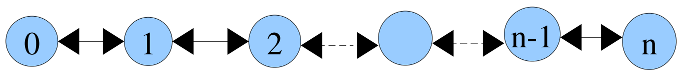
## Home Work-1:

**Question 1: Probability**

A particle moves along the following graph so that at each step it is equally likely to move to any of its neighbors (immediate). From state 0, it can ofcourse go only to state 1. Starting at 0, show that the expected number of steps it takes to reach n is $n^2$.

[Hint: Use recursion on the expected value of an appropriately defined random variable]

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**Question2: Poisson Process**

Trains arrive at a certain station A according to the poisson process with rate a. If you take the train from the station A, it takes a time T, measured from the time at which you enter the train to arrive home. If you walk directly from the train station A, it takes a time W to arrive home. Suppose your policy when arriving at the train station is to wait up to time “r”, and if a train has not yet arrived by that time, then you walk home.

a) Compute the expected time from when you arrive at the station until you reach home.

b) Show that if $W < 1/a + T$, then the expected time of part (a) is minimized by setting $r=0$; if $W > 1/a +T$, then it is minimized by setting r = infinity. When $W = 1/a +T$, all values of r give the same expected time.

c) Explain why we need only consider the cases $r=0$ and r=infinity when minimizing the expected time.

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**Question3: Markov Chains**

Tom is chasing Jerry and moves between location a and b according to a Markov Chain with transition matrix as below. Tom starts at initial location a.

a b
a [0.7 0.3]
b [0.3 0.7]

Jerry unaware of Tom starts at initial location b and moves according to a Markov chain with transition matrix

a b
a [0.4 0.6]

b [0.6 0.4]

This chase ends when Tom catches Jerry. And this happens when they meet in the same location.

a) Show that the progress of the chase except for knowing the location where it ends, can be described by a three-state Markov chain where one absorbing state represents chase ended and the other two that Tom and Jerry are at different locations. Obtain the transition matrix for this chain.
b) Find the probability that at time “n” Tom and Jerry are both at their initial locations.
c) What is the average duration of the chase.

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**Question 4: Markov Chains**

Consider a cube with vertices [000, 001, 010, 100, 110, 101, 011 and 111].
Suppose a fly walks along edges of the cube from vertex to vertex. For an integer n>=0, let $X_n$ denote which vertex the fly is at time n.
Assume X = ( $X_n$ : n>=0) is a discrete time markov chain such that given $X_n$, $X_{n+1}$ is equally likely to be one of the 3 vertices neighboring $X_n$.
a) Sketch the one step transition probability diagram for $X_n$
b) Let $Y_n$ denote the distance of $X_n$ measured in number of hops between vertex 000 and $X_n$. E.g if $X_n$ = 101 then $Y_n$ = 2.
Sketch the one step probability diagram of Markov process Y= ( $Y_n$ : n>=0)
c) Suppose the fly begins at vertex 000 at time zero. Find the expected value of the first passage time (E[T] as defined in class) i.e average time (steps) for the fly to return back to vertex 000 for the first time.

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**Question 5: Stationary Distribution**

Consider the Markov chain with state space S = {0, 1, 2} and transition matrix
P as
[.4 .4 .2]
[.3 .4 .3]
[.2 .4 .4]
Show that this chain has a unique stationary distribution and compute it.