Q1.1 :

For the M.C to be irreducible, it is essential that $a>0$ & $p>0$ (otherwise they can't communicate).

For $a<1$ & $p<1$, the M.C is both irreducible & periodic.

When $p=1$, it is essential that a is not equal to 1. If it were, it would become periodic. Similarly for $a=1$, p should not be equal to 1.

Overall conditions are

$$0<p\leq 1 \quad \& \quad 0<a\leq 1$$

$$\text{except } \alpha = p = 1$$

Q 1.2: Let $A(K)$ be a r.v such that $A(K)=1$ if an arrival occurs in slot K & 0 otherwise.

Let $X(K)=\{A(K), A(K-1)\}$ be a M.C with four states corresponding to 00, 01, 10 & 11.

The probability transition matrix is

$$P=\begin{bmatrix} 0.9 & 0 & 0.1 & 0 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.4 & 0 & 0.6 \\ 0.2 & 0 & 0 & 0.8 \end{bmatrix}$$

Let Π be the steady state prob. matrix

i.e $\Pi = [\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11}]$

The arrival rate is equal to $\pi_{10} + \pi_{11}$

solving $\Pi P = \Pi$ gives $\Pi = [6/11, 1/11, 1/11, 3/11]$

$\therefore$ arrival rate $= 4/11$

Q 1.3

Since departures are immediately replaced by a new customer, the departure rate is the same as the arrival rate to the system.

Since the system is always full, departure rate $= \dfrac{K}{\overline{X}}$ (K servers)

$\therefore$ arrival rate $= K/\overline{X}$

From Little's theorem $W = \dfrac{L}{\lambda}$

when $\lambda = K/\overline{X}$, $L = N$ (given)

$$\therefore \quad W = \frac{N\overline{X}}{K}$$

Q1.4:

The effective arrival rate to this system

$= \lambda(1-P_K)$

effective reject rate $= \lambda \cdot P_K$

arrivals bring 5 Rs profit & rejects loose 1 Rs.

$\therefore$ To break even, we need

$$\cancel{\lambda}(1-P_K)\cdot 5 = \cancel{\lambda}\cdot P_K \cdot 1$$

$$\Rightarrow P_K = 5/6.$$

From class, P_K for a M/M/1/2 system is

$$\frac{(1-\rho)\rho^K}{1-\rho^{K+1}}, \quad K = 2$$

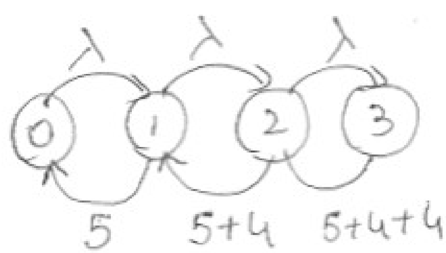
$$= \frac{(1-\rho)\rho^2}{1-\rho^3} = \frac{\rho^2}{1+\rho+\rho^2} = 5/6$$

solving for ρ gives

$$\rho = \frac{\lambda}{\mu} = \frac{5 + \sqrt{45}}{2}$$

Q2:

For this model we have



$\lambda = 6/\text{hr}.$

$\mu_1 = 5, \ \mu_2 = 5+4 = 9$

$\mu_3 = 5+4+4 = 13$

When there is 1 customer in system, he departs at the service rate $(=5)$

When there are 2 customers, the one undergoing service depart at the rate of 5. The other, waiting will depart at the rate of 4. (abandon)

Similarly for 3 customers, it will be $5 + 2 \times 4 = 13$.

Q2.2 This is ~~solution~~ nothing but π_0 (steady state prob of zero customers in the system)

Solving the global balance eq. will give

$$\pi_0 = \frac{65}{219}, \quad \pi_1 = \frac{78}{219}, \quad \pi_2 = \frac{52}{219}, \quad \pi_3 = \frac{24}{219}$$

Q2.3.

$$(\pi_1 + \pi_2 + \pi_3) \cdot 5 = 3.515$$

Q2.4.

$\pi_2 \cdot 4 + \pi_3 \cdot 8$ is the abandonment rate

$= 1.83$

$$\% \text{ abandonment} = \frac{1.83}{6\,(=\lambda)} \times 100 = 30.5\%$$

A quick check

$$\underbrace{(1-\pi_3)\lambda}_{\text{arrival into the system}} = \underbrace{3.515 + 1.83}_{\text{overall departure rate}}$$