

PSet-04, Due on 18/10/2023, 11PM

[20 points], Kindly put in my mailbox near FB-382

1. **Descendent correlators on the torus** We are interested in computing descendant correlators on the torus :

$$\langle \Phi_1(z_1) \dots \Phi_N(z_N) \rangle.$$

Consider an elliptic function

$$g(z) = \sum_{m=0}^{\infty} g_{-n_i+m}^{(i)} (z - z_i)^{-n_i+m}$$

such that it has poles at insertion points. Then write down the Laurent series for

$$F(z) = g(z) \langle T(z) \Phi_1(z_1) \dots \Phi_N(z_N) \rangle$$

in terms of $g_{-n_i+m}^{(i)}$ and $\hat{L}_{-k} \Phi_i(z_i)$ within correlators. Therefore write down the residue at $z = z_i$. [**5 pts**] Now the sum over all residues being zero relates correlation functions of different descendants to each other. As an example choose $g(z) = \mathcal{P}^{(n)}(z - z_i)$, $n \geq 0$ and write down the relation that this gives among torus descendant correlators. [**5 pts**] As a second choice choose $g(z) = \zeta(z - z_0) - \zeta(z - z_i)$ where z_0 is not an insertion point. Find now the relationship among the correlators for this choice. [**5 pts**] Integrate the obtained relation over $\omega_1 = L$ cycle to get rid of the z_0 dependence. Write down the final relationship for $\langle \hat{L}_{-2} \Phi_i(z_i) \dots \rangle$ in terms of lower descendant correlators. [**5 pts**]