Preliminary Test

1. In triangle ABC, find side c if $a = 2$, $b = 3$, $C = \pi/3$. (5)

2. Express the equation of the tangent to the curve $y = x^3$ at the point $(-2, -8)$ in terms of its intercepts on coordinate axes. (10)

3. Show that $g(t) = \sin^2 t - 3t$ decreases on every interval in its domain. How many solutions does the equation $\sin^2 t - 3t = 5$ have? (5)

4. A round hole of radius $\sqrt{3}$ cm is bored through the centre of a solid sphere of radius 3 cm. Find the volume of material removed from the sphere. (15)

5. A parallelogram $PQRS$ has vertices at $P(2, -1, 4)$, $Q(1, 0, -1)$, $R(1, 2, 3)$ and S. Find out

   (a) the coordinates of S,   (b) the cosine of the interior angle at Q,
   (c) the vector projection of $\vec{QP}$ onto $\vec{QR}$,   (d) the area of the parallelogram,
   (e) an equation for the plane of the parallelogram,
   (f) the areas of the orthogonal projections of the parallelogram on the three coordinate planes.

   (30)

6. If $a = 10$ cm and $b = 16$ cm to the nearest millimeter, then what is the maximum possible percentage error in the calculated area $A = \pi ab$ of the ellipse $x^2/a^2 + y^2/b^2 = 1$? (10)

7. Sketch the region of integration for

   $$\int_0^{3/2} \int_{-\sqrt{9-4y^2}}^{\sqrt{9-4y^2}} y\,dx\,dy$$

   and evaluate the integral through change of order or directly. (15)

8. Show the solutions of the equation $|x| + |y| = 1 + x$ in an x-y plot. (10)