Solution to Preliminary Test and Grading Scheme

1. $\sqrt{7}$. $[c^2 = a^2 + b^2 - 2ab\cos C.]$ (5 marks)

2. $\frac{x}{-4/3} + \frac{y}{16} = 1$. [Verify that the point is on the curve. Find slope $\frac{dy}{dx} = 12$ (at that point) and the tangent $y + 8 = 12(x + 2)$. (5 marks)
   Re-arrange the equation to get it in intercept form, or solve $y = 0$ for x-intercept and $x = 0$ for y-intercept.] (5 marks)

3. One. [Show that $g'(t) < 0\ \forall t \in R$, hence g is strictly decreasing. Show existence. Show uniqueness.] (5 marks)
   Comment: Graphical solution is also acceptable, but arguments must be rigorous.

4. $4\pi(9 - 2\sqrt{6})$ or 16.4π or 51.54 cm^3. $[V = \int_0^{2\pi}\int_0^{R_h} 2\sqrt{R^2 - r^2}\, r dr d\theta,\ R = 3,\ R_h = \sqrt{3}$. Sketch domain. (5 marks)
   $V = 4\pi\int_0^{R_h}\sqrt{R^2 - r^2}\, rdr$, substitute $R^2 - r^2 = t^2$. (5 marks)
   Evaluate integral $V = -4\pi\int_R^{\sqrt{R^2 - R_h^2}} t^2 dt.]$ (5 marks)

5. (a) (2,1,8). [Consider $\vec{p} - \vec{q} = \vec{s} - \vec{r}$.] (5 marks)
   (b) $\sqrt{3/5}$. $[\cos Q = \frac{\vec{QP}\cdot\vec{QR}}{\|\vec{QP}\|\ \|\vec{QR}\|}.]$ (5 marks)
   (c) $\frac{9}{5}(\mathbf{j} + 2\mathbf{k})$. [Vector projection $= (\vec{QP}\cdot\hat{QR})\hat{QR}$.] (5 marks)
   (d) $6\sqrt{6}$ square units. [Vector area $= \vec{QP}\times\vec{QR}$.] (5 marks)
   (e) $7x + 2y - z = 8$.
   [Take normal $\mathbf{n}$ in the direction of vector area and $\mathbf{n}\cdot(\mathbf{x} - \mathbf{q})$.] (5 marks)
   Comment: Parametric equation $\mathbf{q} + \alpha(\mathbf{p} - \mathbf{q}) + \beta(\mathbf{r} - \mathbf{q})$ is also acceptable.
   (f) 14, 4, 2 on yz, xz and xy planes. [Take components of vector area.] (5 marks)

6. 1.625%. $[A = \pi ab \Rightarrow \frac{dA}{A} = \frac{da}{a} + \frac{db}{b}$. (5 marks)
   Put $a = 10$, $b = 16$ and $da = db = 0.1$.] (5 marks)

7. 9/2. [Sketch domain: region inside the ellipse $x^2 + 4y^2 = 9$ and above the x-axis. (5 marks)
   By change of order, $I = \int_{-3}^{3}\int_0^{\frac{1}{2}\sqrt{9-x^2}} ydydx$. (5 marks)
   Then, evaluate $I = \int_{-3}^{3}\frac{9-x^2}{8}dx$.] (5 marks)

8. Connect $(\infty, 1)$, $(0, 1)$, $(-1/2, 0)$, $(0, -1)$, $(\infty, -1)$ by straight segments. [Split: for $y \geq 0$, $y = 1 + x - |x|$ and for $y < 0$, $y = -1 - x + |x|$. (5 marks)
   Next, split in x to describe these two functions.] (5 marks)
   Comment: y is undefined for $x < -\frac{1}{2}$.