Question 2

a) Fraction of time machine is not serving customers $= (1-\rho)$

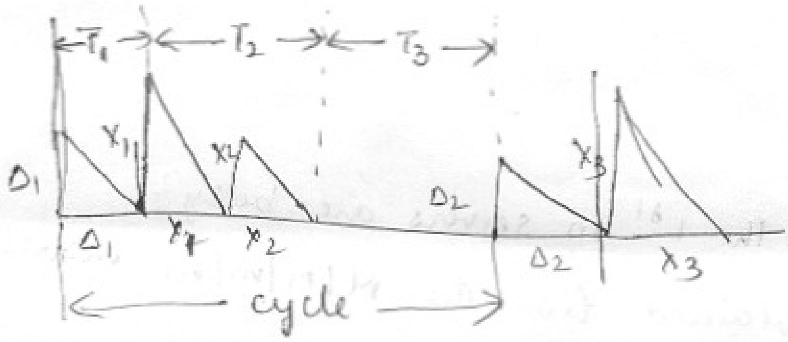
out of this fraction, the fraction of time the server is idle is $\frac{1/\lambda}{1/\lambda + \bar{\Delta}} = \frac{1}{1+\lambda\bar{\Delta}}$

$\therefore$ P{the server is idle} $= (1-\rho) \cdot \frac{1}{1+\lambda\bar{\Delta}}$

b) out of $(1-\rho)$ fraction of time, the fraction of time server is in setup phase is $\frac{\bar{\Delta}}{1/\lambda + \bar{\Delta}} = \frac{\bar{\Delta}\lambda}{1+\lambda\bar{\Delta}}$

$\therefore$ P{the server is in setup phase} $= \frac{(1-\rho)\cdot\bar{\Delta}\lambda}{1+\lambda\bar{\Delta}}$

c)



$$\rho = \frac{T_2}{T_1 + T_2 + T_3}$$

Avg. length of busy period $= T_1 + T_2 = B$

$$\therefore \quad \rho = \frac{T_1 + T_2 - T_1}{T_1 + T_2 + T_3} = \frac{B - T_1}{B + T_3}$$

$T_1 = \overline{\Delta}, \quad T_3 = 1/\lambda$

$$\therefore \quad \rho = \frac{B - \overline{\Delta}}{B + 1/\lambda} \Rightarrow B = \frac{\overline{X} + \overline{\Delta}}{(1-\rho)}, \quad \rho = \lambda \overline{X}$$

d) The waiting time is composed of four components

- If the ~~[illegible]~~ server is busy serving customers $\{P_r = \rho\}$, ~~on~~ on arrival, it has to wait for a mean residual service time ($\frac{\overline{X^2}}{2\overline{X}}$)
- If the ~~[illegible]~~ server is ~~[illegible]~~ idle on arrival (P_r is $\frac{(1-\rho)}{1+\lambda\overline{\Delta}}$), it has to wait on average for setup i.e. $\overline{\Delta}$
- If the server is in setup phase on arrival (P_r is $\frac{\overline{\Delta}\lambda}{1+\lambda\overline{\Delta}}$), it has to wait for a mean residual setup time $\frac{\overline{\Delta^2}}{2\overline{\Delta}}$

- Customers ahead of it in queue

$(N_Q \cdot \bar{X})$

$$W_Q = N_Q \bar{X} + \rho \cdot \frac{\overline{X^2}}{2\bar{X}} + \frac{(1-\rho)}{1+\lambda\bar{\Delta}} \cdot \bar{\Delta}$$

$$+ (1-\rho)\left(\frac{\cancel{\bar{\Delta}}\lambda}{1+\lambda\bar{\Delta}}\right) \cdot \frac{\overline{\Delta^2}}{2\cancel{\bar{\Delta}}}$$

Since $N_Q = \lambda \cdot W_Q$

$$W_Q(1-\rho) = \frac{\lambda\overline{X^2}}{2} + \frac{(1-\rho)}{2(1+\lambda\bar{\Delta})}\left(2\bar{\Delta} + \lambda\overline{\Delta^2}\right)$$

$$\Rightarrow W_Q = \frac{\lambda\overline{X^2}}{2(1-\rho)} + \frac{2\bar{\Delta} + \lambda\overline{\Delta^2}}{2(1+\lambda\bar{\Delta})}$$

Question 2

a) we have $\lambda_1 = r_1 + \lambda_3$

$$\lambda_2 = r_2 + \lambda_1 + 0.5\lambda_2$$

$$\lambda_3 = 0.2\lambda_2$$

Solving, $\lambda_1 = \frac{19}{6}$, $\lambda_2 = \frac{25}{3}$, $\lambda_3 = \frac{5}{3}$

we have $\rho_1 = \frac{19}{54}$, $\rho_2 = \frac{5}{12}$, $\rho_3 = \frac{5}{12}$

$$N_1 = \frac{\rho_1}{1-\rho_1} = \frac{19}{35}, \quad N_2 = \frac{\rho_2}{1-\rho_2} = \frac{5}{7}, \quad N_3 = \frac{5}{7}$$

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$$T = \frac{19/35 + 2 \cdot 5/7}{r_1 + r_2} = \underline{\underline{0.7885}}$$

b) False. Recalculating. $N_1 = \frac{19}{8}$, $N_2 = N_3 = 5$

Thus N is not doubled.

c) False. The ρ's are same as before, however since $r_1 + r_2$ is changed, the expected delay is different.

Question 3 4

From figure 1, we have

$$\overline{\lambda_1} = \overline{\lambda_2} = \overline{\lambda_3} = \lambda \text{ (say)}$$

let $\lambda = \mu$, $\therefore$ $\rho_1 = \rho_2 = 1$, $\rho_3 = 1/2$

a) $$G(M) = \sum_{n_1+n_2+n_3=M} \rho_1^{n_1} \rho_2^{n_2} \rho_3^{n_3} = \sum_{n_1+n_2+n_3=M} \frac{1}{2^{n_3}}$$

$$= \sum_{n_3=0}^{M} \frac{1}{2^{n_3}} \sum_{n_1+n_2=M-n_3} 1$$

$$= \sum_{n_3=0}^{M} \frac{1}{2^{n_3}} \binom{M-n_3+1}{M-n_3} = \sum_{n_3=0}^{M} (M-n_3+1) \frac{1}{2^{n_3}}$$

$$= (M+1) \sum_{n_3=0}^{M} \frac{1}{2^{n_3}} - \frac{1}{2} \sum_{n_3=1}^{M} n_3 \cdot \frac{1}{2^{n_3-1}}$$

have $$\sum_{n=1}^{N} n x^{n-1} = \frac{\partial}{\partial x} \sum_{n=0}^{N} x^n = \frac{N x^{N+1} - (N+1) x^N + 1}{(x-1)^2}$$]

$$\therefore \quad G(M) = (M+1)\frac{(1-(1/2)^{M+1})}{1-1/2} - \frac{1}{2}\,\frac{M(1/2)^{M+1} - (M+1)\left(\frac{1}{2}\right)^{M} +}{(1-1/2)^2}$$

$$= \frac{1+2^{M+1}\cdot M}{2^M} = 2^{-M} + 2M$$

b) $$\lambda_i(M) = \overline{\lambda_i}\cdot\frac{G(M-1)}{G(M)} = \mu\left(\frac{2^{-M+1} + 2(M-1)}{2^{-M} + 2M}\right), \quad i = 1,2,3$$

c)

M	$G(M)$	$\lambda_i(M)$
0	1	0
1	$5/2$	$2/5\,\mu$
2	$17/4$	$10/17\,\mu$
3	$49/8$	$\frac{34}{49}\mu$
4	$129/16$	$\frac{98}{129}\mu$
5	$321/32$	$86/107\,\mu$

d) $$\overline{\lambda_1} = \frac{\overline{\lambda_3}}{2}, \quad \overline{\lambda_2} = \frac{\overline{\lambda_3}}{2}, \quad \overline{\lambda_3} = \overline{\lambda_1} + \overline{\lambda_2}$$

Let $\overline{\lambda_1} = \mu$, $\rho_1 = \rho_2 = 1$, $\rho_3 = 1/2$

Since ρ_i's are same as the network for figure 1, they have same stationary distribution