Practice Problems:

Question 1:

Customers arrive at a fast food restaurant at a rate of 5 per minute and wait to receive their order for an average of 5 minutes. Customers eat in the restaurant with probability 0.5 and carry out their order without eating with probability 0.5. A meal requires an average of 20 minutes. What is the average number of customers in the restaurant?

Question 2:

IITK sports facility has 4 tennis courts. Players arrive at the courts at a Poisson rate of one pair per 10 min and use a court for an exponentially distributed time with mean 40 min. Suppose that a pair of players arrives and finds all courts busy and k other pairs waiting in queue. How long will they have to wait to get a court on the average?

Question 3:

Consider an M/D/1 queue with service time equal to b time units. Suppose further that one is able to determine the system size when time is a multiple of b. Determine the transition matrix of the Markov chain of $\{X_n, n = 0, 1, 2....|X_n$ equal to the system size at time $t = nb\}$.

Question 4:

Consider a three station queueing system (single server at each station) with Poisson input (parameter λ) and exponential service (parameters μ_1, μ_2 and μ_3). There is no capacity limit on the queue in front of the first two stations, but at the third, there is a limit of K allowed (including service). If K are in the third station, then any subsequent arrivals are shunted out of the third station. Find the expected time spent in the system by a customer who completes all three stages of service.

Question 5:

Consider a ring network with nodes 1,2,...,K. In this network, a customer that completes service at node i exits the network with probability p, or it is routed to node i+1 with probability 1-p, for i = 1,2,...,K-1. Customers that complete service at node K, either exit the network, or are routed to node 1, with respective probabilities p and 1-p. At each node, external customers arrive according to a Poisson process with rate γ. The service times at each node are exponentially distributed with rate μ. The arrival processes and the service times at the various nodes are independent.

1. Find the aggregate arrival rates λ_i, $i = 1, 2, ..., K$
2. Find the stationary distribution of the network. Under what conditions does this stationary distribution exist?
3. Find the average time that a customer spends in the network.
4. Is the flow on the link between nodes 1 and 2 Poisson? Justify your answer.

Question 6:

Customers arrive according to poisson process with rate λ to a two server system. The service time of the servers A and B are exponentially distributed with rates μ_A and μ_B, respectively. A customer who finds the system empty on arrival is allocated to the server that has been idle for the longest time. Otherwise, the customer at the head of the queue goes to the first free server.

1. Define the states describing the system as a Markov chain and draw the state-transition diagram.
2. Find the stationary distribution.
3. Is the system reversible? Justify your answer.
4. Find the transition rates of the reversed Markov chain and draw its state-transition diagram.

Question 7:

Customers arrive at a service station according to a Poisson process with rate λ. The service times are exponentially distributed and independent of each other and the arrival process. Queue buffers are infinite and customers are served on a first-come-first-served basis. Consider the following cases:

- Case 1: There are two servers sharing a common queue. Each server provides service at rate μ. A customer that upon arrival finds the system empty is routed randomly to one of the servers. Otherwise, it enters the queue and waits for service. The customer at the head of the queue goes to the first free server.
- Case 2: There is a single server (with a single queue) that provides service at rate 2μ.
- Case 3: There are two servers, each with its own dedicated queue. The service rate of each server is μ. Upon arrival to the combined service system, a customer is routed to the first queue with probability 0.5, or to the second queue with the same probability.

Answer the following questions:

1. For a Case 1 station: Draw the state transition diagram, and derive the stationary distribution. Find the average number of customers in the system and the average time a customer spends in the system. Also find the average number of customers that are queued
2. For a Case 2 station, find the average number of customers in the system and the average time a customer spends in the system.
3. For a Case 3 station, find the average number of customers and the average time a customer spends in the system.
4. Compare the average time delay of these three systems. Is it better to use two identical servers or a single server with double processing power to serve a single queue? If we have two servers, is it better to use dedicated queues, or a common queue for waiting. Explain the results intuitively.

Question 8: In the M/G/1 system with parameters λ and $\bar{X}$, determine the following:

- What is the probability that the server is idle?
- What is the average length of the time between busy periods?
- What is the average length of a busy period?
- What are the average number of customers served in a busy period?

Question 9: Consider an open network of single-server M/M/1 queues as shown in Figure. 1. The mean service times at the queues Q1,Q2,Q3 and Q4 are $1/\mu, 2/\mu, 2/\mu$ and $2/\mu$ seconds respectively. Let λ be the average external arrival rate (Poisson) to Q1. What is the maximum value of λ under which the network will operate in a stable manner?

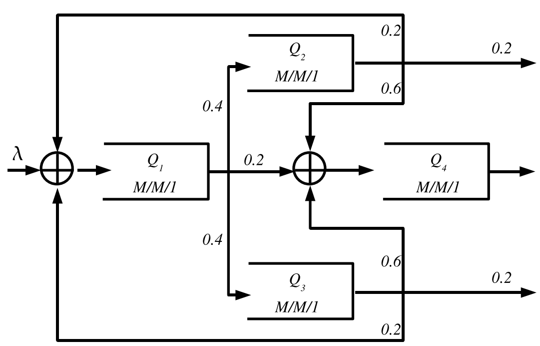

Figure 1:

Question 10:

Two types of packets are transmitted over a data network. 48-bit long control packets are designated for operations such as signaling, congestion notification and routing change information. These packets have a higher priority to any data packet passing through the network elements. Let assume that the data packets are 960 bits long on the average, with an exponential distributed packet length. The transmission links all have a capacity of 9600 bps. Control packets constitute 20% of the total traffic. Assume that the overall traffic utilization over a transmission link is 0.5.

- If non-priority service is used, show that the average waiting time for either type of traffic (control or data packets) is 99 ms.
- If non-preemptive priority is given to the control packets, calculate the waiting times for the two types of packets.

Question 11: Students enter the dining hall for breakfast in equally likely groups of either one or two with a group arrival rate of λ. The first member of the group is served in an exponentially distributed time. The second member if any orders an extra side-dish which requires δ seconds more, where δ is fixed. The mess operates as a single server queue. Find the mean delay that an arriving student will encounter before being served.