PSet-02 & 03, Due on 27/09/2023, 11PM

[42+35 points], Kindly put in my mailbox near FB-382

1. Derive the z^{-1} term in the $T(z)\phi(0)$ operator product expansion, using the same annulus construction as done in class for dilatations to generate the $1/z^2$ term. In this case, use translations, which means α^μ is independent of z, thus get the term from a statement on the contour integral of just $T(z)\phi(0)$. [2 pts]

2. (a) Find the Schwartzian for the transformation:

$$z \to \frac{az+b}{cz+d}.$$

   [1 pts]

   (b) Given $\langle T \rangle = 0$ on the full complex plane, find its value under the map : $z \to e^{2\pi z/\beta}$. [2 pts]

3. Consider the CFT Hamiltonian (with space being on a circle of circumference L) : $H = \int_0^L h(x)dx$.

   (a) This in terms of generators is proportional to $L_0 + \bar{L}_0$. Derive this. [1 pts]

   (b) Let us now deform the Hamiltonian to

$$H_1 = \int_0^L 2\sin^2\left(\frac{\pi\, x}{L}\right) h(x)dx.$$

   Write the Hamiltonian that you get in terms of the Virasoro generators. [3 pts]

   (c) After going to the plane, find a further conformal transformation, such that in the new frame the Hamiltonian once again becomes proportional to $L_0' + \bar{L}_0'$. [5 pts]

   (d) Consider the evolution operator $U(T) = e^{-iH_1(T-t_0)}e^{-iHt_0}$. Find the correlator :

$$\left\langle \Big(U(T)^\dagger\Big)^n O_{h,\bar{h}}(z_1) \Big(U(T)\Big)^n O_{h,\bar{h}}(z_2) \right\rangle.$$

   [10 pts]

4. Write a `Mathematica` or a `Python` code from scratch, that will take as an input a series of positive and negative integers in the format, e.g.:

```
>> 2, 3, 5, 7, -4, -10, -3
```

And output the result of the correlator:

$$\langle 0|L_2 L_3 L_5 L_7 L_{-4} L_{-10} L_{-3}|0\rangle.$$

Please email the code to `sarkara@iitk.ac.in`. [5 pts]

5. Express in terms of local fields, $\phi_n^{(k_1,k_2\dots)}(z)$ and its derivatives the result of infinitesimal conformal transformation parametrized by $\epsilon(z)$ on :

$$\prod_{k=1}^{N}\Big(L_{-k}(z)\Big)^{j_k}\phi_n(z).$$

[13 pts]

PSet-03

1. Write a `Mathematica` or a `Python` code from scratch, that will take as an input three sets of negative integers in the format, e.g.:

```
>> -4, -3, -2
>> -2
>> -5, -7
```

Next it will take as an input the values of conformal dimensions of three primary operators, e.g.,

```
>> 2.62, 1.32
>> 1, 0
>> 23.54, 0.23
```

And output the result of the correlator modulo the OPE coefficient:

$$\langle \phi_1^{(-4,-3,-2)}|\hat{L}_{-2}\phi_2(1)|\phi_3^{(-5,-7)}\rangle.$$

In the above example the conformal dimensions are : $h_1 = 2.62, \bar{h}_1 = 1.32$ and so on. Remember to take into account the correct definition of the conjugate state, i.e., along with appropriate normalizations. Please email the code to `sarkara@iitk.ac.in`. [20 pts]

2. Write a code that will compute the (holomorphic) Kac matrix and the determinant at level N for highest weight h which will be taken as an input from the user. Please email the code to `sarkara@iitk.ac.in`. Estimate the complexity of your code. [10 pts]

3. Consider a state :

$$|\psi_n\rangle = L_{-2n}|h\rangle + aL_{-n}^2|h\rangle.$$

Derive the value (real) of a which minimizes the norm of this state. [5 pts]