

Informal commentary on my research papers

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If you are a potential PhD student considering IIT Kanpur, then you might want to see what my research papers are about. Here is an informal commentary on them. Some of the earlier papers are not discussed much.

My papers span a rather broad range of topics. It means fewer papers at the end of the day, a steeper learning curve for each paper, and no big reputation built in any one area. But it is more interesting.

I have not said much about the papers from my own PhD and postdoc work.

1. S. Rakshit and A. Chatterjee. Scalar generalization of Newtonian restitution for simultaneous impact. *International Journal of Mechanical Sciences*, in press. [DOI:10.1016/j.ijmecsci.2015.08.019]

I thought about single-point rigid body impact for my PhD (1993-97). When Sourav Rakshit (now at IIT Madras) was briefly here as a postdoc, I got interested in simultaneous multiple impact.

Complete physical realism, as in accurate prediction of future out-of-sample experimental outcomes with real solid objects, is highly unlikely in rigid body impact models. Existing impact models struggle with more elementary requirements, like not predicting kinetic energy increases, or interpenetration between the rigid bodies, or unreasonable amounts or directions of frictional contact impulses.

In this context, we have a simple, serendipitous, and in my opinion aesthetically pleasing model. It is a new generalization of Newtonian restitution, and a single scalar inequality for all contacts together; and with some other modeling assumptions its predictions match some experiments better than comparable models do.

Clever people have made basic errors in modeling rigid body impacts (like ending up predicting kinetic energy increases, or interpenetration of the bodies, or clearly unreasonable frictional impulses), and in that sense I find our model – which avoids such errors – pleasing.

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2. S. Biswas and A. Chatterjee. A two-state hysteresis model from high-dimensional friction. *Royal Society Open Science*, vol. 2, 2015, 150188. <http://dx.doi.org/10.1098/rsos.150188>
 3. S. Biswas and A. Chatterjee. A reduced-order model from high dimensional frictional hysteresis. *Proceedings of the Royal Society of London A*, vol. 470, 2014, 20130817. [DOI: 10.1098/rspa.2013.0817]

Hysteresis is a rate-independent, highly irreversible, complex and nonlinear type of behavior seen in plasticity, material energy dissipation, magnetism, and other areas. For relatively simpler mechanical elements like nonlinear springs, we can write $F = f(x)$, and fit f with polynomials and the like.

But hysteretic responses involve corners at every direction reversal, and energy dissipating loops (both big and wide, and small and narrow). Just *describing* hysteretic responses is a lot harder. It typically involves nonlinear differential equations with nonsmoothness (Bouc-Wen model) or a geometrical construction that cannot be reduced to ODEs (Preisach model).

We have used an underlying, large, physically-based, frictional system for our initial approach; used a new type of approximation and analysis; and obtained a completely new type of low-order (read: “simple”) system of equations that shows a nice hysteretic response. These, upon fitting a few parameters, can be used for modeling hysteretic components in larger systems (like actuators with hysteresis, or material dissipation in structures). As of October 2015, this is Saurabh Biswas’s ongoing PhD thesis work.

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4. N. Sharma, T. Vimal and A. Chatterjee. Unexpectedly low angular extent of journal bearing pressures: experiment and theory. *Zeitschrift für angewandte Mathematik und Physik (ZAMP)*, vol. 66(2), 2015, 455-471. [DOI: 10.1007/s00033-014-0409-6]

Textbook solutions (i.e., based on simplified models) of journal bearings suggest that the angular extent of the pressure-bearing film is something like 150-180 degrees. But experiments with a cheap setup where the clearance was a bit higher than usual showed angular extents like 50 degrees, and no standard textbook solution predicts anything close. We developed new simple solutions that do predict such small-extent

pressures, and we faced some resistance from journals when we tried to publish them, too. But look at the experimental data, look at our numerical results, and judge for yourself. An initially puzzling observation, reasonably and plausibly explained with simple theory.

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5. B. U. Taskar, D. Dasgupta, V. Nagarajan, S. Chakraborty, A. Chatterjee and O. P. Sha. CFD aided modeling of anti-rolling tanks towards more accurate ship dynamics. *Ocean Engineering*, vol. 92, 2014, 296-303. [DOI: 10.1016/j.oceaneng.2014.09.035]

Ships have a rolling oscillation that can be damped using U-shaped water-filled tanks whose internal oscillation frequency is tuned to that of the ship. The dissipation rate in the anti-rolling tank is a key quantity for effective damping of the ship's oscillations. That damping was estimated in this paper using CFD and some small tricks. The CFD was done by others; I contributed a couple of useful insights to the tricks.

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6. P. Jana and A. Chatterjee. Computational prediction of modal damping ratios in thin-walled structures. *Journal of Sound and Vibration*, vol. 333(26), 2014, 7125-7134. [DOI: 10.1016/j.jsv.2014.08.028]
7. P. Jana and A. Chatterjee. An internal damping formula derived from dispersed elasto-plastic flaws with Weibull-distributed strengths. *International Journal of Mechanical Sciences*, vol. 87, 2014, 137149. [DOI: 10.1016/j.ijmecsci.2014.06.007]
8. P. Jana and A. Chatterjee. Modal damping in vibrating objects via dissipation from dispersed frictional microcracks. *Proceedings of the Royal Society of London A*, vol. 469(2152), 2013, Article number 20120685. [DOI: 10.1098/rspa.2012.0685]

Usual finite element analyses of structures yield natural frequencies and mode shapes, but the modal damping ratios have to be specified by the analyst. To the extent that shape might determine this modal damping (which to some extent it does), computational approaches such as this lead to nice background theory. Prasun Jana got his PhD with me. These papers represent his thesis work.

The key idea is this. The actual dissipation in a cyclically deformed material under arbitrary triaxial stress states depends on complex microscopic physics that is not well enough understood for accurate quantitative prediction. Somewhat simpler empirical constitutive models are needed. We developed such empirical models by assuming some simplified and idealized microscopic dissipation physics, and integrating it appropriately to average out the microscopic detail and get a net macroscopic dissipation. The microscopic dissipation mechanisms studied were (a) frictional microcracks and (b) elastic-then-perfectly-plastic inclusions.

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9. A. Chatterjee. A simple wage-talent curve illustrates several aspects of higher technical education. *Current Science*, vol. 107(2), 2014, 189-194.
10. A. Chatterjee. Better rank assignment in multiple-choice entrance exams. *Current Science*, vol. 105(2), 2013, 193-200.

Some musings on topics relevant to the Indian higher education system.

The first one considers some consequences of a wage-talent curve used by Milind Sohoni of IIT Bombay. Easier to write, and to read, than most of my other papers!

I am still thinking about the second one, considering better approaches.

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11. S. Srivastava and A. Chatterjee. Planar oscillations of a boat in a tank. *International Journal of Mechanical Sciences*, vol. 79, 2014, 152-161. [DOI: 10.1016/j.ijmecsci.2013.11.019]

A boat in water has a well-enough-defined roll oscillation frequency: tip the boat a little to one side, and watch it rock. Mathematically, when we seek natural frequencies of oscillations of, e.g., beams or plates or spring-mass systems, we assume a purely sinusoidal solution. Here, if you put the boat in open water and attempt a planar (2D) analysis, then waves carry energy away to infinity and there *is* no purely sinusoidal solution. If you put the boat in a finite but somewhat large tank, you get a sinusoidal solution, and the frequency varies very little with the size of the tank. The procedure for finding the solution is somewhat interesting (a direct BEM approach I learned from Atanu Mohanty), with small oscillations of an incompressible and inviscid liquid interacting with a rocking rigid body. This represents Shashwat Srivastava's masters thesis.

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12. S. Das and A. Chatterjee. Numerical stability analysis of linear incommensurate fractional order systems. *ASME Journal of Computational and Nonlinear Dynamics*, vol. 8(4), 2013, 041012:1-6. [DOI: 10.1115/1.4023966]

13. S. Das and A. Chatterjee. Simple recipe for accurate solution of fractional order equations. *ASME Journal of Computational and Nonlinear Dynamics*, vol. 8(3), 2013, 031007:1-7. [DOI: 10.1115/1.4023009]
14. S. J. Singh and A. Chatterjee. Unified Galerkin- and DAE-based approximation of fractional order systems. *ASME Journal of Computational and Nonlinear Dynamics*, vol. 6(2), 2011, art. no. 021010. [DOI:10.1115/1.4002516]
15. S. J. Singh and A. Chatterjee. Beyond fractional derivatives: local approximation of other convolution integrals. *Proceedings of the Royal Society of London A*, vol. 466, 2010, 563 - 581. [DOI: 10.1098/rspa.2009.0378]
16. S. J. Singh and A. Chatterjee. DAE-based solution of nonlinear multiterm fractional integrodifferential equations. *Journal Européen des Systèmes Automatisés*, vol. 42(6-8), 2008, 677-688.
17. S. J. Singh and A. Chatterjee. Galerkin projections and finite elements for fractional order derivatives. *Nonlinear Dynamics*, vol. 45, 2006, 83-206.

Some people are interested in fractional order derivatives, and differential equations that contain them. These derivatives, unlike integer order derivatives, involve convolution integrals and so there is the technical difficulty of carrying along the entire history of the solution for evaluating the convolution integral afresh at each time step. Different numerical methods have been proposed for bypassing this difficulty, and I do believe ours is simple, accurate, and now reduced to an easily implementable recipe. Satwinder Jit Singh's PhD thesis and Sambit Das's masters are represented here.

Om Prakash Agrawal of SIU had done something similar for fractional derivatives before us, and we did not know about it; but we have carried our initially-similar approach to a more satisfactory point, I think.

18. A. Bhattacharjee and A. Chatterjee. Dissipation in the Bouc-Wen model: small amplitude, large amplitude and two-frequency forcing. *Journal of Sound and Vibration*, vol. 332(7), 2013, 1807-1819. [DOI: 10.1016/j.jsv.2012.10.026]

Our first attempt to study hysteresis. Our understanding has evolved since then. Arindam Bhattacharjee's masters thesis is represented here.

19. A. Chatterjee and A. Chatterjee. Use of the Fréchet distribution for UPV measurements in concrete. *Non-destructive Testing and Evaluation International*, vol. 52, 2012, 122128. [DOI: 10.1016/j.ndteint.2012.07.003]
20. V. R. Dabiru and A. Chatterjee. A linear S-N curve with load dependent variance and explicit failure probability. *Journal of Testing and Evaluation*, Paper ID: JTE104419, 2012. [DOI: 10.1520/JTE104419]

Engineering applications of statistics. Not a primary interest for me, but something that I think is important. I need to know more statistics, as do most other people.

In the first one, we studied the statistical distribution of ultrasonic pulse velocities in concrete structures. Even in a perfectly healthy structure, the randomness of concrete makes these measurements effectively random variables. 100 measurements at 100 different locations have different values (effectively random). We found a reasonable way to describe the statistics of these values. Possible applications are in structural health monitoring as well as statistical quality control.

In the second one, we proposed a way to incorporate probability of failure in an $S-N$ curve while keeping the curve as a straight line for a chosen probability value, and ensuring reasonable behavior for other probability values (they cannot all be straight lines without being parallel, and experiment shows they are not parallel).

21. K. Nandakumar, M. Wiercigroch and A. Chatterjee. Optimum energy extraction from rotational motion in a parametrically excited pendulum. *Mechanics Research Communications*, vol. 43, 2012, 7-14. [DOI: 10.1016/j.mechrescom.2012.03.003]

Some people are trying to extract energy from waves by mounting whirling pendulums on vertically bobbing floats. The analysis shows that it may be possible to optimize the pendulum motion to extract slightly higher amounts of energy.

22. V. M. Karanam and A. Chatterjee. Common underlying steering curves for motorcycles in steady turns. *Vehicle System Dynamics*, vol. 49(6), 2011, 931-948. [DOI: 10.1080/00423114.2010.483282]

Dimensional analysis, simple asymptotics, and ADAMS simulation combine to unify steer angle and steer torque curves for a given motorcycle. Mangaraju Karanam, a former student, works for TVS Motor Co.

Some other papers with Nandakumar:

23. A. Basak, K. Nandakumar and A. Chatterjee. Decoupled three dimensional finite element computation of thermoelastic damping using Zener's approximation. *Meccanica*, vol. 46(2), 2011, 371-381. [DOI: 10.1007/s11012-010-9318-8]

Heated bodies expand in volume. Conversely, compressed bodies heat up due to thermoelastic coupling. Vibrating bodies with spatially inhomogeneous strains can thus lead to irreversible heat flows, seen externally as thermoelastic damping. This paper has a simple finite element implementation of something that was done analytically for simpler geometries decades ago. Anup Basak moved from IISc to IIT Kanpur to get his PhD in serious solid mechanics with Anurag Gupta. This was a lighter one-off for him, and represents his masters thesis. Nandakumar helped him in the early stages.

24. K. Nandakumar, P. Wahi and A. Chatterjee. Infinite dimensional slow modulations in a delayed model for orthogonal cutting vibrations. *Nonlinear Dynamics*, vol. 62, 2010, 705-716. [DOI: 10.1007/s11071-010-9755-x]

This was an almost direct application of a method of multiple scales analysis for large delays that I developed with Sovan Das (see below). The delay appears due to the regenerative effect present in machining (specifically turning on a lathe), of which we studied a very simple model. For the regenerative effect, see my earlier papers with Pankaj Wahi.

25. K. Nandakumar and A. Chatterjee. Nonlinear secondary whirl of an overhung rotor. *Proceedings of the Royal Society of London A*, vol. 466, 2010, 283 - 301. [DOI: 10.1098/rspa.2009.0262]

Rotors have things called gravity critical speeds, at about half the synchronous whirl speed. We studied the dynamics near gravity critical speeds with nonlinear terms and perturbation methods. Multiple scales analysis leads to somewhat pretty bifurcation diagrams and maybe a bit of new understanding.

26. K. Nandakumar and A. Chatterjee. Continuation of limit cycles near homoclinic points using splines in phase space. *Nonlinear Dynamics*, vol. 57(3), 2009, 383 - 399. [DOI: 10.1007/s11071-008-9449-9]

A somewhat offbeat numerical analysis problem within bifurcation analysis. If you have a numerically determined periodic solution, then you can use numerical continuation to find a family of solutions as a parameter is varied. However, if the solution of interest approaches a homoclinic orbit, then the time period goes to infinity with most of the action within a bounded interval, and some special treatment is needed. This papers develops one way to treat these near-homoclinic solutions.

Papers 24 through 26 are from Nandakumar's PhD thesis.

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27. V. R. Dabiru, V. R. Ranganath, U. Ramamurty and A. Chatterjee. Variable stress ratio in cumulative fatigue damage: Experiments and comparison of three models. *Proc. IMechE Part C, Journal of Mechanical Engineering Science*, vol. 224(2), 2010, 271 - 282. [DOI: 10.1243/09544062JMES1579]

Fatigue testing and data analysis for comparing three models. The load level is not constant during the test, but varies in blocks of piecewise constant load levels.

One of the models compared was the following:

28. N. Patil, P. Mahadevan and A. Chatterjee. A constructive empirical theory for metal fatigue under block cyclic loading. *Proceedings of the Royal Society of London A*, vol. 464(2093), 2008, 1161-1179. [DOI: 10.1098/rspa.2007.0109]

In this paper we made a simple (constructive) assumption. All fatigue test results in the form of $S-N$ curves in handbooks seem to be straight lines on log log plots. So we assumed that, for specimens at all stages of damage, the $S-N$ curves would remain straight lines on log log plots. The mathematical statement of this assumption led to a functional equation. These are often either too hard to solve, or lead to solutions that are too restricted or too unrestricted to be useful. Here we got lucky when we combined the assumption with a simple power law evolution model, and obtained a simple new fatigue damage evolution model with a small number of fitted coefficients, including Miner's rule as a special case (which industry uses heavily).

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29. A. Roy and A. Chatterjee. Vibrations of a beam in variable contact with a flat surface. *ASME Journal of Vibration and Acoustics*, vol. 131(4), 2009, 041010. [DOI: 10.1115/1.3086930].

Take a long, floppy, cantilever beam fixed to a wall, not far above the floor. Some part of the beam stays above ground, and the rest of the sagging beam lies along the floor. Now disturb it slightly. What are small oscillations like? This is a free boundary problem, and represents Arjun Roy's masters thesis.

Although the noncontacting length of the beam changes as it oscillates, there is a way to linearize the equations. A good reduced order model is found as well (unconventional SDOF).

The similar problem where there is no gravity, but reversible adhesion between the beam and the substrate, was also studied. There were some small differences between the two cases.

30. I. Chakraborty, A. K. Mohanty and A. Chatterjee. Localized waves along a line of masses on a plate: propagation and sub-exponential attenuation. *Proceedings of the Royal Society of London A*, vol. 464, 2008, 2229-2246. [DOI: 10.1098/rspa.2008.0051]

An academic problem; seems very classical when I look at the paper again. If you have a periodic structure with no dissipation, it may nevertheless attenuate waves in some frequencies. These waves simply fail to propagate even though there is no damping (think of a perfect mirror: no damping, all reflection, no propagation through).

Usual studies of such structures show that the wave amplitude decays exponentially *in space*. But the system studied here shows power-law attenuation, which is not well known. The methods used are somewhat new for this field (Atanu Mohanty explained the power law and the observed exponent). Ishita Chakraborty's masters thesis.

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31. P. Mahadevan, C. S. Jog and A. Chatterjee. Modal projections for synchronous rotor whirl. *Proceedings of the Royal Society of London A*, vol. 464(2095), 2008, 1739-1760. [DOI: 10.1098/rspa.2007.0139]

We had trouble getting this paper accepted, but got lucky after it went to an adjudicator. The goal was to find the critical speeds of axisymmetric but arbitrary rotors, i.e., not just disks on shafts but things like bottles and funnels. All rotor analyses we knew of added on *ad hoc* gyroscopic terms that were easy to calculate for the disks-on-shafts rotors but not the general rotors we were interested in.

It took us time (which perhaps it should not have, but hindsight is 20-20) to realize that the gyroscopic terms were macroscopic net effects of continuum level pre-stresses set up due to the spin in the rotor. The reviewers claimed that it was *not possible* to compute whirl speeds of rotors without gyroscopic terms, and would not accept our explanation. The adjudicator (may he or she live long and prosper) ruled in our favor. The title of the paper, unfortunately, does not well advertise what I think was the key insight offered. This paper represents Pradeep Mahadevan's PhD thesis. C. S. Jog wrote code and helped with a formulation that helped us verify our results and become sure we were right.

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32. P. Wahi and A. Chatterjee. Self-interrupted regenerative metal cutting in turning. *International Journal of Non-Linear Mechanics*, vol. 43, 2008, 111-123.

If you cut material on a lathe, then the cutting tool vibrates due to forces from the chip. The chip forces depend on chip thickness, which in turn depends on both the instantaneous position of the cutting tool as well as the position of the tool one workpiece revolution ago. This is the famous regenerative effect.

Simple models for tool vibrations in this system predict possible loss of stability as some parameters (like average chip thickness) are varied; the loss of stability is immediately followed by large oscillations (a subcritical Hopf bifurcation). Geometric nonlinearities probably do not play a strong role because tool oscillation amplitudes may not be large. But these amplitudes may be large enough to cause loss of contact with the workpiece, which in turn helps to keep them bounded. Pankaj Wahi did a detailed study of this system, including the loss of contact, as a part of his PhD work.

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33. R. Mourya and A. Chatterjee. Anomalous frictional behavior in collisions of thin disks revisited. *ASME Journal of Applied Mechanics*, vol. 75, 2008, 024501-3. [DOI: 10.1115/1.2793131]

34. D. Joshi, P. Mahadevan, A. Marathe and A. Chatterjee. Unimportance of geometric nonlinearity in analysis of flanged joints with metal-to-metal contact. *International Journal of Pressure Vessels and Piping*, vol. 84(7), 2007, 405-411.

The above two papers: just some work done and reported. Not much to say.

35. A. K. Mohanty, K. Chakraborty and A. Chatterjee. A combinatorial optimization problem for high order PODs with few sensors. *ASME Journal of Vibration and Acoustics*, vol. 129(2), 2007, 252-255.

This problem is really hard, and we did not solve it. We found some heuristic suboptimal solutions, better than nothing.

36. P. R. Basu-Mandal, A. Chatterjee and J. Papadopoulos. Hands-free circular motions of a benchmark bicycle. *Proceedings of the Royal Society of London A*, vol. 463, 2007, 1983-2003.

A long, dense, and difficult paper to write. Some people have liked it. We wrote the equations of motion for a bicycle in two ways that led to matching results. Then we sought circular motions of the hands-free bicycle (rigid rider attached to frame; thin no-slip wheels; no propulsion and no frictional dissipation). We reported various results to many places of decimals, and our position was that all reported digits were correct (hence “benchmark”). Pradipta Basu-Mandal’s PhD thesis, with much help in the writing and some technical guidance as well from the remarkable Jim Papadopoulos. I teach rotations in dynamics in a certain way (my notes are on my web page), and that way helped me do this work reliably with Pradipta.

37. P. K. Tallapragada, A. K. Mohanty, A. Chatterjee and A. G. Menon. Geometry optimization of axially symmetric ion traps. *International Journal of Mass Spectrometry*, vol. 264(1), 2007, 38-52.

38. N. Rajanbabu, A. Chatterjee and A. G. Menon. Motional coherence during resonance ejection of ions from Paul traps. *International Journal of Mass Spectrometry*, vol. 261, 2007, 159-169.

39. N. Rajanbabu, A. Marathe, A. Chatterjee and A. G. Menon. Multiple scales analysis of early and delayed boundary ejection in Paul traps. *International Journal of Mass Spectrometry*, vol. 261, 2007, 170-182.

Three papers with A. G. Menon (who taught us about Paul traps) and his students. The first was based on field calculations for different trap geometries, led by Atanu Mohanty, using a simple and direct BEM code. The next two were analyses of weakly nonlinear oscillations, with significant inputs from me. All three were pretty collaborative on the whole. Rajanbabu was primarily Menon’s student (though formally jointly guided by me). Amol Marathe (now at BITS Pilani) was my PhD student.

40. A. Chatterjee and D. Chatterjee. Analytical investigation of hydrodynamic cavitation control by ultrasonics. *Nonlinear Dynamics*, vol. 46(1-2), 2006, 179-194.

Dhiman Chatterjee (now at IIT Madras) had finished his PhD with Vijay Arakeri. He did experiments where he found that imposing an ultrasonic pressure field could suppress growth in tiny bubbles. The governing equation for a single bubble, which he presented in his seminar, is deeply nonlinear and analytically difficult to attack. But under some approximations, we got meaningful solutions using the method of multiple scales.

41. S. Gorthi, A. Mohanty and A. Chatterjee. Cantilever beam electrostatic MEMS actuators beyond pull-in. *Journal of Micromechanics and Microengineering*, vol. 16, 2006, 1800-1810.

Subrahmanyam Gorthi was Atanu Mohanty’s masters student. If you take a MEMS beam actuator and apply a DC voltage, then as the voltage is increased the beam suddenly gets pulled in and a portion of it becomes flat on the substrate (somewhat like the beam with Arjun Roy above). Assuming a thin dielectric layer exists between the beam and the substrate, we computed various solutions and transition points for different voltages (increasing and decreasing). Sometimes a portion of the beam is flat on the substrate; and sometimes only its tip touches the substrate and pivots there; and so on. Subrahmanyam continued elsewhere for his PhD.

42. A. Marathe and A. Chatterjee. Asymmetric Mathieu equations. *Proceedings of the Royal Society of London A*, vol. 462 (2070), 2006, 1643-1659.

43. A. Marathe and A. Chatterjee. Wave attenuation in weakly nonlinear periodic structures using harmonic balance and multiple scales. *Journal of Sound and Vibration*, vol. 289(4-5), 2005, 871-888.

Two more papers with Amol Marathe; part of his PhD thesis.

The first one was a study of a parametrically forced oscillator whose stiffnesses were different on either side of zero. The Mathieu equation describes, among other things, the dynamics of small lateral motions of a pendulum with a base which is driven vertically in a sinusoidal manner; for that system, imagine that the pendulum is also restricted by a one-side spring. Amol computed detailed stability diagrams through forward simulation, point by point, dividing the job among several PCs (most of them in Bangalore, but at least one in Chennai, operated based on telephoned instructions from Bangalore, by Pradeep Mahadevan’s parents). We were pleased when we demonstrated that numerically computed periodic solutions could be used to easily find the same stability boundaries (as for the usual Mathieu equation, but here there are many more instability regions).

The second one started with me thinking about multiple scales for maps. We worked it out for a weakly nonlinear periodic structure. Later I recalled I had read about the basic method in a book by Mark

Holmes, and then (apparently) forgotten about it for a while.

44. P. Wahi and A. Chatterjee. Asymptotics for the characteristic roots of delayed dynamic systems. *ASME Journal of Applied Mechanics*, vol. 72(4), 2005, 475-483.

45. P. Wahi and A. Chatterjee. Regenerative tool chatter near a codimension-2 Hopf point using multiple scales. *Nonlinear Dynamics*, vol. 40(4), 2005, 323-338.

Two more papers with Pankaj Wahi. Part of his PhD work. The first uses simple asymptotics based on the largeness of the roots themselves. The second studies the vicinity of a double-Hopf point wherein the two frequencies are resonant. Some of my earlier analytical work.

46. P. Wahi and A. Chatterjee. Galerkin projections for delay differential equations. *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 127(1), 2005, 80-87.

Reduced order modeling of delay differential equations (DDEs). DDEs are formally infinite-dimensional, but the dynamics can be effectively low-dimensional. We found a way to write a simple partial differential equation (PDE) that was equivalent to the DDE, and then make low-order approximations for the PDE that led to a small number of ODEs. Also a part of Pankaj Wahi's PhD work.

47. K. Nandakumar and A. Chatterjee. Higher order pseudoaveraging via harmonic balance for strongly nonlinear oscillations. *ASME Journal of Vibration and Acoustics*, vol. 127(4), 2005, 416-419.

Some of my earlier analytical work. Part of Nandakumar's masters thesis.

48. P. Wahi and A. Chatterjee. Averaging oscillations with small fractional damping and delayed terms. *Nonlinear Dynamics*, vol. 38(1-4), 2004, 3-22.

More early analytical work. Part of Pankaj Wahi's PhD thesis.

49. A. Chatterjee. Statistical origins of fractional derivatives in viscoelasticity. *Journal of Sound and Vibration*, vol. 284(3-5), 2005, 1239-1245.

Some materials show frequency dependent responses that suggest fractional order derivatives in the constitutive relation. Fractional order derivatives imply convolution integrals, and hence a memory within the system of past deformations. This is due to internal variables that evolve on multiple time scales, whose net external effect looks like a fractional derivative. I gave a simple demonstration and explanation of this idea, suitable for engineering audiences. The idea is not quite new, but I like the article.

50. K. Nandakumar and A. Chatterjee. Resonance, parameter estimation, and modal interactions in a strongly nonlinear benchmark oscillator. *Nonlinear Dynamics*, vol. 40(2), 2005, 149-167.

A simple benchmark experiment with a strongly nonlinear oscillator. We found that getting a *quantitatively* accurate model is rather difficult if you do not know the correct functional form to use. Part of Nandakumar's masters thesis.

51. K. Nandakumar and A. Chatterjee. The simplest resonance capture problem, using harmonic balance based averaging. *Nonlinear Dynamics*, vol. 37, 2004, 271-284.

Another example of my earlier analytical work, with Nandakumar. Consider a strongly nonlinear conservative oscillator with light damping and weak periodic forcing, and start the oscillator from some large-amplitude initial condition. Initially, damping causes the amplitude to decay. Later, at some amplitude, the natural frequency of the oscillator can match the forcing frequency. Then the oscillator could get captured into resonance and stay at that amplitude, or escape from resonance and go down to a significantly smaller amplitude. The calculations involve approximate realizations of known asymptotic methods.

52. S. L. Das and A. Chatterjee. Second order multiple scales for oscillators with large delay. *Nonlinear Dynamics*, vol. 39, 2005, 375-394.

A harmonic oscillator perturbed by a small term with a large delay. The second order multiple scales analysis gave good results, and required some minor cleverness in teasing the required approximation out. This came out of Sovan Das's masters thesis.

53. A. Chatterjee. Mathematics in Engineering. *Current Science*, vol. 88(3), 2005, 405-414.
An invited article.
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54. S. J. Singh and A. Chatterjee. Non-intrusive measurement of contact forces during vibration dominated impacts. *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 126(3), 2004, 489-497.
In an impact, the contact forces cannot be measured by inserting force transducers; that would change the force being measured. An indirect method was considered in this paper, based on strain gauge measurements away from the contact point. Saiwinder Jit Singh's masters thesis.
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55. A. Chatterjee. The short time impulse response of Euler Bernoulli beams. *ASME Journal of Applied Mechanics*, vol. 71, 2004, 208-218.
A mathematical study of the beam equation. What happens over short times, soon after the beam is struck?
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56. G. T. Abraham, A. Chatterjee and A. G. Menon. Escape velocity and resonant ion dynamics in Paul trap mass spectrometers. *International Journal of Mass Spectrometry*, vol. 231(1), 2004, 1-16.
Another with Menon on Paul traps. Part of Glomin Abraham's masters thesis.
57. G. T. Abraham and A. Chatterjee. Approximate asymptotics for a nonlinear Mathieu equation using harmonic balance based averaging. *Nonlinear Dynamics*, vol. 31, 2003, 347-365.
Another with Glomin Abraham, motivated by Paul traps.
58. A. Chatterjee. Harmonic balance based averaging: Approximate realizations of an asymptotic technique. *Nonlinear Dynamics*, vol. 32, 2003, 323-343.
This paper presented the basic analytical idea used in the previous one.
59. S. L. Das and A. Chatterjee. Multiple scales via Galerkin projections: Approximate asymptotics for strongly nonlinear oscillations. *Nonlinear Dynamics*, vol. 32, 2003, 161-186.
An application to multiple scales what the previous paper did for averaging. This was interesting analytically, and used later in papers with Amol Marathe.
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60. V. R. Sonti and A. Chatterjee. Acausality alleviation via nonlinear future prediction in feedforward control of vibrations. *International Journal of Acoustics and Vibration*, vol. 8(3), 2003, 181-189.
A paper with my former colleague Sonti, based on a small project we tried for ISRO. Did not lead to much, but I think the basic idea is interesting. Suppose there is a noise source far away; it is possible to produce sounds from a nearby speaker such that the net noise level *at your ear* is zero. This is called active noise cancellation. But if the control speaker is far away and the noise source is close by, then the speed of sound limits the method and the noise cancellation does not work: the correction signal cannot arrive in time to cancel the disturbance. This paper presented an attempt to use and address these ideas in a structural control setting, where the disturbance had some predictability although it was not perfectly periodic.
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61. A. Chatterjee and J. P. Cusumano. Asymptotic parameter estimation via implicit averaging on a nonlinear extended system. *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 125, 2003, 11-18.
62. A. Chatterjee, J. P. Cusumano and D. Chelidze. Optimal tracking of parameter drift in a chaotic system: Experiment and theory. *Journal of Sound and Vibration*, vol. 250(5), 2002, 877-901.
63. D. Chelidze, J. P. Cusumano and A. Chatterjee. Dynamical systems approach to damage evolution tracking, Part 1: The experimental method. *ASME Journal of Vibration and Acoustics*, vol. 124, 2002, 250-257.
64. J. P. Cusumano, D. Chelidze and A. Chatterjee. Dynamical systems approach to damage evolution tracking, Part 2: Model-based validation and physical interpretation. *ASME Journal of Vibration and Acoustics*, vol. 124, 2002, 258-264.
65. J. P. Cusumano and A. Chatterjee. Steps towards a qualitative dynamics of damage evolution. *International Journal of Solids and Structures*, vol. 37, 2000, 6397-6417.
66. A. Chatterjee, J. P. Cusumano and J. D. Zolock. On contact-induced standing waves in rotating tires: Experiment and theory. *Journal of Sound and Vibration*, vol. 227(5), 1999, 1049-1081.

67. A. Chatterjee. Asymptotic solution for solitary waves in a chain of elastic spheres. *Physical Review E*, vol. 59(5), 1999, 5912-5919.

68. A. Chatterjee. On the realism of complementarity conditions in rigid body collisions. *Nonlinear Dynamics*, vol. 20, 1999, 159-168.

Papers on work from when I was a postdoc at Penn State with Joe Cusumano. David Chelidze was a PhD student there. John Zolock was a masters student. The last paper grew out of my PhD work with Andy Ruina.

69. A. Chatterjee, R. Pratap, C. K. Reddy and A. Ruina. Persistent passive hopping and juggling is possible even with plastic collisions. *International Journal of Robotics Research*, vol. 21(7), 2002, 621-634.

Hopping things can in principle keep bouncing with no loss of energy, if each impact with the ground occurs at zero speed (gentle landings). A study of such a system.

70. S. L. Das and A. Chatterjee. Multiple scales without center manifold reductions for delay differential equations near Hopf bifurcations. *Nonlinear Dynamics*, vol. 30, 2002, 323-335.

71. S. L. Das and A. Chatterjee. An alternative stability analysis for the simplest walker. *Nonlinear Dynamics*, vol. 28(3), 2002, 273-284.

Early analytical work with Sovan Das. Part of his masters thesis.

72. A. Chatterjee. An introduction to the proper orthogonal decomposition. *Current Science*, vol. 78(7), 2000, 808-817.

Rudra Pratap (see 69 above) invited me to write this one. Several people seem to like it.

73. M. Garcia, A. Chatterjee and A. Ruina. Efficiency, speed, and scaling of two-dimensional passive-dynamic walking. *Dynamics and Stability of Systems*, vol. 15(2), 2000, 75-99.

74. M. Garcia and A. Chatterjee. Small slope implies low speed for McGeer's passive walking machines. *Dynamics and Stability of Systems*, vol. 15(2), 2000, 139-157.

75. J. Calsamiglia, S. W. Kennedy, A. Chatterjee, A. Ruina and J. Jenkins. Anomalous frictional behavior in collisions of thin disks. *ASME Journal of Applied Mechanics*, vol. 66, 1999, 146-152.

76. A. Chatterjee and A. Ruina. Two interpretations of rigidity in rigid body collisions. *ASME Journal of Applied Mechanics*, vol. 65, 1998, 894-900.

77. A. Chatterjee and A. Ruina. A new algebraic rigid body collision law based on impulse space considerations. *ASME Journal of Applied Mechanics*, vol. 65, 1998, 939-951.

78. M. Garcia, A. Chatterjee, A. Ruina and M. J. Coleman. The simplest walking model: Stability, complexity, and scaling. *ASME Journal of Biomechanical Engineering*, vol. 120, 1998, 281-288.

79. M. J. Coleman, A. Chatterjee and A. Ruina. Motions of a rimless spoked wheel: A simple three-dimensional system with impacts. *Dynamics and Stability of Systems*, vol. 12(3), 1997, 139-160.

Papers on rigid body impact (my PhD work) and on walking machines (the PhD work of Mariano Garcia and Mike Coleman), all under Andy Ruina. The work was done when I was at Cornell, though the impact papers were written up later.

80. N. Fitz-Coy and A. Chatterjee. Actuator placement in multi-degree-of-freedom vibration simulators. *Shock and Vibration*, vol. 1(3), 1994, 279-287.

My first paper. From my masters thesis at the University of Florida, with Norman Fitz-Coy. Some matrix algebra and a small optimization problem in there.
