Homework-4:

Question 1:

Consider an n-class, non-preemptive priority system

1. Show that the sum $\sum_{k=1}^{n} \rho_k W_Q^k$ is independent of the priority order of classes and in fact given by,

   $\sum_{k=1}^{n} \rho_k W_Q^k = \frac{R\rho}{1-\rho}$

   where $\rho = \rho_1 + \rho_2 + \ldots + \rho_n$.

2. Suppose that there is a cost c_k per unit time for each class k customer that waits in queue. Show that cost is minimized when classes are ordered so that

   $\frac{\overline{X_1}}{c_1} \leq \frac{\overline{X_2}}{c_2} \leq \ldots \leq \frac{\overline{X_n}}{c_n}$

   [Hint: Express cost as $\sum_{k=1}^{n}(c_k/\overline{X_k})(\rho_k W_Q^k)$ and use part a. Also use the fact that interchanging the order of any two adjacent classes leaves the waiting time of all other classes unchanged.]

Question 2:

Consider a preemptive resume priority system with n priority classes. All priority classes have exponentially distributed service times with common mean $1/\mu$.

1. Let $W_Q^{[K]}$ be the average time in queue averaged over the first k priority classes. Show that this quantity is the same as the waiting time in queue of an M/M/1 system with arrival rate $\lambda_1 + \ldots + \lambda_k$ and mean service time $1/\mu$.

2. Given we know the waiting time in queue of an M/M/1 system, use the above result to obtain the average waiting time in queue of a k^{th} priority class customer.

Question 3:

Consider an M/G/1 queue at equilibrium, where the server goes on vacations of random length whenever the system becomes empty. Model this system using Imbedded Markov Chain (just as we did in class for an M/G/1 system) i.e. determine the number of customers left behind in the system at departure instants. (Hint: You will need to define a new term that captures the number of customers waiting in service when a busy period starts.)