Efficient measurement of high-dimensional quantum states

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Outline

Orbital Angular Momentum of light

• What it means?
• Generation and detection of OAM states of light

Efficient measurement of states of light

• in the orbital angular momentum (OAM) basis.
• in the transverse momentum basis.
Orbital angular momentum (OAM) of light

Paraxial Helmholtz Equation

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik \frac{\partial}{\partial z} \right) E(x, y, z) = 0 \rightarrow \left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + 2ik \frac{\partial}{\partial z} \right) E(\rho, \phi, z) = 0
\]

Solutions are of the form: \( E(\rho, \phi, z) = \psi(\rho, \phi, z)e^{ikz} \)

Laguerre-Gaussian (LG) modes are solutions to paraxial Helmholtz Equation

\[
LG_p^l(\rho, \phi, z) = \frac{C}{(1 + z^2/z_R^2)^{1/2}} \exp \left[ i(2p + l + 1)\tan^{-1} \left( \frac{z}{z_R} \right) \right] \left[ \frac{\rho/2}{w(z)} \right]^l L_p^l \left[ \frac{2\rho^2}{w^2(z)} \right] \times \exp \left[ -\frac{\rho^2}{w^2(z)} \right] \exp \left[ -\frac{ik^2\rho^2z}{2(z^2 + z_R^2)} \right] e^{-il\phi}
\]

Phase of the LG mode with \( p=0 \)

Yao and Padgett, Advances in optics and photonics, 3, 161 (2011)
Orbital angular momentum (OAM) of light

**Paraxial Helmholtz Equation**

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\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik \frac{\partial}{\partial z}\right) E(x, y, z) = 0 \rightarrow \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho}\right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + 2ik \frac{\partial}{\partial z}\right) E(\rho, \phi, z) = 0
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\]

\[
\times \exp \left[-\frac{\rho^2}{w^2(z)}\right] \exp \left[-\frac{ik^2\rho^2z}{2(z^2 + z_R^2)}\right] e^{-il\phi}
\]

**Orbital angular momentum per photon in an LG mode:** \(\hbar l\)

\[
\frac{J_z}{W} = \frac{\hbar l}{\hbar \omega}
\]

Allen et al., PRA 45, 8185 (1992)
Orbital angular momentum (OAM) of light

Paraxial Helmholtz Equation

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\]

Laguerre-Gaussian (LG) modes form a complete set

\[
\sum_{p=0}^{\infty} \sum_{l=-\infty}^{\infty} \rho LG_p^* \rho (\rho, \phi, z) LG_p^l (\rho', \phi', z) = \delta (\phi - \phi') \delta (\rho - \rho')
\]

\[
\int_0^{2\pi} \int_0^\infty \rho d\rho d\phi LG_p^* \rho (\rho, \phi, z) LG_p^l (\rho, \phi, z) = \delta_{l\mu} \delta_{pp'}
\]
Orbital angular momentum (OAM) of light

Paraxial Helmholtz Equation

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\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik \frac{\partial}{\partial z} \right) E(x, y, z) = 0 \rightarrow \left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + 2ik \frac{\partial}{\partial z} \right) E(\rho, \phi, z) = 0
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\times \exp \left[ -\frac{\rho^2}{w^2(z)} \right] \exp \left[ -\frac{ik^2 \rho^2 z}{2(z^2 + z_R^2)} \right] e^{-il\phi}
\]

In many cases, one is concerned only with the OAM index:

\[
LG_p^l(\rho, \phi, z) \rightarrow \psi_l(\phi) = \frac{1}{\sqrt{2\pi}} e^{-il\phi} \equiv \langle \phi | l \rangle
\]

OAM modes form a complete basis

\[
\sum_{l=-\infty}^{\infty} |l\rangle\langle l'| = \delta(\phi - \phi') \quad \text{with} \quad \langle l|l'\rangle = \delta_{ll'}
\]
Orbital angular momentum (OAM) of light

Since OAM modes $\psi_l(\phi) = \frac{1}{\sqrt{2\pi}} e^{-il\phi}$ form a complete basis, we have:

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi},$$

Where, $$\alpha_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \psi(\phi) e^{il\phi} d\phi ,$$

Angular Fourier Relationship

Barnett and Pegg, PRA 41, 3427 (1990)

Angular position

Orbital Angular momentum

OAM provides an infinite dimensional discrete basis

(i) higher allowed error rate in cryptography
(ii) higher transmission bandwidth
(iii) Supersensitive angle measurements
(iv) Fundamental tests of quantum mechanics

Consequences of Angular Fourier Relationship

\[ \alpha_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \psi(\phi)e^{il\phi} \, d\phi = \frac{1}{\sqrt{2\pi}} \int_{-\pi/2}^{\pi/2} e^{il\phi} \, d\phi \]

So,

\[ |\alpha_l|^2 = \left( \frac{\alpha}{2\pi} \right)^2 \text{sinc}^2 \left( \frac{l\alpha}{2} \right) \]

Angular Diffraction

\[ \alpha_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \psi(\phi)e^{il\phi} \, d\phi = \frac{1}{\sqrt{2\pi}} \int_{-\alpha/2}^{\alpha/2} e^{il\phi} \, d\phi + \frac{1}{\sqrt{2\pi}} \int_{-\alpha/2}^{\beta+\alpha/2} e^{il\phi} \, d\phi \]

So,

\[ |\alpha_l|^2 = \left( \frac{\alpha}{2\pi} \right)^2 \text{sinc}^2 \left( \frac{l\alpha}{2} \right) 2[1 + \cos l\beta] \]

Angular double-slit Interference

\[ \alpha = \pi/10 \]
\[ \beta = \pi/4 \]
How to generate OAM modes?

1. Using an spiral phase plate:

   Yao and Padgett, Advances in optics and photonics, 3, 161 (2011)

2. Using an SLM

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Efficient measurement of states of light

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- in the transverse momentum basis.
States in OAM basis: \( \langle \phi | l \rangle = e^{-il\phi} \)

State in the OAM basis (classical)
\[
\psi (\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi}
\]

State in the OAM basis (quantum)
\[
|\psi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l |l\rangle
\]

Pure States

\[
W(\phi_1, \phi_2) = \langle \psi(\phi_1)\psi^*(\phi_2) \rangle_e
\]

\[
= \frac{1}{2\pi} \sum_{l_1, l_2 = -\infty}^{\infty} \langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e e^{-i(l_1\phi_1 - l_2\phi_2)}
\]

Mixed States

When different OAM eigenmodes are uncorrelated.
\( \langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e = S_{l_1} \delta_{l_1, l_2} \)

\[
W(\phi_1, \phi_2) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il(\phi_1 - \phi_2)}
\]

Diagonal Mixed States

\[
W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il\Delta\phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi
\]

Angular Wiener-Khintchine theorem

A K Jha, G S Agarwal, R W Boyd, PRA 84, 063847 (2011)

Aim: Measure the angular correlation function \( W(\phi_1, \phi_2) \)

For diagonal states it yields the OAM spectrum
Existing methods for measuring OAM spectrum of Light


ψ_{in}(φ) → SLM → l=0 → SM fiber → hologram for OAM=−l′

ψ_{in}(φ) → angular double slit

A K Jha, G S Agarwal, R W Boyd, PRA 84, 063847 (2011)
M Malik et al., PRA 86, 063806 (2012).
**Existing methods for measuring OAM spectrum of Light**

State in the OAM basis

$$\psi_{\text{in}}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi}$$

Uncorrelated eigenmodes:

$$\langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e = S_{l_1} \delta_{l_1,l_2}$$

$$W(\Delta \phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il\Delta \phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta \phi) e^{il\Delta \phi} d\phi$$

$$\psi_{\text{out}}(\phi) = \sqrt{k_1} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il(\phi + \phi_1) + i\omega t_1}$$

$$+ \sqrt{k_2} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il(\phi + \phi_2) + i\omega t_2}$$

$$I_{\text{out}}(\phi) = \langle \psi_{\text{out}}(\phi) \psi_{\text{out}}^*(\phi) \rangle_e$$

$$= \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(\Delta \phi) \cos \delta$$

$$\delta \equiv \omega(t_1 - t_2)$$

Visibility:

$$V = \frac{[I_{\text{out}}(\phi)]_{\text{max}} - [I_{\text{out}}(\phi)]_{\text{min}}}{[I_{\text{out}}(\phi)]_{\text{max}} + [I_{\text{out}}(\phi)]_{\text{min}}}$$

$$= \frac{4\pi \sqrt{k_1 k_2}}{k_1 + k_2} W(\Delta \phi) \propto W(\Delta \phi)$$

- By measuring $V$, $W(\Delta \phi)$ can be measured
- From $W(\Delta \phi)$, $S_l$ can be computed

Existing methods for measuring OAM spectrum of Light

\[ I_{\text{out}}(\phi) = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2 W(\Delta\phi)} \cos \delta \]

Limitations:
- Efficiency/purity issues
- Too much loss
- Stringent alignment requirements
- Sensitive to background noise and other experimental parameters
Measuring Orbital Angular Momentum of Light (A new scheme)

State in the OAM basis

\[ \psi_{\text{in}} (\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \]

Uncorrelated eigenmodes:

\[ W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il\Delta\phi} \quad \Rightarrow \quad S_l = \frac{\pi}{-\pi} \int W(\Delta\phi) e^{il\Delta\phi} d\phi \]

A reflection flips the wave-front along the reflection axis

\[ \psi_{\text{out}} (\phi) = \sqrt{k_1} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi+i\omega t_1} \]

\[ + \sqrt{k_2} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{il\phi+i\omega t_2} \]
Measuring Orbital Angular Momentum of Light (A new scheme)

\[ \psi_{\text{in}}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \]

State in the OAM basis

Uncorrelated eigenmodes:

\[ W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il\Delta\phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi \]

\[ \psi_{\text{out}}(\phi) = \sqrt{k_1} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi + i\omega t_1} \]

\[ + \sqrt{k_2} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{il\phi + i\omega t_2} \]

\[ I_{\text{out}}(\phi) = \langle \psi_{\text{out}}(\phi) \psi^*_{\text{out}}(\phi) \rangle_e \]

\[ = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos \delta \]

\[ \delta \equiv \omega(t_1 - t_2) \]

- \( W(2\phi) \) gets encoded in the interferogram.
  So, a single-shot measurement of \( I_{\text{out}}(\phi) \) yields \( W(2\phi) \)
- From \( W(\Delta\phi) \), \( S_l \) can be computed, in a single shot manner.
- Still sensitive to background noise and other experimental parameters
Measuring OAM spectrum of Light (in a noise-insensitive manner)

State in the OAM basis

\[ \psi_{\text{in}}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \]

Uncorrelated eigenmodes:

\[ W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il\Delta\phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta\phi)e^{il\Delta\phi} d\phi \]

\[ I_{\text{out}}(\phi) = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos \delta \]

With noise:

\[ I^\delta_{\text{out}}(\phi) = I^\delta_{\text{n}}(\phi) + \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos \delta \]

\[ \Delta I_{\text{out}}(\phi) \equiv I^\delta_{\text{out}}(\phi) - I^\delta_{\text{out}}(\phi) \]

\[ \Delta I_{\text{out}}(\phi) = \Delta I_{\text{n}}(\phi) + 2\sqrt{k_1 k_2} W(2\phi) (\cos \delta_c - \cos \delta_d) \]

If shot-to-shot noise is the same: \[ \Delta I_{\text{n}}(\phi) = 0 \]

Then:

\[ \Delta I_{\text{out}}(\phi) = 2\sqrt{k_1 k_2} (\cos \delta_c - \cos \delta_d) W(2\phi) \propto W(2\phi) \]

- \[ \Delta I_{\text{out}}(\phi) \] has the same functional form as \[ W(2\phi) \].
- So by measuring \[ \Delta I_{\text{out}}(\phi) \] the spectrum \[ S_l \] can be obtained in a single-shot as well as in a noise-insensitive manner

Experimental measurement of OAM spectrum of Light (classical)

$$\Delta I_{\text{out}}(\phi) \propto W(2\phi)$$

$$\propto \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il\Delta \phi}$$

Experimental measurement of OAM spectrum of Light (classical)

\[ \Delta I_{\text{out}}(\phi) \propto W(2\phi) \]
\[ \propto \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l e^{-il\Delta \phi} \]
Measuring Orbital Angular Momentum of Light (Quantum)

**OAM-Entangled State:**

\[ |\psi\rangle = \sum_{l=-\infty}^{\infty} \sqrt{S_l} \ |l\rangle_s |\bar{l}\rangle_i \]

- \( S_l \) is called the angular Schmidt spectrum
- Very important to have an accurate measurement of \( S_l \)
- The current methods involve coincidence measurements, which is very difficult.

Angular coherence function of the signal photon is

\[ W_s(\phi_1, \phi_2) \rightarrow W_s(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l \ e^{-i l \Delta\phi} \]

- The OAM spectrum of signal photon is same as the angular Schmidt spectrum of the entangled state

A K Jha, G S Agarwal, R W Boyd, PRA 84, 063847 (2011)

G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; *Nature Communications, 8, 1054 (2017)*
Measuring Orbital Angular Momentum of Light (Quantum)


OAM-Entangled State:

\[
|\psi\rangle = \sum_{l=-\infty}^{\infty} \sqrt{S_l} \ |l\rangle_{s} |l\rangle_{i}
\]

\[
W_{s}(\Delta \phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l \ e^{-il\Delta \phi}
\]

Schmidt Number

\[
K = \frac{1}{\sum_{l} S_{l}^{2}}
\]

\[
K = 82.1
\]
Measuring Orbital Angular Momentum of Light (Quantum)

G. Kulkarni, S. Aarav, L. Taneja, and A. K. Jha, Arxiv:1712.03355
States in transverse momentum basis: $\langle \rho | q \rangle = e^{i q \cdot \rho}$

State in the transverse momentum basis

$$V(\rho, z) = \int_{-\infty}^{\infty} a(q) e^{i q \cdot \rho} e^{-\frac{i q^2 z}{2 k_0 z}} dq$$

When the eigenmodes are uncorrelated.

$$\langle a^*(q_1) a(q_2) \rangle_e = I(q_1) \delta(q_1 - q_2)$$

Diagonal Mixed States

$$W(\rho_1, \rho_2, z) = \langle V^*(\rho_1, z) V(\rho_2, z) \rangle_e$$

$$\rightarrow W(\Delta \rho) = \int_{-\infty}^{\infty} I(q) e^{-i q \cdot \Delta \rho} dq$$

Spatial Wiener-Khintchine theorem

- Such partially coherent fields have propagation-invariant spatial correlation function
- The correlation function is the Fourier transform of the spectral intensity
- Partially coherence fields are extremely important for imaging through scattering, etc.


Aim: Measure the spatial correlation function $W(\phi_1, \phi_2)$

For diagonal states it yields the spectral intensity
Efficient generation of spatially partially coherent field

How to produce partially coherent field?

• Most light sources (sunlight, light bulbs, etc.)

• Take a spatially coherent field and introduce randomness to it.

• Start from a planar incoherent source

\[ \begin{align*}
\langle V^* (\rho'_1) V (\rho'_2) \rangle_e &= I_s (\rho'_1) \delta (\rho'_1 - \rho'_2) \\
\langle a^* (q_1) a (q_2) \rangle_e &= I (q_1) \delta (q_1 - q_2)
\end{align*} \]

**Diagonal Mixed States**

\[ W (\Delta \rho) = \int_{-\infty}^{\infty} I (q) e^{-i q \cdot \Delta \rho} \, dq \]

S. Aarav, A. Bhattacharjee, H. Wanare, and A. K. Jha, PRA 96, 033815 (2017)
Efficient generation of spatially partially coherent field

How do we measure the spatial correlation function?

$$\langle V^*(\rho'_1)V(\rho'_2) \rangle_e = I_s(\rho'_1)\delta(\rho'_1 - \rho'_2)$$

$$\langle a^*(q_1)a(q_2) \rangle_e = I(q_1)\delta(q_1 - q_2)$$

Diagonal Mixed States

$$W(\Delta \rho) = \int_{-\infty}^{\infty} I(q)e^{-i \cdot q \cdot \Delta \rho} dq$$

S. Aarav, A. Bhattacharjee, H. Wanare, and A. K. Jha, PRA 96, 033815 (2017)
Efficient generation of spatially partially coherent field

How do we measure the spatial correlation function?

S. Aarav, A. Bhattacharjee, H. Wanare, and A. K. Jha, PRA 96, 033815 (2017)
Conventional methods for measuring spatially partially coherent field

This is not a very efficient method for measuring spatial correlation functions.

How do we measure the spatial correlation function?
Single-shot technique for measuring the spatial correlation function

A reflection flips the wave-front along the reflection axis.

A converging lens flips the wavefront in both x- and y-directions.

A. Bhattacharjee, S. Aarav, and A. K. Jha, Arxiv:1712.04517
A converging lens flips the wavefront in both x- and y-directions.

A. Bhattacharjee, S. Aarav, and A. K. Jha, Arxiv:1712.04517
Single-shot technique for measuring the spatial correlation function

A converging lens flips the wavefront in both x- and y-directions.

A. Bhattacharjee, S. Aarav, and A. K. Jha, Arxiv:1712.04517
Conclusions

• Demonstrated a single-shot technique for measuring the angular correlation function.

• For diagonal mixed states, the angular correlation function yields the OAM spectrum through a Fourier transform.

• The technique can be used for measuring the angular Schmidt spectrum of OAM-entangled states in a single-shot manner without requiring coincidence detection.

• Extended the technique for measuring the spatial correlation function in a single-shot manner.
Acknowledgements

• Initiation grant from IIT Kanpur

• Research grant from SERB, DST

Thank you for your attention
Experimental measurement of OAM spectrum of Light (classical)

\[
\psi_{\text{out}}(\phi) = \sqrt{k_1} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi + i\omega t_1} \\
+ \sqrt{k_2} \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{il\phi + i\omega t_2}
\]

\[I_{\text{out}}(\phi) = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos \delta\]

G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; *Nature Communications, 8, 1054 (2017)*