bar{n}_b[k] = \sqrt{p_r[k]}g_b[k]n_r[k] + n_b[k]$ and $\bar{n}_u^{(2)}[k] = \sqrt{p_r[k]}g_u[k]n_r[k] + n_u^{(2)}[k]$ are the effective noise at the BS and the RU in the BC phase, where $n_b[k] \sim \mathcal{CN}(0,1)$ and $n_u^{(2)}[k] \sim \mathcal{CN}(0,1)$. The BS uses its self-data to cancel the BI from its receive signal $y_b[k]$ in (3) as follows

$$\tilde{y}_b[k] = \sqrt{p_r[k]}g_b[k]h_u[k]x_u[k] + \bar{n}_b[k]. \quad (4)$$

The RU uses the overheard signal in (1) to cancels its BI

$$\tilde{y}_u[k] = y_u^{(2)}[k] - \frac{\sqrt{p_r[k]}g_u[k]h_u[k]}{h_0[k]}y_u^{(1)}[k]$$
$$= \sqrt{p_r[k]}g_u[k]h_b[k]x_b[k] + \bar{n}_u^{(2)}[k] - \frac{\sqrt{p_r[k]}g_u[k]h_u[k]}{h_0[k]}n_u^{(1)}[k]. \quad (5)$$

If RU can overhear a noise-free TU signal, it can directly cancel the BI in (3), which is, however, impractical. We, therefore, consider a practical scenario where RU overhears a noisy version of the TU transmit signal, which is given in (1). The RU then cancels the interference as shown in (5).

The rate of the kth subband for the TU←Relay←BS and BS←Relay←RU links using (4) and (5) respectively are $R_i[k] = \frac{1}{2}\log_2\Big(1 + \text{SNR}_i[k]\Big)$, for$i \in \{u, b\}$where

$$\text{SNR}_b[k] = \frac{|h_u[k]g_b[k]|^2 p_r[k]p_u[k]}{1 + |g_b[k]|^2 p_r[k]}$$
$$\text{SNR}_u[k] = \frac{|g_u[k]h_b[k]|^2 p_r[k]p_b[k]}{1 + |g_u[k]|^2 p_r[k] + |\frac{g_u[k]h_u[k]}{h_o[k]}|^2 p_r[k]}. \quad (6)$$

The scaling of $1/2$ is because of half-duplex relay which requires two channel uses to receive and transmit signals. The sum-rate now is $R_{\text{sum}} = \sum_{k=1}^{K}(R_b[k] + R_u[k])$.

Implementation issues: The relay requires the channels $h_b[k]$ and $h_u[k]$, which it estimates using pilots transmitted by the BS and TUE, respectively. The BS requires the composite channels $g_b[k]h_b[k]$ and $g_b[k]h_u[k]$ to cancel the BI, and to decode data. The relay can help BS estimate them by sending pilots precoded with $h_b[k]$ and $h_u[k]$, respectively. The RU requires the composite channels $g_u[k]h_b[k]$ and $\frac{g_u[k]h_u[k]}{h_o[k]}$ to decode data and to cancel the BI. The relay can help RU estimate $g_u[k]h_b[k]$ and $g_u[k]h_u[k]$ by sending pilots precoded with $h_b[k]$ and $h_u[k]$, respectively. The TU can help RU estimate $h_o[k]$ by sending un-precoded pilot. The relay also requires the channels $g_b[k]$ and $g_u[k]$ to solve the optimization, which it can estimate using the techniques from [9]. As it is easy for the relay to estimate channel of all the links, it executes the optimization and distributes the power variables.

Power consumption model: The consumed system power $P_{total} = P_c + P_{tx}$, where P_c is the circuit power and P_{tx} is the transmission power. The circuit power P_c consists of the i) per-band signal processing power of users TU, RU, and the BS denoted as $P_s^t[k]$, $P_s^r[k]$, $P_s^b[k]$ respectively, which linearly scales with number of subbands; and ii) fixed power P_f^b and P_f^u, required to operate the electronic devices of the BS and the user, respectively. The circuit power P_c is therefore [10] $P_c = \sum_{k=1}^{K}(P_s^b[k] + P_s^t[k] + P_s^r[k]) + P_f^b + P_f^u$. The transmission power P_{tx} is given as [10] $P_{tx} = \sum_{k=1}^{K}\frac{1}{\xi}(p_r[k] + p_u[k] + p_b[k])$. Here, $\xi \in (0,1)$ is the power amplifier efficiency.

Remark 1: It is possible to multiple users in the system. Each RU will then experience BI from multiple TUs. It is non-trivial to extend the overhearing technique for a RU to cancel the BI from multiple TUs. A new solution needs to designed to handle BI for OFDM-based multi-user ncTWR, and is an important topic for future research.

III. GEE MAXIMIZATION USING QUADRATIC TRANSFORM

The GEE is defined as the ratio of the network sum-rate and its total power consumption [6] i.e.,

$\text{GEE} = \frac{R_{\text{sum}}}{\sum_{k=1}^{K} \frac{1}{\xi}(p_r[k]+p_b[k]+p_u[k])+P_c}$. Here P_c is the system circuit power. We will now jointly allocate power across the subbands and the transmit nodes, TU, RU and the BS, to optimize the GEE. For the sake of notational convenience, we first stack the power variables and define an optimization power vector as $\mathbf{p}_j = (p_j[1], \ldots, p_j[K])^T$ for $j = r, b, u$. The GEE optimization problem can now be formulated as follows.

$$\mathbf{P1}: \underset{\mathbf{p}_r,\mathbf{p}_b,\mathbf{p}_u}{\text{Max}} \frac{R_{\text{sum}}}{\sum_{k=1}^{K} \frac{1}{\xi}(p_r[k]+p_b[k]+p_u[k])+P_c} \tag{7a}$$

$$\text{s.t. } (2), \sum_{k=1}^{K} p_b[k] \le P_b \quad \sum_{k=1}^{K} p_u[k] \le P_u. \tag{7b}$$

The three constraints are on the maximum transmit power of the relay, TU and BS, respectively. The GEE optimizes the system energy to achieve a given sum-rate [10]. Further, the individual transmit constraint make sure that they are not violated while optimizing the system power. The GEE is a function of the non-concave sum-rate, and the consumed power. We will next show it can be optimized using QT. To accomplish this objective, we use the following proposition from [6] which will decouple its numerator and denominator.

Proposition 1: Consider a function of ratio problem

$$\underset{x}{\text{Max}} f\left(\frac{u(x)}{v(x)}\right) \quad \text{subject to } x \in \chi, \tag{8}$$

where χ is a convex set, and $f_k(\cdot)$ is a non-negative function. The function in the numerator and the denominator are defined such that $u(x)$: $\mathbb{R}^n \leftarrow \mathbb{R}^+$ and $v(x)$: $\mathbb{R}^n \leftarrow \mathbb{R}^{++}$. Using QT, the above problem in (8) can be equivalently written as

$$\underset{x,y}{\text{Max}} f\left(2\sqrt{u(x)}y - y^2 v(x)\right) \text{subject to } x \in \chi, y \in \mathbb{R}. \tag{9}$$

Proposition 1 assumes that the functions $u(x)$ and $v(x)$ are non-negative and positive, respectively. We next state another result from [6] which assumes a specific structure on the functions $u(x)$ and $v(x)$. This proposition, as shown next, will allow us to calculate a stationary (x, y) of (9) by iteratively solving a concave problem and a closed-form equation.

Proposition 2: For the function of ratio problem in (8), if each ratio $\frac{u(x)}{v(x)}$ is in the concave-convex form *i.e.,* each $u(x)$ is concave and $v(x)$ is convex, and further assuming that the function f is non-decreasing and concave, then for a given y, the problem (9) is a concave problem in x and for a given x, the optimal value of y can be obtained in a closed form as $y^* = \frac{\sqrt{u(x)}}{v(x)}$. By iteratively optimizing x and y, the problem (9) converges to a stationary point of (8) with a non decreasing objective value after every iteration.

We now solve problem **P1** using the QT in Proposition 1. The GEE optimization, even after applying QT, is not straightforward because it contains non-concave terms. We will then use first-order Taylor series result to linearly approximate these non-concave terms. Using Proposition 1, we now rewrite **P1** using QT as

$$\mathbf{P2}: \underset{\mathbf{p}_r,\mathbf{p}_b,\mathbf{p}_u,\mathbf{y}}{\text{Max}} 2y\sqrt{R_{sum}} - y^2 \sum_{k=1}^{K} \frac{1}{\xi}(p_r[k]+p_b[k]+p_u[k]) + P_c$$
$$\text{s.t. } (2), (7b) \quad \text{and} \quad y \in \mathbb{R}.$$

Here, y is a auxiliary variable which decouples the numerator and denominator of GEE. We observe from objective in **P2**, the term R_{sum} contains two fractional terms in the form of $\text{SNR}_b[k]$ and $\text{SNR}_u[k]$. Hence using the proposition 1 twice, we rewrite the **P2** as

$$\begin{aligned}\mathbf{P3}: \underset{\mathbf{p}_r,\mathbf{p}_b,\mathbf{p}_u,y,\mathbf{w},\mathbf{z}}{\text{Max}} & 2y\Big(\sum_{k=1}^{K} \frac{1}{2}\log_2(1+2w_k\sqrt{C_k} - w_k^2 D_k) \\ & + \sum_{k=1}^{K} \frac{1}{2}\log_2(1+2z_k\sqrt{E_k} - z_k^2 F_k)\Big)^{\frac{1}{2}} \\ & - y^2 \sum_{k=1}^{K} \frac{1}{\xi}(p_r[k]+p_b[k]+p_u[k]) + P_c\end{aligned} \tag{11a}$$

$$\text{s.t. } (2), (7b), y \in \mathbb{R}, \mathbf{w} \in \mathbb{R}^K \quad \text{and} \quad \mathbf{z} \in \mathbb{R}^K.$$

Here $\mathbf{w} = [w_1, \cdots, w_K]$ and $\mathbf{z} = [z_1, \cdots, z_K]$, where each w_k and z_k decouples the numerator and denominator of each $R_b[k]$ and $R_u[k]$ respectively. For notational convenience, we use C_k, D_k and E_k, F_k for $k = 1, \cdots, K$ to denote the numerator and denominator of $\text{SNR}_b[k]$ and $\text{SNR}_u[k]$ in (6), respectively. To summarize, we apply QT in Proposition 1 twice to transform **P1** to **P3**. From Proposition 2, **P3** is a concave maximization problem if C_k, E_k and D_k, F_k are concave and convex functions respectively.

We see that the terms C_k and E_k are non-concave in optimization variables $p_r[k]$, $p_u[k]$ and $p_b[k]$. Further the constraint (7b) is convex but the LHS of (2) is non-convex. We use first-order Taylor series approximation to linearize $p_r[k]p_b[k]$ and $p_r[k]p_u[k]$ as affine function in $p_r[k], p_b[k], p_u[k]$, which will help us convexify the objective and the constraint, using the following lemma, whose proof is relegated to Appendix.

Lemma 1: The non-convex terms $p_r[k]p_b[k]$ and $p_r[k]p_u[k]$ can be linearly approximated as $p_r[k]p_b[k] = \tilde{p}_r[k]\tilde{p}_b[k] + \tilde{p}_b[k](p_r[k] - \tilde{p}_r[k]) + \tilde{p}_r[k](p_b[k] - \tilde{p}_b[k])$ and $p_r[k]p_u[k] = \tilde{p}_r[k]\tilde{p}_u[k] + \tilde{p}_u[k](p_r[k] - \tilde{p}_r[k]) + \tilde{p}_r[k](p_u[k] - \tilde{p}_u[k])$ where $\tilde{p}_r[k]$, $\tilde{p}_b[k]$ and $\tilde{p}_u[k]$ are initial values of $p_r[k]$, $p_b[k]$ and $p_u[k]$, respectively.

We have used the fact that a convex function can be lower-bounded by its first-order Taylor series approximation [11]. The bound, as discussed in [11], is a tight one, and is a commonly used heuristic to approximate the problem as convex which leads close-to-optimal results. To convexify the objective, we now use Lemma 1 for affine approximation of C_k and E_k in (11a).

$$\begin{aligned}\widehat{C}_k = |h_u[k]g_b[k]|^2 \Big(& \tilde{p}_r[k]\tilde{p}_u[k] + \tilde{p}_u[k](p_r[k] - \tilde{p}_r[k]) \\ & + \tilde{p}_r[k](p_u[k] - \tilde{p}_u[k])\Big)\end{aligned} \tag{12}$$

$$\begin{aligned}\widehat{E}_k = |g_u[k]h_b[k]|^2 \Big(& \tilde{p}_r[k]\tilde{p}_b[k] + \tilde{p}_b[k](p_r[k] - \tilde{p}_r[k]) \\ & + \tilde{p}_r[k](p_b[k] - \tilde{p}_b[k])\Big).\end{aligned} \tag{13}$$

Hence both $\widehat{C}_k$ and $\widehat{E}_k$ are concave (affine) and D_k and F_k are convex which makes the objective in **P3** to concave. The constraint (2) is now modified using the Lemma 1 as

$$\begin{aligned}\sum_{k=1}^{K} & |h_b[k]|^2 \Big(\tilde{p}_r[k]\tilde{p}_b[k] + \tilde{p}_b[k](p_r[k] - \tilde{p}_r[k]) + \tilde{p}_r[k](p_b[k] - \tilde{p}_b[k])\Big) \\ & + |h_u[k]|^2 \Big(\tilde{p}_r[k]\tilde{p}_u[k] + \tilde{p}_u[k](p_r[k] - \tilde{p}_r[k]) \\ & + \tilde{p}_r[k](p_u[k] - \tilde{p}_u[k])\Big) + p_r[k] \le P_r.\end{aligned} \tag{14}$$

Algorithm 1 GEE using Quadratic Programming

Input: Tolerance $\epsilon > 0$, and the maximum iterations L.

Output: Optimal power allocation variables $\mathbf{p}_r, \mathbf{p}_b$ and $\mathbf{p}_u$.

1 *Initialization*: Allocate equal power across all K subbands to calculate initial feasible values of $\mathbf{p}_r, \mathbf{p}_b$ and $\mathbf{p}_u$, denoted as, $\mathbf{p}_r^1, \mathbf{p}_b^1$ and $\mathbf{p}_u^1$, respectively.

2 **for** $m \leftarrow 1$ ***to*** L **do**

3 Given a feasible $\mathbf{p}_i^m$,$\forall i = r, b, u$ compute y, $\mathbf{w}$ and $\mathbf{z}$ $\forall k = 1, \cdots, K$ from (16a), (16b) and (16c).

4 Solve the following problem to calculate $\mathbf{p}_r, \mathbf{p}_b, \mathbf{p}_u$

$$\underset{\mathbf{p}_r,\mathbf{p}_b,\mathbf{p}_u}{\text{Max}}\ 2y\Big(\sum_{k=1}^{K}\frac{1}{2}\log_2(1+2w_k\sqrt{\widehat{C}_k}-w_k^2D_k) + \sum_{k=1}^{K}\frac{1}{2}\log_2(1+2z_k\sqrt{\widehat{E}_k}-z_k^2F_k)\Big)^{\frac{1}{2}} - y^2\sum_{k=1}^{K}\frac{1}{\xi}(p_r[k]+p_b[k]+p_u[k])+P_c \quad (17a)$$

$$\text{s.t.} \quad (14), (7b), y \in \mathbb{R}, \mathbf{w} \in \mathbb{R}^K \quad \text{and} \quad \mathbf{z} \in \mathbb{R}^K.$$

5 Do until convergence **if** $\max|\mathbf{p}_r - \mathbf{p}_r^m| \le \epsilon$ *and* $\max|\mathbf{p}_b - \mathbf{p}_b^m| \le \epsilon$ *and* $\max |\mathbf{p}_u - \mathbf{p}_u^m| \le \epsilon$ **then** ;

6 break **else** $\mathbf{p}_r^{m+1} = \mathbf{p}_r$, $\mathbf{p}_b^{m+1} = \mathbf{p}_b$ and $\mathbf{p}_u^{m+1} = \mathbf{p}_u$;

7 **return** $\mathbf{p}_r, \mathbf{p}_b$ *and* $\mathbf{p}_u$.

The left side of the above constraint in (14) is now affine. Using (12), (13) and (14), we rewrite **P3** as **P4**.

$$\textbf{P4}: \underset{\mathbf{p}_r,\mathbf{p}_b,\mathbf{p}_u,y,\mathbf{w},\mathbf{z}}{\text{Max}}\ 2y\Big(\sum_{k=1}^{K}\frac{1}{2}\log_2(1+2w_k\sqrt{\widehat{C}_k}-w_k^2D_k) + \sum_{k=1}^{K}\frac{1}{2}\log_2(1+2z_k\sqrt{\widehat{E}_k}-z_k^2F_k)\Big)^{\frac{1}{2}} - y^2\sum_{k=1}^{K}\frac{1}{\xi}(p_r[k]+p_b[k]+p_u[k])+P_c \quad (15a)$$

$$\text{s.t.} \ (14), (7b), y \in \mathbb{R}, \mathbf{w} \in \mathbb{R}^K \quad \text{and} \quad \mathbf{z} \in \mathbb{R}^K.$$

For a given y, $\mathbf{w}$ and $\mathbf{z}$, the problem **P4** is now concave in $\mathbf{p}_r, \mathbf{p}_b, \mathbf{p}_u$. After calculating $\mathbf{p}_r, \mathbf{p}_b, \mathbf{p}_u$, we calculate the optimal values of auxiliary variables y, $\mathbf{w}$ and $\mathbf{z}$ by using Proposition 2 as

$$2y = \frac{\sqrt{R_{sum}}}{\sum_{k=1}^{K}\frac{1}{\xi}(p_r[k]+p_b[k]+p_u[k])+P_c} \quad (16a)$$

$$w_k = \frac{\sqrt{|h_u[k]g_b[k]|^2 p_r[k]p_u[k]}}{1+|g_b[k]|^2 p_r[k]} \quad (16b)$$

$$z_k = \frac{\sqrt{|g_u[k]h_b[k]|^2 p_r[k]p_b[k]}}{1+|g_u[k]|^2 p_r[k] + |\frac{g_u[k]h_u[k]}{h_o[k]}|^2 p_r[k]}. \quad (16c)$$

Here $\mathbf{w} = [w_1, \cdots, w_K]$ and $\mathbf{z} = [z_1, \cdots, z_K]$. We iteratively calculate $\mathbf{p}_r, \mathbf{p}_b, \mathbf{p}_u$ by first solving **P3** for a given y, $\mathbf{w}$ and $\mathbf{z}$ and then calculate y, $\mathbf{w}$ and $\mathbf{z}$ from (16a), (16b), (16c), respectively. The process is summarized in Algorithm 1.

The convergence criterion $|p - p^m| <= \epsilon$ will also ensure GEE convergence.

Remark 2: The computational complexity of Algorithm 1 is dominated by step-3 where we solve the optimization with N variables, which has a complexity of $\mathcal{O}(N^3)$. For our problem, which has 3 power variables $\mathbf{p}_r, \mathbf{p}_b$, and $\mathbf{p}_u$ with K subbands each, computational complexity per-iteration is $\mathcal{O}(27K^3)$.

Convergence analysis: The auxiliary variables y, $\mathbf{w}$, $\mathbf{z}$ are determined by (16a), (16b) and (16c) respectively, using $\mathbf{p}_r^m$, $\mathbf{p}_b^m$ and $\mathbf{p}_u^m$. The objective in **P1**, **P3** and **P4** are written as $f_o(\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m)$, $f_q(\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m, y, \mathbf{w}, \mathbf{z})$ and $\tilde{f}_q(\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m, y, \mathbf{w}, \mathbf{z})$ respectively at the m-th iteration. First, we state a useful lemma, which can be easily verified.

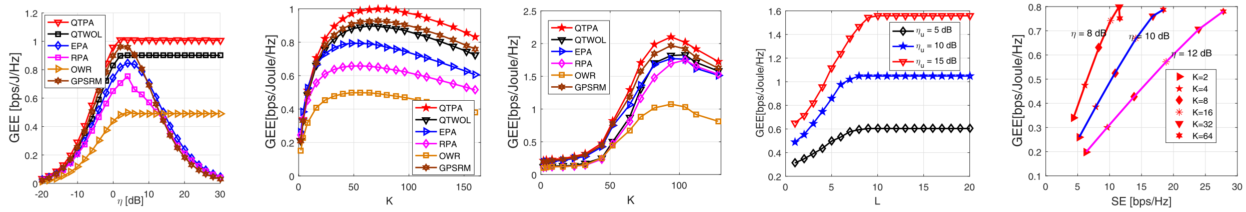
Lemma 2: $f_o(\mathbf{p}_r, \mathbf{p}_b, \mathbf{p}_u) \ge f_q(\mathbf{p}_r, \mathbf{p}_b, \mathbf{p}_u, y, \mathbf{w}, \mathbf{z})$, with equality iff $y, \mathbf{w}, \mathbf{z}$ satisfy (16a), (16b) and (16c) respectively. We now prove the convergence.

$$\begin{aligned} & f_o(\mathbf{p}_r^{m+1}, \mathbf{p}_b^{m+1}, \mathbf{p}_u^{m+1}) \\ & =(a) f_q(\mathbf{p}_r^{m+1}, \mathbf{p}_b^{m+1}, \mathbf{p}_u^{m+1}, y^{m+1}, \mathbf{w}^{m+1}, \mathbf{z}^{m+1} | \mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m) \\ & \ge(b) \tilde{f}_q(\mathbf{p}_r^{m+1}, \mathbf{p}_b^{m+1}, \mathbf{p}_u^{m+1}, y^{m+1}, \mathbf{w}^{m+1}, \mathbf{z}^{m+1} | \mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m) \\ & \ge(c) \tilde{f}_q(\mathbf{p}_r^{m+1}, \mathbf{p}_b^{m+1}, \mathbf{p}_u^{m+1}, y^m, \mathbf{w}^m, \mathbf{z}^m | \mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m) \\ & \ge(d) \tilde{f}_q(\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m, y^m, \mathbf{w}^m, \mathbf{z}^m | \mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m) \\ & =(e) f_q(\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m, y^m, \mathbf{w}^m, \mathbf{z}^m | \mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m) \\ & =(f) f_o(\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m). \end{aligned}$$

In the above equations, the notation $f(\ldots|\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m)$ imply "for a given value of $\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m$". Equality in (a) is due to Lemma 2. (b) is because a convex function can be lower bounded by its first order Taylor series approximation [11]. (c) is because the updates of auxiliary variables y, $\mathbf{w}$, $\mathbf{z}$ in (16a), (16b) and (16c) respectively, maximize $\tilde{f}_q$, with other variables being fixed. (d) is because the updates of $\mathbf{p}_r$, $\mathbf{p}_b$, $\mathbf{p}_u$ maximize $\tilde{f}_q$, with other variables being fixed. Equality in (e) is because Taylor-series-approximated $\tilde{f}_q$ and f_q are equal at $\mathbf{p}_r^m, \mathbf{p}_b^m, \mathbf{p}_u^m$ [11]. Equality in (f) is because of Lemma 2. The objective f_o is monotonically nondecreasing after each iteration. As the value of f_o is bounded from above, the algorithm must converge to a local optimum.

IV. Simulation Results

We now numerically investigate the GEE achieved using the proposed QT-base algorithm (denoted as QTPA) for an OFDM AF ncTWR with K subbands. We compare the performance of the QTPA algorithm with i) scheme where overhearing link is ignored (labelled as QTWOL); ii) equal-power allocation (EPA) [8]; iii) random power allocation (RPA) [8]; iv) one way relaying (labelled as OWR) [7]; and v) geometric programming-based sum-rate maximization algorithm in [5] (labelled as GPSRM); The GPSRM scheme maximizes the sum-rate using geometric programming and uses the optimal power so obtained for calculating GEE. The QTWOL scheme uses the proposed algorithm to maximize GEE but ignores the overhearing link. The conventional one way relaying also uses the proposed algorithm to maximize the GEE but the BS now requires four time slots to serve TU and RU - two to receive data from TU and two to send data to RU. The RPA scheme randomly allocates power across the subbands at the TU, Relay and BS nodes to satisfy their individual power constraints. We assume, similar to [2], that the channels links are distributed as $\mathcal{CN}(0, \eta_i)$, where $i = u$ denotes the Relay $rightarrow$ RU link, $i = b$ denotes the BS$rightarrow$Relay link, and $i = o$ denotes the TU $\leftarrow$ RU overhearing link. The channel variance η_i for $\{i \in u, b, o\}$ denote the channel



(a) (b) (c) (d) (e)

Fig. 2. a) GEE vs η for $\eta_b = 10$ dB,$\eta_u = 10$ dB, $\eta_o = 5$ dB and $K = 16$; b) GEE versus K for fixed η, η_b, η_u and η_o; and c) GEE versus K and d) GEE versus L for $\eta_b = 10$ dB, $\eta_o = 5$ dB, $\eta = 10$ dB and $K = 16$; e) GEE vs sum-rate for $\eta_b = 10$ dB,$\eta_u = 10$ dB, $\eta_o = 5$ dB.

gains of the respective links. For notational convenience in simulations, we set $P_r = P_b = P_u = \eta$.

We first investigate in Fig. 2a the GEE obtained by varying η, which is the maximum power available at the TU, relay and the BS. We also fix the noise power, and assume it to be unity. We also fix $\eta_u = 10$ dB, $\eta_b = 10$ dB, and $\eta_o = 5$ dB with respect to the noise power. Also, $P_s^b = 40$ mW, $P_s^t = 5 - 30$ mW, $P_s^r = 5 - 30$ mW per subband and $P_f^u =$ 50 mW and $P_f^b = 2000$ mW. We also fix tolerance $\epsilon = 10^{-3}$, $L = 30$ in Algorithm 1 and $K = 16$ subbands. We observe from Fig. 2a that the proposed QTPA algorithm outperforms all the aforementioned state-of-the-art techniques. We also observe that with the proposed algorithm, the GEE increases till $\eta = 5$ dB, and remains constant after that. This is because power of $\eta = 5$ dB allows the system to attain the maximum GEE, and any additional power used by the system will only decrease the GEE for EPA, RPA and GPSRM schemes as the system keeps using the available power for $\eta > 5$ dB.

We next vary the number of subbands K in Fig. 2b and Fig. 2c, and plot the GEE achieved. In Fig. 2b, we fix $\eta_u = \eta_b = \eta = 10$ dB and $\eta_o = 5$ dB for all K values, whereas in Fig. 2c we start by considering $\eta = 2$ dB for $K = 1$ subband and then double it with each $K = 20$ subband increment. We observe that the proposed QTPA algorithm, for different subband values, yields higher GEE than other techniques. In Fig. 2b, for $K = 70$ subbands, it yields 9%, 16%, 22%, and 35% bits/Joule/Hz better GEE than the GPSRM, the QTWOL, the EPA and the RPA, respectively. In Fig. 2|c, for $K = 90$ subbands, the QTPA algorithm yields 9%, 15%, 21%, and 24% bits/Joule/Hz higher GEE than the GPSRM, the QTWOL, the EPA and the RPA, respectively. In Fig. 2b and Fig. 2c, the GEE decreases with increase in subbands K as the circuit power depends on the number of subbands. The GEE decreases after a certain K value as the total power consumed by the system now dominates the increase in sum-rate.

We see from Fig. 2d, where GEE versus number of iterations is plotted, that the algorithm converges in quite a few iterations. This also shows that the QT-based algorithm does not increase complexity. We next plot in Fig. 2e the system sum-rate-GEE relationship for different η values. We see that for a fixed η, increasing the number of sub-bands K, increases both sum-rate and GEE. Further, for a given K value, increasing η value, reduces both GEE and SE.

V. Conclusion

We used a novel quadratic transformation to develop a joint power allocation algorithm to maximize the global energy efficiency (GEE) metric for an OFDM-based non-concurrent two-way relaying (ncTWR). We showed that the proposed algorithm not only uses lesser than the maximum available power but also achieves as high as 35% average GEE over other state-of-the-art algorithms. We also showed that with the increase in the number of subbands, the ncTWR GEE decreases as the circuit power depends on number of subbands.

Appendix

An arbitrary function f(x,y) in two variables x and y, using the first-order Taylor series approximation [11], can be approximated around a point (a, b) as

$$f(x, y) = f(a, b) + \left[\frac{\partial f(x, y)}{\partial x}\right]_{a,b} (x - a) + \left[\frac{\partial f(x, y)}{\partial x}\right]_{a,b} (x - b).$$

We apply the above approximation for the terms $p_r[k]p_b[k]$ and $p_r[k]p_u[k]$ around the points $\tilde{p}_r[k]$, $\tilde{p_u}[k]$ and $\tilde{p_b}[k]$, and $p_r[k]p_b[k] \approx \tilde{p}_r[k]\tilde{p}_b[k] + \tilde{p}_b[k](p_r[k] - \tilde{p}_r[k]) + \tilde{p}_r[k](p_b[k] - \tilde{p}_b[k])$, $p_r[k]p_u[k] \approx \tilde{p}_r[k]\tilde{p}_u[k] + \tilde{p}_u[k](p_r[k] - \tilde{p}_r[k]) + \tilde{p}_r[k](p_u[k] - \tilde{p}_u[k])$.

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