Quiz - 1

1) $P(X_{n+1}=j \mid X_n=i, X_{n-1}=i_{n-1}, \ldots, X_1=i_1)$

$$= P(X_{n+1}=j \mid X_n=i)$$

If for all n, $\{X_n : n \geq 1\}$ satisfies the above property it is a Markov Chain.

Example scenario where the stochastic process satisfies the model specification but is not Markov.

Consider a deterministic sequence of tempos & trucks
(T_m) (T_c)

$$(\underbrace{T_m T_c T_m T_c T_m T_c T_m T_m \underbrace{T_c T_c \cdots T_c T_c T_c}_{\text{length} = 37}}_{\text{length} = 45})$$ that repeats itself.

$5\ T_m$ & $40\ T_c$

- only one of the five T_m's is followed by a T_m
  (four out of every five tempos followed by a truck)
- 4 of the T_c are followed by a T_m i.e 4 out of $40 = \frac{1}{10}$

Its obvious that ~~to~~ this example does not satisfy Markov property.

2) $P = \begin{bmatrix} 0.2 & 0.8 \\ 0.1 & 0.9 \end{bmatrix}$, 0 – T_m
1 – T_c

3) Since the chain is homogeneous Markov, the two step transition matrix will give the answer.

$P^2 = \begin{bmatrix} 0.12 & 0.88 \\ 0.11 & 0.89 \end{bmatrix}$ $\underline{P^2_{11} = 0.89}$

4) We need to solve for steady state prob.

i.e $\pi = \pi \cdot P$.

$\pi = (1/9, 8/9)$ $\pi_1 =$

$\longrightarrow$ 8/9 is the answer.

5) This is the mean recurrence time of State 1.

$$= \frac{1}{\pi_1} = \frac{9}{8}$$