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Acoustic metamaterials hold great potential for attenuation of low frequency acoustic emissions. However, a fundamental challenge is achieving high transmission loss over a broad frequency range. In this work, we report a double negative acoustic metastructure for absorption of low frequency acoustic emissions in an aircraft. This is achieved by utilizing a periodic array of hexagonal cells interconnected with a neck and mounted with an elastic membrane on both ends. An average transmission loss of 56 dB under 500 Hz and an overall absorption of over 48% have been realized experimentally. The negative mass density is derived from the dipolar resonances created as a result of the in-phase movement of the membranes. Further, the negative bulk modulus is ascribed to the combined effect of out-of-phase acceleration of the membranes and the Helmholtz resonator. The proposed metastructure enables absorption of low frequency acoustic emissions with improved functionality that is highly desirable for varied applications. Published by AIP Publishing. https://doi.org/10.1063/1.5022602

Aircraft cabin noise insulation has become imperative to avert health hazards posed to the passengers.¹ The fundamental sources of cabin noise in an airplane while in flight have been identified as the engine and also the turbulent flow of air around the fuselage, which typically generate low frequency excitations. The average sound pressure level measured in the 10–500 Hz frequency range in modern aircraft is approximately 90 dB(A).¹,² This implies that an average reduction of 15 dB(A) or more is required for compliance with the NIOSH 1972 standards.³ Sound insulation has been practiced by the use of traditional porous and fibrous materials,⁴ micro-perforated panels,⁵ gradient index materials,⁶ etc. These materials follow the mass-density law⁷ and necessitate a thick absorbing material for attenuation of low frequency noise creating enhanced payload issues for the airplane. In order to mitigate the above problem, extensive studies have been carried out on sound attenuation through deployment of acoustic metamaterials.

Acoustic metamaterials are artificially engineered structures with properties that may be used for controlling sound wave propagation within them.⁸ The functionality of such metamaterials is solely dependent on the geometry of the structure which is designed to achieve an overall negative set of constituent parameters like mass density and/or bulk modulus. Physically negative constituent parameters of a system signify anomalous response realized at certain frequencies where the system accelerates in a direction opposite to the external sound pressure (negative mass density) and/or expands the medium upon compression (negative bulk modulus).⁹ Negative effective mass density has been achieved through rubber coated solid spheres,¹⁰ membrane-type acoustic metamaterials,¹¹,¹² etc. while an effective bulk modulus has been demonstrated in subwavelength Helmholtz resonators,¹³,¹⁴ balloon-like soft resonating structures,¹⁵ an array of split hollow spheres,¹⁶ etc. In the recent past, research has focused heavily on adapting the concept of negativity of constituent parameters of a structured medium for confinement and dissipation of sound. The double negativity phenomenon can be achieved through an overlap in the frequency response of structures with rotational and polarizing symmetry.¹⁷ The coupling of structures exhibiting such different symmetries is complex due to their distinct response to acoustic waves.¹⁵,¹⁶ Double negative effective acoustic parameters have been observed by fabricating structures such as a composite of an array of thin membranes and side holes,¹⁸ 2D elastic metamaterials,¹⁹ ultraslow-fluid-like particles,²⁰ an array of bubble-contained-water spheres,²¹ perforated hollow steel tubes,²² coupled membranes,²³ 2D anisotropic elastic metamaterials,²⁴ bi-layer plate-type metamaterials,²⁵ etc. Albeit the substantial amount of work done in this field, experimental realization of this phenomenon still remains a technical challenge.

Resonant elements sized at subwavelength scales can endow structures that may realize negative effective parameters in their operating frequency range. Depending on the symmetry of the resonant elements, monopolar and dipolar resonance effects may be created owing to the negative bulk modulus and mass density, respectively. Metastructures with elements exhibiting single negative effective acoustic parameters operate in a very narrow bandgap, generally around the resonant frequency region. On the contrary, metastructures with double negative acoustic parameters offer sound attenuation in a large bandgap.

This study reports a one-dimensional (1D) acoustic metastructure capable of exhibiting double negative effective acoustic parameters. The double negativity characteristics
have been achieved by utilizing a combination of thin elastic membranes and a Helmholtz resonator structure. A unit cell of the metastructure typically comprises two hexagonal cells interconnected with a small neck and mounted with an elastically stretched membrane on both ends. The membranes and the rigid frame are made of natural rubber (Young’s modulus, $E_m = 4$ MPa, density, $\rho_m = 930$ kg/m$^3$, and Poisson’s ratio, $\nu = 0.49$) and 3-D printed polyamide [PA 2200, density ($\rho_p$) $= 440$ kg/m$^3$], respectively. The membrane tension was measured using a previously reported procedure$^{26}$ and was calculated to be 2.1 MPa. The hexagonal cells act as individual Helmholtz resonators sharing a common neck. The in-phase acceleration of the membranes with respect to each other leads to a negative mass density, and the increase in the dynamic volume of the Helmholtz resonator structure with respect to the applied pressure field results in a negative bulk modulus. Double negativity is achieved in these structures by realizing an overlap of resonant frequencies of the above elements. The acoustic metastructure is realized using multiple unit cells placed in parallel to each other [Figs. 1(a) and 1(b)], to establish a proof-of-concept. Figures 1(c) and 1(d) show the 3-D printed metastructure, depicting the twin elastically stretched membranes and the bilayer hexagonal geometry.

The membranes and the Helmholtz resonator structure serve in combination to realize the corresponding dipolar and monopolar resonances. The dipole resonant frequency of the elastic membrane$^{27}$ is obtained as $f_{rm} = 0.2347 \frac{E_m}{\rho_m (1-\nu^2)} = 720.7$ Hz. It should be emphasized that unlike the conventional Helmholtz resonators, the base of the hexagonal cavity resonator in the current configuration is not rigid and instead possesses an elastic behaviour. Thus, when air enters the cavity via the neck region, the membrane develops a paraboloidal shape. Taking this effect into consideration, the monopole resonant frequency of the Helmholtz resonator has been obtained as 1098.52 Hz (supplementary material Note S1). Hereafter, an individual unit cell is modelled and simulated (supplementary material Note S2) since the final metastructure is a periodic repetition of the unit cell. Figure 2(a) reports the simulated transmission loss spectrum (blue solid line) in a range of 25–1600 Hz under normal incidence of sound waves. Two resonant dips ensued at frequencies 745 Hz and 1205 Hz as a result of the in-phase and out-of-phase movement of the membranes. It can be further seen that the second resonant dip overlaps with the calculated resonant frequency of the Helmholtz resonator (deviation < 10%).

The experimental transmission loss is overlaid in the same plot [Fig. 2(a)] through red solid dots. The experimental data closely follow the trend of the simulated results. A good agreement is observed for the experimentally obtained

![FIG. 1. Geometrical design of the proposed metastructure. (a) Schematic of the unit cell. (b) The respective dimensions of the unit cell are hexagon side length ($L$) $= 3.5$ mm, neck length ($l_n$) $= 1$ mm, neck radius ($r_n$) $= 0.25$ mm, thickness of the metastructure ($t$) $= 24$ mm, wall thickness ($t_w$) $= 0.5$ mm, membrane thickness ($t_m$) $= 0.25$ mm, and radius of membrane ($r_m$) $= 3.5$ mm. (c) Photograph of the fabricated metastructure by means of 3D printing. (d) Photograph of the top view of the metastructure without the membrane.](image)

![FIG. 2. Sound transmission loss and absorption and reflection coefficients of the acoustic metastructure. (a) The spectra of sound transmission loss as a function of frequency obtained from experimentations (red solid curve) and simulations (blue solid curve). The dashed olive and pink curves show the simulated transmission loss for different damping losses. (b) The measured reflection coefficient $R$ (black solid curve), absorption coefficient $\frac{1}{R} - |R|^2$ (red solid curve), and the positions of the absorption peak frequencies (745 and 1205 Hz) predicted by simulations (blue arrows).](image)
resonant frequencies (708 Hz and 1168 Hz) which lie within 2%–6% of the theoretically predicted values. The average sound transmission loss is found to be 56.58 dB for frequencies under 500 Hz and 36.97 dB over the entire 25–1600 Hz range. These minor discrepancies in the resonant frequencies can be attributed to the asymmetric vibrational modes of the membranes (supplementary material Note S3). Furthermore, there is a baseline shift in the experimentally observed loss spectrum. If a membrane damping loss factor of \( \eta = 0.4 \) is taken into consideration, the simulated data show good agreement with the experimental results [Fig. 2(a)]. It has further been observed that an increase in the loss factor leads to an increase in transmission loss without introducing any substantial shifts in the resonant frequencies. However, the sharp dip features at the resonant frequencies observed in the experimental results are not exhibited when loss factor is applied. Additionally, we hypothesize the further difference in the transmission loss in terms of overall periodicity of the metastructure. The periodic unit cell configuration results in inter-cavity interactions and near-field coupling effect due to the vibration of walls demarcating the individual unit cells. Laser Doppler vibrometry has been used to investigate the displacement effect of the cavity side wall with respect to frequency (supplementary material Note S3). It is observed that the displacement reaches a maximum (0.048 and 0.023 \( \mu m \)) near the resonant frequencies. This transverse coupling of sound between the cavities may be a major cause of the experimental baseline shifting of the loss spectrum as discussed earlier. A theoretical modelling of this interaction between the neighbouring unit cells could propose a methodology to achieve a transmission loss akin to the experimental results.

The frequency spectrum of the absorption and reflection coefficients of the metastructure is reported in Fig. 2(b). An absorption coefficient level of 0.78 and 0.91 is achieved at the respective resonant frequencies (708 and 1168 Hz) whilst the overall absorption is recorded as 48%. Such a high level of absorption by the metastructure can be accredited to the incorporation of the Helmholtz resonator array serially with a pair of vibrating elastically stretched membranes. The influx and efflux of sound through the neck region lead to the dissipation of acoustic energy. Several other mechanisms may be responsible for the sound absorption phenomenon. Viscous dissipative losses like friction between the air and the internal wall of the metastructure due to wall roughness, compression of air leading to a temperature rise within the metastructure, etc. can also be explored as potential reasons for the absorption phenomenon. However, the above-mentioned mechanisms play a minor role especially with such significantly high levels of absorption.

The double negativity of the proposed metastructure has been explored in terms of the acoustic effective parameters. The effective bulk modulus was simulated as a function of angular frequency using the expression,\(^{11}\)

\[
E_{\text{eff}}(\omega) = E_0 \left[1 - \frac{F_{\text{bulk}}}{\omega^2 - \omega_0^2 \pm i \Gamma} \right]^{-1}
\]

where \( E_0 \) and \( \omega_0 \) are the bulk modulus of ambient air and resonant angular frequency of the resonator, \( F \) is the geometrical factor, and \( \Gamma \) is the dissipation loss in the resonating structure. Assuming ideal conditions, dissipation loss was considered to be zero. The effective mass of the acoustic metastructure was obtained using the formula,\(^{11}\)

\[
\rho_{\text{eff}} = \frac{\langle \sigma_z \rangle}{\langle a_z \rangle}
\]

where \( < > \) denotes the volumetric average of the metastructure, and \( \sigma_z \) and \( a_z \) are the stress and acceleration acting normal to the membrane at rest. The simulated responses of the effective bulk modulus and effective mass are illustrated in Figs. 3(a) and 3(c), respectively. The \( \rho_{\text{eff}} \) switches to negative values at the dipolar resonant frequency ranging from 745 to 1600 Hz as a consequence of the in-phase motion of the membranes [inset Fig. 4(a)]. In a similar fashion, at the monopole frequency of the Helmholtz resonator, \( E_{\text{eff}} \) realizes a negative value in the frequency spectrum of 1105 to 1545 Hz. This flipping of signs can be attributed to the energy accumulation within the resonator. Near the resonant frequency, this energy results in an out-of-phase instantaneous displacement of the centre of mass of the resonator with an increase in the driving pressure field. A negative bulk modulus is also offered due to the monopolar symmetry of the out-of-phase motion of the membranes [inset Fig. 4(b)]. The single negative \( E_{\text{eff}} \) dip confirms the overlap of the resonant frequencies originated because of the motion between membranes and the Helmholtz resonator. Further, single negative bandgaps have been found in two regions of the spectrum, 745 to 1105 Hz and 1545 to 1600 Hz. The first region demonstrates a negative value of mass density with both membranes oscillating in phase. The negative value of the bulk modulus as obtained in the second region corresponds to the out-of-phase motion of the membranes coupled with the Helmholtz resonant behaviour of the top hexagonal cavity. Additionally, a wide double negative bandgap (light grey) is observed in the frequency range of 1105 Hz to 1545 Hz. In this region, \( \rho_{\text{eff}} \) and \( E_{\text{eff}} \) become negative due to in-phase vibration of the membranes coupled to the corresponding Helmholtz resonance effect generated by the bottom hexagonal cavity. The double negativity of the metastructure was further validated by calculating the experimental effective parameters\(^{15,30}\) [Figs. 3(b) and 3(d)]. As is evident from the figures, a wide double negative bandgap (light grey) is observed between 708 Hz and 1168 Hz. A difference in the simulated and experimental double negative frequency region is observed owing to the difference in the resonant dip frequencies as described earlier. The normal displacement of the membrane on the transmission side was measured using a laser Doppler vibrometer. Figure 4 shows a comparison between the simulated and experimentally measured membrane displacements at the respective resonant frequencies. The insets show the simulated mode shapes of the two membranes.

The sound transmission loss spectra can also be explained from the effective parameter’s viewpoint. In the double negative region, the acoustic impedance of the homogenized structure, \( Z = \sqrt{\rho_{\text{eff}} E_{\text{eff}}} \), matches that of air.\(^{31}\) This implies a low sound transmission loss according to the equation,\(^{32}\)

\[
\text{STL} = 20 \log_{10} \left| 1 + \frac{Z_\text{air}}{Z} \right|
\]

where \( Z_\text{air} \) is the acoustic impedance of the metastructure and \( Z \) is the acoustic impedance of air. The low transmission loss can also be attributed to the propagating sound wave, which can be explained in terms of the effective wave vector \( k = \omega \sqrt{\frac{\rho_{\text{eff}}}{\rho_{\text{air}}}} \), which becomes real in the double negative region. The high sound transmission loss at frequencies below the first resonant dip can be justified by the increase in \( \rho_{\text{eff}} \) at lower frequencies. At these frequencies, the
The decaying length of the evanescent waves decreases according to the relation \( \Delta d \propto \left| \frac{1}{\rho_{\text{eff}}} \right|^{1/2} \), resulting in a higher loss. Therefore, by tuning the resonant frequencies of the membrane and the Helmholtz resonator, double negativity can be achieved in a wide bandgap region.

The proposed metastructure has been designed for sound attenuation purposes specifically for the reduction of airplane cabin noise. In order to ascertain the feasibility of the designed acoustic metastructure for aeronautical applications, the metastructure’s tunability was further studied.

Realization of the proposed metastructure for practical applications requires it to have a tunable operating frequency range. As mentioned earlier, such metastructures are strongly dependent on their geometrical parameters. Rigorous simulations were carried out to identify the effect of different geometrical parameters on the two resonant frequencies (supplementary material Note S4). It was observed that while the first resonant frequency could be tuned by altering either the neck length or neck radius, the second resonant frequency was primarily dependent on the hexagonal cavity height and the membrane thickness. Hence, different combinations of the above geometrical parameters could help achieve a flexible and wide band low frequency operating range.

Moreover, design of experiments was carried out to tune the sound transmission loss of the metastructure. Next, the Taguchi method and genetic algorithm were used to identify the optimized geometrical parameters to achieve maximum average sound transmission loss over the frequency range of 25–1600 Hz. In addition, the analysis of variance (ANOVA) method was incorporated in the study to recognize the most significant parameters. It was seen that the hexagonal side length was the most significant parameter, \( p < 0.05 \) (supplementary material Note S5). Also, at the expense of weight, increasing the membrane thickness could result in an increase in the sound transmission loss.

We have reported an acoustic metastructure for attenuation of low frequency aircraft acoustic emissions. An average transmission loss of 45 dB(A) (i.e., 56.58 dB) has been attained for frequencies below 500 Hz which satisfactorily conforms to the NIOSH standards. It has been demonstrated that coupling between membranes and the Helmholtz resonator can lead to the realization of a wide double negative acoustic bandgap. The subwavelength metastructure is capable of exhibiting both monopolar and dipolar resonances.

FIG. 3. Experimental and simulation results of effective acoustic parameters. Effective bulk modulus having a negative value in the frequency range from (a) 1105 to 1545 Hz (simulated) (b) up to 1168 Hz (experimental) due to monopolar resonance. Effective dynamic mass having a negative value in the frequency range from (c) 745 to 1600 Hz (simulated) and (d) 708 to 1600 Hz (experimental) due to dipolar resonance. The insets in (a) and (c) show the effective parameter spectra over the double negative frequency range of 1105 Hz to 1545 Hz. The grey-shaded area illustrates the double negative region.

FIG. 4. Experimental and simulation results of the displacement profiles of the membranes as a function of frequency. The normal displacement of the top and bottom membranes at (a) 708 and 745 Hz and (b) 1168 and 1205 Hz. The insets show the simulated vibrational modes of the two membranes at the resonant frequencies.
The high transmission loss in conjunction with a flexible operating frequency range makes the metastructure suitable for lowering of aircraft cabin noise.

See supplementary material for the calculation of the resonant frequency of the Helmholtz resonator (Note S1), membrane and side wall displacement of the metastructure (Note S2), resonant frequency tunability (Note S3), and sound transmission loss enhancement (Note S4).

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