Winning Strategies and Complexity of Nim-Type Computer Game on Plane [1]

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1. Introduction

Nim is a mathematical game involving two players taking turns removing objects from distinct heaps. The basic nim game consists of two players playing with distinct piles of stones removing some stones in each turn.

The author had previously introduced a Nim game with a heap of N stones, where each player is allowed to take at most m stones, provided that he does not repeat the last move of his opponent. The player taking the last stone is the winner and the player who cannot make a feasible move is the looser. This paper is an extension to that. In this paper the author introduces a new Nim-type strategy game on plane.

2. Description

A two player nim game on plane involves choice of a random point on plane say, \((S, L)\) and moves by any player on it can only reduce it to \((S, L-u), (S-u, L), (S-u, L-u)\) \(u > 0\). If a player reduces the point to \((0, 0)\) the other player looses. Thus \((0, 0)\) is a winning position. Similarly it can be proved that \((1, 2), (3, 5)\) are winning positions. There are many properties of winning positions which the author describes. He further demonstrates that the winning strategies of this two-person game are determined a system of equations with two unknown integer sequences and using them he formulates a recursive algorithm to calculate the winning positions less than a particular \(n\). He then improves the algorithm to give an \(O(\log\log n)\) bound for calculating the winning
states. The author also describes and analyzes some other variants of the Game. [2] [3]

3. Review
The paper is very mathematical and straight to the point. The description of the problem is concise and may be hard to understand for those who are unfamiliar with the topic. Some of the proofs given in the paper involve the reader to fill in details which can be quite challenging. It is advised to read some prior materials on the subject for a better understanding of the problem and algorithm described. But for the mathematics lovers the paper is a gem and one would thoroughly enjoy it.

References