Exploring Energy-Time Entanglement Using Geometric Phase

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Background: Geometric Phase

- Geometric Phase (Berry’s Phase)

- Pancharatnam’s Phase

- Connection
  M.V. Berry, J. Mod. Opt. 34, 1401 (1987)
  Ramaseshan and Nityananda, Curr. Sci. 55, 1225 (1986)

- Example:
  Tomita and Chiao, PRL 57, 937 (1986)
  Bhandari and Samuel, PRL 60, 1211 (1988)

Pancharatnam’s phase is the geometric phase in polarization optics.
Background: Energy-Time Entanglement

\[ \omega_p = \omega_s + \omega_i \]

Energy conservation

\[ |\psi\rangle = \int d\omega \phi(\omega) |\omega\rangle_s |\omega_p - \omega\rangle_i \]

Energy-Time Entanglement

\[ |\psi\rangle = \int dt f(t) |t\rangle_s |t\rangle_i \]

- Bell inequality of Energy and Time

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left[ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i \right] \]

State

\[ R_{AB} = C [1 + \cos(\phi_s + \phi_i)] \]

Coincidence count rate

Violation of CHSH Bell inequality

Brendel et al., PRL 66, 1142 (1991)
Kwiat et al., PRA 47, R2472 (1993)
Strekalov et al., PRA 54, R1 (1996)
Barreiro et al., PRL 95, 260501 (2005)

Dynamic phase based violation

Franson, PRL 62, 2205 (1989)
Geometric phase based violation

State:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left[ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i \right] \]

Coincidence count rate:

\[ R_{AB} = C \left\{ 1 - \cos \left[ k_0 (x_s + x_i) + 2\beta_s + 2\beta_i \right] \right\} \]

\[ \text{dynamic phase} \quad \text{geometric phase} \]

Jha, O’sullivan, Chan, and Boyd, PRA 77, 021801(R) (2008)
Geometric phase based violation

State: \[ |\psi\rangle = \frac{1}{\sqrt{2}}[ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i ] \]

Coincidence count rate: \[ R_{AB} = C\{1 - \cos[k_0(x_s + x_i) + 2\beta_s + 2\beta_i]\} \]

\[ V = 77\% \quad (> 70.7\%) \]
\[ S = 2\sqrt{2V} = 2.18 \pm 0.04 \]

Violation by 5 standard deviations

Jha, Malik, and Boyd, PRL 101, 180405 (2008)
Conclusions

- Energy-Time Entanglement can be explored using geometric phase

- Potential benefits for quantum information science
  - Geometric phase is non-dispersive (wavelength independent)
  - Ease in introducing small phase shifts
Acknowledgments

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http://www.optics.rochester.edu/~boyd
Background: Bell’s Inequality

- **Bell’s inequality**  J.S. Bell, Physics 1, 195 (1964)
  For showing the incompatibility of local hidden variable interpretations of quantum mechanics

- **CHSH Bell inequality**  Clauser, Horne, Shimony and Holt, PRL 23, 880 (1969)
  Generalization for realizable experiments

- **Experimental violations, polarization entanglement**

\[ |\psi\rangle = \frac{1}{\sqrt{2}} [ |H\rangle_s |H\rangle_i + |V\rangle_s |V\rangle_i ] \]

  Ou and Mandel, PRL 61, 50 (1988)
  Shih and Alley, PRL 61, 2921 (1988)
  Kwiat et al., PRL 75, 4337 (1995)
  Kwiat et al., PRA 60, R773 (1999)
Background: Franson Interferometer

Bell inequality of Energy and Time

Franson, PRL 62, 2205 (1989)

State of the Photons: \[ |\psi\rangle = \frac{1}{\sqrt{2}} [ |l\rangle_s |l\rangle_i + |s\rangle_s |s\rangle_i ] \]

Coincidence count rate: \[ R_{AB} = C [1 + \cos(\phi_s + \phi_i)] \]

Experimental violations:
- Brendel et al., PRL 66, 1142 (1991)
- Kwiat et al., PRA 47, R2472 (1993)
- Strekalov et al., PRA 54, R1 (1996)
- Barreiro et al., PRL 95, 260501 (2005)

Dynamic phase based Violation of the Bell inequality
Experimental Results

Visibility: \( V = 77\% \ (> 70.7\% \) 

Bell Parameter: \( S = 2\sqrt{2V} = 2.18 \pm 0.04 \)

Violation by 5 standard deviations