Homework-2:

Question1: Birth-Death Process

Consider a shop consisting of M machines and a single mechanic. Suppose the amount of time a machine runs before breaking down is exponential with parameter a. When a breakdown occurs, a request is sent to the mechanic. Requests are buffered. It takes an exponentially distributed amount of time with parameter m for the mechanic to fix the machine. If X(t) is the number of machines up at time t, model X(t) as a birth-death process i.e specify the infinitesimal generator matrix (A).

Question2: Birth-Death Process

A communication node **A** receives Poisson packet traffic from two other nodes, 1 and 2, at rates λ_1 and λ_2 respectively and transmits it on a first-come-first-serve basis using a link with Capacity C bits/sec. The two input streams are assumed independent and their packet lengths are identically and exponentially distributed with mean L bits. A packet from node 1 is always accepted by **A**. A packet from node 2 is accepted only if the number of packets in **A** (in queue or under transmission) is less than a given number $K > 0$; otherwise, it is assumed lost.

1. What is the range of values of λ_1 and λ_2 for which the expected number of packets in **A** will stay bounded as time increases?
2. For λ_1 and λ_2 in the range of part(1), find the steady state probabilities of having n packets in **A**.
3. Find the average time needed by a packet from node 1 to clear **A** once it enters **A**.
4. Repeat part(3) for node 2.

Question 3: Birth-Death Process

Ram and Shyam operate a barber shop. The shop has two chairs where they can server customers separately. There is also an extra chair where another customer can wait. The waiting customer is served by the first barber to be free. Arriving customers finding 3 customers in shop, leave immediately.

Suppose customers arrive according to poisson process at a rate of 8/hr. Service times are mutually independent and independent of arrival process. Service time is exponentially distributed with mean 15 min.

1. Model the above process as a birth-death process and find the steady-state distributions.
2. What portion of the time the barber shop is empty in the long run?
3. What is the expected number of customers in the barber shop in steady state?
4. What is the long run portion of arrivals that get served?
5. How much more business would they do (what portion of customers will now be served compared to before) if instead of 2 barbers and 1 waiting space, they had 3 barbers and no extra waiting space.

Question 4: M/M/1

Consider an M/M/1 system with parameters λ, μ. Upon arrival to this system, customers estimate their approximate waiting time W ($= k/\mu$, when the arrival finds k in the system). They join the queue with probability $e^{-\alpha W}$ (or leave the system with probability $1 - e^{-\alpha W}$). Assume $\alpha \geq 0$.

1. Find the system steady state probability that there are k customers in the system.
2. Under what conditions will the steady state solution hold.
3. For $\alpha \to \infty$, find the average number in the system.

Question 5: M/M/∞

Consider an M/M/∞ queue with servers numbered 1,2,...... There is an additional restriction that upon arrival a customer will choose the lowest-numbered server that is idle at that time. Find the fraction of time each server is busy.