# EXTRUSION OF SQUARE BILLET THROUGH COSINE DIE

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## ABSTRACT

An upper-bound analysis is proposed for the extrusion of square sections from square billets through cosine dies having prescribed profiles. Kinematically-admissible velocity fields for the purpose are derived using the dual-stream-function technique. It is shown that a cosine-shaped die with zero entry and exit angles yields the lowest extrusion pressure in the absence of friction, whilst the best upper-bound is provided by a straight tapered die under sticking-friction conditions. The internal work of deformation, however, is still found to be minimum for a straight die for frictionless extrusion.


## INTRODUCTION

Extrusion is the process by which a block of metal is reduced in cross-section by forcing it to flow through a die orifice under high pressure. Extrusion dies are used in the industries for high production rate and accuracy in the metal forming process. There are many factors that affect the extrusion process like die profile, friction factor, extrusion pressure and temperature. The extrusion process is carried out conventionally by shear faced die. But shear faced dies have many practical problems such as dead metal zone, breaking of whiskers, more redundant work and above all the design of shear die is done based on experience and made by trial and error methods. But these methods are approximate and time-consuming methods. The profile of the extrusion dies is the important parameter to optimize the extrusion pressure. In earlier work, Nagpal and Altan [1] used the dual stream functions to obtain upper bound solutions for the extrusion of an ellipse from cylindrical billets. Yang and Han [2] proposed an analytic method for estimating extrusion pressure for arbitrarily curved dies using upper bound solution. The three-dimensional approach for obtaining optimal die shape which produce minimal stress in the extrusion is explained elsewhere. Yang et al. [4] analyzed the forward extrusion of composite rods through curved dies using flow function concept. Narayanasamy, et al. [6] proposed an analytical method for designing the streamlined extrusion dies. In this paper, the extrusion die is assumed to have the cosine profile and an upper bound analysis is proposed for the extrusion of circular section from circular billets. The material flow in the extrusion die does not remain on the same radial plane which contains the longitudinal axis, so that a three-dimensional approach is proposed in this paper.

A number of analytical studies have been carried out during the past few years to compute the deformation loads for the extrusion/drawing of metals through curved dies. Such studies were initiated due to the use of these dies rendering the deformation more homogeneous with consequent reduction in the deformation load. Hence, for metals that are either difficult to form or where the temperature rise is to be minimized to protect the metallurgical structure of the deformed product, these dies can be used with advantage. The geometry of an ideal streamlined wire-drawing die ol perfect efficiency was first proposed by Richmond and Devenpeek [7], the proposed die profile being sigmoidal with zero entry and exit angles, so that no tangential velocity discontinuities were introduced. Further, using slip-line-field analysis, it has been shown that for such dies the load is equal to that for homogeneous compression .

In the preseht investigation, an attempt has been made to derive upper bounds for the extrusion of square sections from square billets using curved dies of prescribed profiles.The dies examined are cosine, elliptic, circular, parabolic and hyperbolic in shape, and kinematically-admissible velocity fields for all these cases have been obtained using the dual-stream-function method proposed by Nagpal and Altan [1].Upper- bound extrusion loads for these dies are computed for a number of reductions and for different friction conditions at the die-metal interface. It is observed that although a cosine-shaped die yields the least upper bound for frictionless extrusion, the internal work of deformation for this die is not necessarily the minimum. Further, for high friction conditions, the best results are still provided by a straight-tapered die.

## DUAL STREAM FUNCTIONS

For ideal fluid flow in three dimensions, Yih[8] suggested the use of two stream functions in place of one as in the case of a two-dimensional flow. Each stream function represents a class of surfaces called stream surfaces. The intersection line of two stream surfaces, one taken from each class, is a three-dimensional stream line.

Let ø1(x, y, z) and ø2 (x, y, z) be two continuous functions satisfying the boundary conditions on velocity. These two functions, therefore, can be treated as a pair of dual stream functions. Following Yih , the velocity components can be derived from these stream functions using the equations:

$$V(x) = (\partial ø2/\partial y)(\partial ø1/\partial z) - (\partial ø1/\partial y)(\partial ø2/\partial z) \tag{1}$$

$$V(y) = (\partial ø2/\partial z)(\partial ø1/\partial x) - (\partial ø1/\partial z)(\partial ø2/\partial x) \tag{2}$$

$$V(z) = (\partial ø2/\partial x)(\partial ø1/\partial y) - (\partial ø1/\partial x)(\partial ø2/\partial y) \tag{3}$$

It can be verified easily that the velocity components determined in the above-mentioned manner identically satisfy the incompressibility condition:

$$(\partial V(x)/\partial x) + (\partial V(y)/\partial y) + (\partial V(z)/\partial z) = 0 \tag{4}$$

Thus, analysis of the flow field for any three-dimensional metal deformation problem reduces the determination of the corresponding dual stream functions satisfying the boundary conditions on velocity.

Let F(z) be the die-profile function such that the die faces in the x-z and y-z planes are represented by x = F(z) and y = F(z), respectively. The function F(z) must satisfy the conditions that F(z) = W at z = 0 and F(z) = A at x = L, where W and A are the semi-widths of the billet and product, respectively, and L is the die length . Further let the dual stream functions ø1 and ø2 be chosen as shown below:

$$ø1 = x/F(z) \tag{5}$$

$$ø2 = W^2 V(b) y/F(z) \tag{6}$$

V(b) is the biilet velocity. It can be verified easily that:(i)ø1= 0 on the plane x = 0 and ø1 = - 1 on the die surface x = F(z); and (ii) ø2 = 0 on the plane y = 0 and on the die surface y = F(z). Such constant values ensure that surfaces x = 0, x = F(z), y = 0 and y = F(z) are stream surfaces and, as such, velocity components normal to these surfaces vanish.

Thus, ø1 and ø2 defined in the above-mentioned manner satisfy all velocity boundary conditions. Hence, they are valid stream functions to generate a kinematically-admissible velocity field. from equation (1),(2),(3),(4), the velocity components in the deformation region are:

$$V(x) = W^2V(b)xF'/F^3 \tag{7}$$

$$V(y) = W^2V(b)yF'/F^3 \tag{8}$$

$$V(z) = W^2V(b)/F^2 \tag{9}$$

where F= F(z) and F'= dF/dz.

THE UPPER-BOUND

The upper-bound theorem states that amongst all kinematically-admissible velocity fields the actual one minimises the expression:

$$J = (2\sigma_o/\sqrt{3})\int\sqrt{(\varepsilon_{ij}\varepsilon_{ij})}dV + (\sigma_o/\sqrt{3})\int\Delta V_s dS + (m\sigma_o/\sqrt{3})\int\Delta V_{sf}dS_r \tag{10}$$

where J is the power dissipation rate
the strain-rate components for the proposed flow lield are written as:

$$\varepsilon_{xx} = (W^2V(b)F')/F^3 \tag{11}$$

$$\varepsilon_{yy} = (W^2V(b)F')/F^3 \tag{12}$$

$$\varepsilon_{zz} = (-2W^2V(b)F')/F^3 \tag{13}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = 0 \tag{14}$$

$$\varepsilon_{yz} = \varepsilon_{zy} = (1/2)W^2V(b)y[(F"/F3) - (3(F')^2/F^4)] \tag{15}$$

$$\varepsilon_{zx} = \varepsilon_{zx} = (1/2)W^2V(b)x[(F"/F3) - (3(F')^2/F^4)] \tag{16}$$

Using Eq. (11-16), J can be evaluated from Eq. (10) when the die-profile function, F is known. For any reduction and friction factor m, J then can be minimized vdth respect to appropriate parameters to yield the best upper bound.

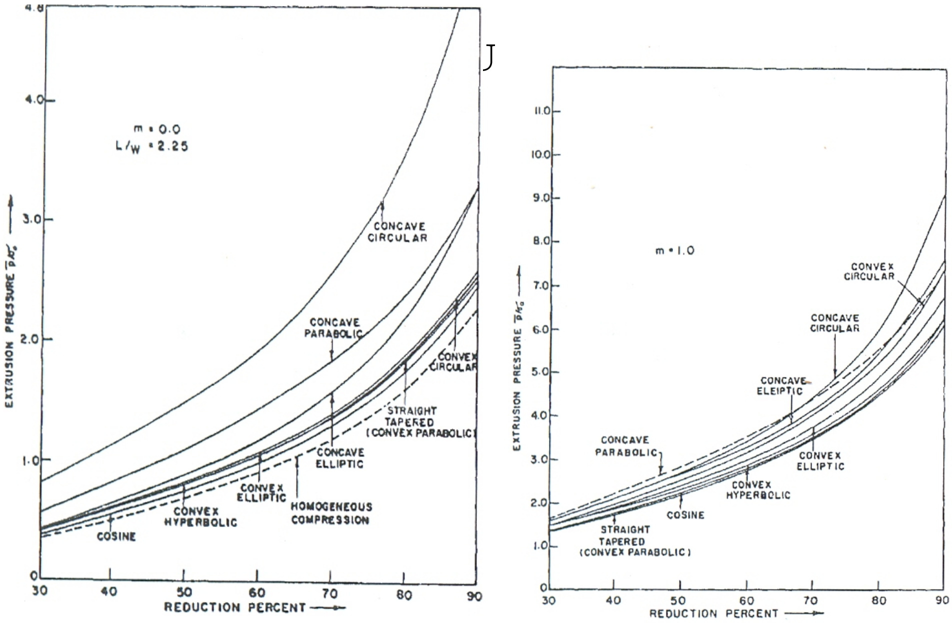


Figure 1: Variation of extrusion pressure with percentage reduction for smoth(m=0) rough die(m=1).

Upper bound solution is theoretically applied for the predication of plastic deformation work (WI) for extrusion of circular billet to circular shape. Similar work was carried out by Gunasekera and Hoshino[5], for extruding polygonal section from circular billet, and by Maity et al[10] to extrude square billet to square shape. Comparison is made between the performance of the extrusion dies designed based on straightly converging dies, concave circular with cosine dies under no frictional condition. It is observed that under no friction condition the cosine profile based die consumes less extrusion pressure compared to straightly converging and concave circular profile based dies. Hence, it is noted that cosine profile based die is superior to straightly converging and concave circular dies. The power consumed due to plastic deformation (WI), die surface friction (WS) and total power consumption are computed for the cosine die Comparison is also made between straightly converging dies and cosine profile based die to study the

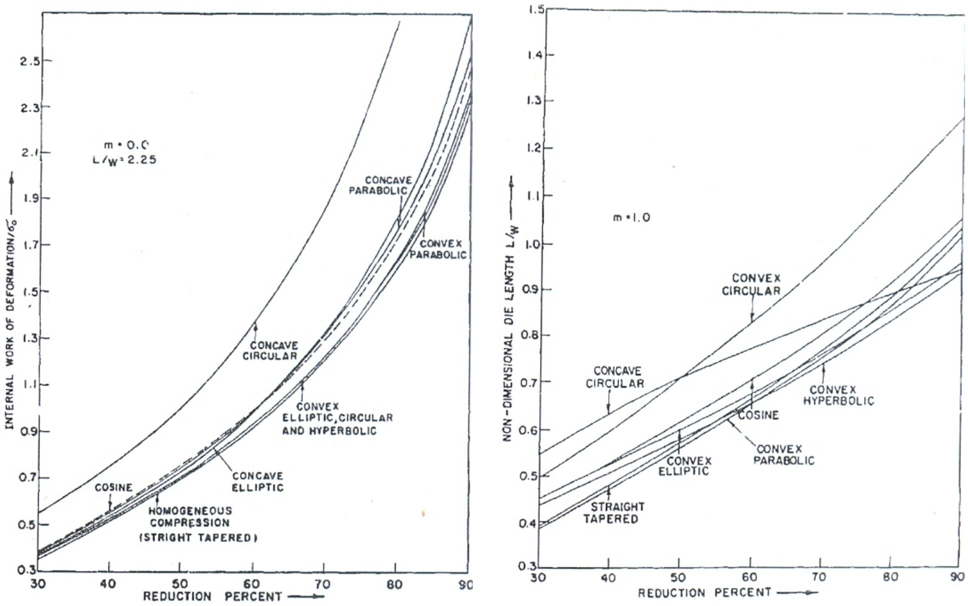


Figure 2: Variation of internal work of decipation for smoth(m=0) non-diamentional length for rough die(m=1) with percentage reduction .

variation of relative extrusion pressure with respect to relative die length. It is further observed that the cosine profile based die needs lower total power consumption, plastic deformation and die surface friction for all the values of relative die length compared to straightly converging die.
Upper-bound loads for the extrusion of square sections from square billets have been computed using the dual-stream-function method for a number of concave and convex dies. It is seen that a cosine die yields the lowest extrusion pressure under frictionless conditions(m = 0), whilst under sticking-friction conditions (m =1.0) a straight-tapered die provides the least pressure.The internal work of deformation is found to be minimum and nearly equal to that for homogeneous compression for a straight tapered die for m = 0. It is also seen that the upper bounds calculated for concave dies are always greater than those for convex dies, due to the greater deformation volumes enclosed by these latter dies.

# CONCLUSION

An upper bound solution has been developed for three-dimensional extrusion of circular sections from a circular bar through cosine die. It is concluded that the extrusion dies based on cosine profile is superior to straightly converging and concave circular profile. It is further observed that the cosine profile based die needs lower plastic deformation work, die surface friction and total power consumption compared to straightly converging die for all the values of relative die length compared to straightly converging die.also It is seen that a cosine die yields the lowest extrusion pressure under frictionless conditions(m = 0), whilst under sticking-friction conditions (m =1.0) a straight-tapered die provides the least pressure.The internal work of deformation is found to be minimum and nearly equal to that for homogeneous compression for a straight tapered die for m = 0. It is also seen that the upper bounds calculated for concave dies are always greater than those for convex dies, due to the greater deformation volumes enclosed by these latter dies.

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