# Non-Relativistic Conformal Field Theories

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In this project report, we will do a comparative analysis of three non-relativistic (NR) conformal algebras which are the Schrödinger algebra, Galilean conformal algebra (GCA) and conformal Carroll algebra (CCA). This will be followed by the derivation of the two-point correlators in NR-CFTs which respect the abovementioned algebras. In the end, we will discuss OPEs in Schrödinger CFTs.

## I. INTRODUCTION

Non-relativistic conformal field theories (NR-CFT) play a central role in understanding several condensed matter systems [1]. By utilizing the scaling symmetry present in NR (non-relativistic) systems, it becomes possible to provide an explanation for critical phenomena—for example, ferromagnetic phase transition, NR fermions at unitarity, helium near superfluid transitions [2]. It was shown in [3] that the NR Naviers-Stokes equation is invariant under Galilean conformal algebra (GCA), a branch of NR-CFT. In [4], the authors identify modified Mellin amplitude with the time-dependent correlation functions of primaries in a Carrollian CFT, another branch of NR-CFT. This result establishes a connection between the two lines of research in flat holography. This brief introduction serves as a compelling reason to study NR-CFTs.

## II. SCHRÖDINGER ALGEBRA

The Schrödinger symmetry group [5–7] is defined by the following transformation rules

$$\vec{r} \longrightarrow \vec{r'} = \frac{\mathcal{R}\vec{r} + \vec{v}t + \vec{a}}{\gamma t + \delta}, \qquad t \longrightarrow t' = \frac{\alpha t + \beta}{\gamma t + \delta}, \qquad \alpha\delta - \beta\gamma = 1 \tag{1}$$

where $\alpha, \beta, \gamma, \delta, \vec{v}$ and $\vec{a}$ are real parameters and $\mathcal{R}$ is the rotation matrix in d dimensions. Niederer [6] showed that this forms the group of symmetries of free Schrödinger wave operator in d+1 dimensions. The generators of Schrödinger algebra in d+1 dimensions are given in table I. It is important to note that in the Schrödinger group, the special conformal transformation functions as a scalar operator, which leads to a reduced number of generators in comparison to the relativistic conformal group. Therefore, we can anticipate that the correlation functions will not be determined exactly.

| Symmetries | Generators |
|---|---|
| Rotations | $J_{ij} = -(x_i\partial_j - x_j\partial_i)$ |
| Translations | $P_i = \partial_i$ |
| Galilean boosts | $B_i = t\partial_i$ |
| Hamiltonian | $H = -\partial_t$ |
| Dilatations | $D = -(2t\partial_t + x_i\partial_i)$ |
| Schrödinger SCT | $K = -(tx_i\partial_i + t^2\partial_t)$ |

TABLE I: Generators of Schrödinger Algebra

## III. SCHRÖDINGER VIRASORO ALGEBRA

There exists an infinite-dimensional extension to the Schrödinger algebra called the Schrödinger-Virasoro algebra [7]. In d+1 dimensions with $\mathcal{R} = 1$, the generators are given by

$$X_n = -t^{n+1}\partial_t - \frac{n+1}{2}t^n r^i \partial_{r^i} - \frac{n(n+1)}{4}\mathcal{M}t^{n-1}r_i^2 \tag{2}$$

$$Y^i_m = -t^{m+\frac{1}{2}}\partial_{r_i} - \left(m + \frac{1}{2}\right)\mathcal{M}t^{m-\frac{1}{2}}r^i \tag{3}$$

$$M_n = -t^n\mathcal{M}. \tag{4}$$

Here, m takes half-integer values and n takes integer values. When $\mathcal{M} = -im$, this gives the infinite-dimensional extension of the Schrödinger algebra, where $m$ is interpreted as the mass. The Lie algebra of the Schrödinger group is spanned by $\{X_{\pm1,0}, Y_{\pm\frac{1}{2}}, M_0\}$. The non-trivial commutation relations are given as

$$\begin{aligned} [X_n, X_m] &= (n-m)X_{n+m} \\ [X_n, Y^i_m] &= \left(\frac{n}{2} - m\right)Y^i_{n+m} \\ [X_n, M_m] &= -mM_{m+n} \\ [Y^i_n, Y^j_m] &= (n-m)M_{n+m}\delta^{ij} \end{aligned} \tag{5}$$

## IV. GALILEAN CONFORMAL ALGEBRA

Scaling the coordinates in a particular manner contracts the relativistic conformal group to the Galilean conformal group, we will refer to this scaling as the NR limit.

$$t \to t \text{ and } x_i \to \epsilon x_i \text{ in the limit } \epsilon \to 0. \tag{6}$$

This scaling limit is equivalent to taking $c \to \infty$. Here, the number of generators do not change on performing the contraction, unlike the Schrödinger algebra. [1, 8]. The generators of Galilean conformal algebra in d+1 dimensions are given in table II.

| Symmetries | Generators |
|---|---|
| Rotations | $J_{ij} = -(x_i\partial_j - x_j\partial_i)$ |
| Translations | $P_i = \partial_i$ |
| Galilean boosts | $B_i = t\partial_i$ |
| Hamiltonian | $H = -\partial_t$ |
| Dilatations | $D = -(t\partial_t + x_i\partial_i)$ |
| Galilean temporal SCT | $K = -(2tx_i\partial_i + t^2\partial_t)$ |
| Galilean spatial SCT | $K_i = t^2\partial_i$ |

TABLE II: Generators of Galilean Conformal Algebra

## V. VIRASORO-KAC-MOODY TYPE ALGEBRA

There exists an extension of GCA that is infinite-dimensional. The extension of GCA is similar to the Virasoro-Kac-Moody type algebra [1, 8].

$$L^{(n)} = -(n+1)t^n x_i\partial_i - t^{n+1}\partial_t, \tag{7}$$

$$M_i^{(n)} = t^{n+1}\partial_i, \tag{8}$$

$$J_a^{(n)} := J_{ij}^{(n)} = -t^n(x_i\partial_j - x_j\partial_i). \tag{9}$$

Here $n$ takes integer values and $a$ labels the generators of the spatial rotation group $SO(d)$. The non-trivial commutation relations are given as

$$\begin{aligned} [L^{(n)}, L^{(m)}] &= (n-m)L^{(n+m)} \\ [L^{(n)}, J_a^{(m)}] &= -nJ_a^{(m+n)} \\ [J_a^{(n)}, J_b^{(m)}] &= f_{ab}{}^c J_c^{(n+m)} \\ [L^{(n)}, M_i^{(m)}] &= (n-m)M_i^{(n+m)} \end{aligned} \tag{10}$$

## VI. VIRASORO TO GCA IN 2D

The two-dimensional relativistic conformal algebra is an infinite-dimensional algebraic structure that comprises two copies of the Virasoro algebra. Using the NR contraction as portrayed above one can derive the infinite-dimensional GCA in 1+1 dimensions from the Virasoro algebra. The generators of the Witt algebra are given by

$$\mathcal{L}_n = -z^{n+1}\partial_z \quad \text{and} \quad \bar{\mathcal{L}}_n = -\bar{z}^{n+1}\partial_{\bar{z}}. \tag{11}$$

To perform the abovementioned contraction, consider the coordinate transformation given as $z = t+x$ and $\bar{z} = t-x$. One can obtain the generators of 2D GCA by-

1. Expressing $\mathcal{L}_n$ and $\bar{\mathcal{L}}_n$ in $(x,t)$ coordinates.
2. Considering the combinations $\mathcal{L}_n+\bar{\mathcal{L}}_n$ and $\mathcal{L}_n-\bar{\mathcal{L}}_n$.
3. On scaling $\epsilon \to 0$,

$$\mathcal{L}_n + \bar{\mathcal{L}}_n \to L_n \quad \text{and} \quad \epsilon(\mathcal{L}_n - \bar{\mathcal{L}}_n) \to -M_n. \tag{12}$$

At the quantum level, the two Virasoro algebras have a central extension

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n,-m}, \tag{13}$$

$$[\bar{\mathcal{L}}_n, \bar{\mathcal{L}}_m] = (n-m)\bar{\mathcal{L}}_{n+m} + \frac{\bar{c}}{12}n(n^2-1)\delta_{n,-m}. \tag{14}$$

Using the abovementioned linear combination of the Virasoro generators one can derive the centrally extended GCA in 2D.

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + C_1 n(n^2-1)\delta_{n,-m}, \\ [L_n, M_m] &= (n-m)M_{n+m} + C_2 n(n^2-1)\delta_{n,-m}, \\ [M_n, M_m] &= 0 \\ \text{where} \quad C_1 &= \frac{c+\bar{c}}{12} \quad \text{and} \quad C_2 = \epsilon\frac{\bar{c}-c}{12}. \end{aligned} \tag{15}$$

For non-zero $C_2$, $\bar{c} - c \propto \mathcal{O}(\frac{1}{\epsilon}) + \mathcal{O}(\epsilon)$, the second proportionality can be motivated from the form of $\mathcal{L}_n - \bar{\mathcal{L}}_n$ in $(x,t)$ coordinates and for finite $C_1$, $\bar{c} + c \propto \mathcal{O}(1)$. For the proportionalities to hold, we require $c$ and $\bar{c}$ to be large and of opposite sign. Hence, the original 2D relativistic CFT cannot be unitary.

## VII. CONFORMAL CARROLL ALGEBRA

The conformal Carroll algebra (CCA) is obtained from the relativistic conformal algebra by scaling the coordinates in the opposite manner [4], that is,

$$t \to \epsilon t \text{ and } x_i \to x_i \text{ in the limit } \epsilon \to 0 \tag{16}$$

This scaling limit is equivalent to taking $c \to 0$, we will refer to this scaling as the ultra-relativistic (UR) limit. The generators of the algebra are given in table III.

| Symmetries | Generators |
|---|---|
| Rotations | $J_{ij} = -(x_i\partial_j - x_j\partial_i)$ |
| Translations | $P_i = \partial_i$ |
| Carrollian Boosts | $B_i = -x_i\partial_t$ |
| Hamiltonian | $H = \partial_t$ |
| Dilatations | $D = -(t\partial_t + x_i\partial_i)$ |
| Carrollian SCT (Temporal) | $K = x_i x_i\partial_t$ |
| Carrollian SCT (Spatial) | $K_i = -2x_i(t\partial_t + x_i\partial_i) + x_j x_j\partial_i$ |

TABLE III: Generators of Conformal Carroll Algebra

## VIII. BMS ALGEBRA

Conformal Carrollian isometries are isomorphic to BMS (Bondi Metzner Sachs) symmetries in one higher dimension [9, 10]. Bondi, van der Burgh, Metzner and Sachs discovered the symmetries of the asymptotically flat 4D spacetimes. This forms an infinite-dimensional group and is known as the BMS group. The BMS algebra of a 4D asymptotically flat spacetime at the null boundary is given as

$$\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m}, \\
[\bar{L}_n, \bar{L}_m] &= (n-m)\bar{L}_{n+m}, \\
[M_{r,s}, M_{t,u}] &= 0, \\
[L_n, M_{r,s}] &= \left(\frac{n+1}{2} - r\right) M_{r+n,s}, \\
[\bar{L}_n, M_{r,s}] &= \left(\frac{n+1}{2} - s\right) M_{r,n+s}.
\end{aligned} \tag{17}$$

Hence, the 3D CCA has an infinite-dimensional extension and the generators are given as

$$\begin{aligned}
&M_{00} = H, \quad M_{01} = B_x + iB_y, \quad M_{10} = B_x - iB_y, \\
&M_{11} = K, \quad L_0 = \frac{1}{2}(D + iJ), \quad L_{-1} = \frac{1}{2}(P_x + iP_y), \\
&L_1 = \frac{1}{2}(K_x + iK_y), \quad \bar{L}_0 = \frac{1}{2}(D - iJ), \\
&\bar{L}_{-1} = \frac{1}{2}(P_x - iP_y), \quad \bar{L}_1 = \frac{1}{2}(K_x - iK_y).
\end{aligned} \tag{18}$$

Here, the BMS generators of 4D asymptotically flat spacetimes are on the LHS and the finite-dimensional conformal Carroll generators are on the RHS. In the UR limit, the Virasoro algebra gives the CCA in 2D [11–13]. The derivation is similar to the one in section VI

## IX. CORRELATION FUNCTIONS

In this section, we will report the two-point correlation function for all three cases. Refer to table IV for the correlation functions and the appendix for all the derivations.

## X. A COMPARATIVE ANALYSIS OF ALL THE CORRELATION FUNCTIONS

1. All except conformal Carroll two-point correlators (the second channel) vanish for unequal scaling dimensions.

2. The exponential in the correlators (except CCA) arises due to invariance under Galilean boost.

3. The Schrödinger three-point function is not completely fixed, unlike the GCA and CCA correlators. The correlator is given in terms of cross-ratios as given in the argument of the H function. This is because Schrödinger algebra has less number of generators.

4. The Carroll correlators in the second channel are ultralocal and vanish if the spins do not add up to zero. Similarly, there is an additional mass superselection rule in the Schrödinger correlators.

## XI. OPE IN SCHRÖDINGER CFT

In this section, we analyze the OPEs of primary scalar operators in Schrödinger CFT [14]. This section does not follow the same algebraic interpretation as in the previous section. Refer [15] for the algebraic interpretation. The OPE of two operators can be expanded as

$$\begin{aligned}
\mathcal{O}_2(0)\mathcal{O}_3(x) = (&C_0(x) + C_1^i(x)\partial_i + C_2(x)\partial_t \\
&+ C_3^{jk}(x)\partial_j\partial_k \ldots.)\mathcal{O}_1(0).
\end{aligned} \tag{19}$$

Commuting both sides with $K_i$ and $C$ and rearranging yields $C_1^i(x)$, $C_2(x)$ and $C_3^{jk}(x)$ in terms of $C_0(x)$. The rest of the coefficients can be found in the similar manner. The expression for $C_2(x)$ is explicitly given below

$$\begin{aligned}
C_2(x) = &\frac{i}{N_1(2\Delta_1 - d)}[t^2(-2iN_1\partial_t + \partial^2) - 2iN_2 t x_i \partial_i \\
&+ i(N_3 d - 2N_1\Delta_3)t + N_3 N_2 x^2]C_0(x)
\end{aligned} \tag{20}$$

where $N_2 + N_3 = N_1$.
It is seen that if $\mathcal{O}_1$ has dimension $d/2$ then the equation can be treated as a restriction on $C_0(x)$. Therefore, for the OPE of any two primary operators, the coefficients of an operator of dimension $d/2$ can be calculated exactly. In the theory of fermions at unitarity, the aforementioned fact is utilized. Here, $\mathcal{O}_1$ refers to a fermion field $\psi$ with $N_1 = -1$ and $\Delta_1 = d/2$. By utilizing equation (20) and a scale-invariant form of $C_0(x,t)$, it becomes possible to calculate the exact expression of $C_0$, which, in turn, enables the determination of all the other coefficients.


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| Algebra | Two-point Correlator |
|---|---|
| Schrödinger Algebra | $G^{(2)}(r,t) = \delta_{x_1,x_2}\delta_{\mathcal{M}_1,\mathcal{M}_2}\phi_0 t^{-x} exp\left(-\frac{\mathcal{M}_1 r^2}{2t}\right)$ |
| Galilean Conformal Algebra | $G^{(2)}(r_i,\tau) = \delta_{\Delta_1,\Delta_2}\delta_{\xi_1^i,\xi_2^i} C^{(2)}\tau^{-\Delta} exp\left(\frac{\xi_i r^i}{\tau}\right)$ |
| Conformal Carroll Algebra | $G^{(2)}(u,z,\bar{z},u',z',\bar{z}') = \frac{C\delta^2(z-z')}{(u-u')^{\Delta+\Delta'-2}}\delta_{\sigma+\sigma',0}$ |

TABLE IV: Two-point Correlators


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## Appendix

### Derivation of Correlation Functions

*Schrödinger Algebra*

The content covered in this section follows from [7]. Consider a 1+1 dimensional field (in this case spin = 0). $\phi(r,t)$ with scaling dimension $x$ and mass $\mathcal{M}$. The action of the generators on the field is as follows:

$$[X_n,\phi(r,t)] = \left(t^{n+1}\partial_t + \frac{n+1}{2}t^n r\partial_r + \frac{n(n+1)}{4}\mathcal{M}t^{n-1}r^2 + (n+1)t^n\frac{x}{2}\right)\phi(r,t)$$
$$[Y_m,\phi(r,t)] = \left(t^{m+\frac{1}{2}}\partial_r + \left(m+\frac{1}{2}\right)\mathcal{M}t^{m-\frac{1}{2}}r\right)\phi(r,t). \tag{21}$$

$\phi(r,t)$ is a *primary field* if it follows the above equations for all integers $n$ and half integers $m$ and *quasi-primary* if it satisfies the above equations for $n=\pm1,0$ and $m=\pm\frac{1}{2}$.

Consider the two-point function given as

$$F(r_1,r_2;t_1,t_2) = \langle\phi_1(r_1,t_1)\phi_1^*(r_2,t_2)\rangle. \tag{22}$$

Space and time translation symmetry restrict $F$ as a function of $r=r_1-r_2$ and $t=t_1-t_2$. Invariance under scaling ($X_0$) requires

$$\left(t\partial_t + \frac{1}{2}r\partial_r + x\right)F(r,t) = 0, \tag{23}$$

with $x=\frac{1}{2}(x_1+x_2)$. The solution of $F(r,t)$ can be written as

$$F(r,t) = t^{-x}G\left(\frac{r^2}{t}\right). \tag{24}$$

Invariance under boosts ($Y_{1/2}$) imposes two conditions

$$\mathcal{M}_1 - \mathcal{M}_2 = 0 \quad \text{and} \quad (t\partial_r + \mathcal{M}_1 r)F(r,t) = 0. \tag{25}$$

Combining this with (26) we get,

$$G(u) = G_0 exp\left(-\frac{\mathcal{M}_1}{2}u\right). \tag{26}$$

Invariance under Schrödinger SCT ($X_1$) gives three conditions

$$x = x_1 = x_2, \quad \mathcal{M}_1 - \mathcal{M}_2 = 0 \text{ and } (\partial_u + \frac{1}{2}\mathcal{M}_1)G = 0. \tag{27}$$

These equations determine the two-point function up to a normalization constant $\phi_0$

$$F(r,t) = \delta_{x_1,x_2}\delta_{\mathcal{M}_1,\mathcal{M}_2}\phi_0 t^{-x} exp\left(-\frac{\mathcal{M}_1 r^2}{2t}\right). \tag{28}$$

Using the same analysis the three-point function $\langle\phi(r_a,t_a)\phi(r_b,t_b)\phi^*(r_c,t_c)\rangle$ can be determined for three scalars up to a function H.

$$\begin{aligned} F(r,s,\tau,\sigma) =& \delta_{\mathcal{M}_a+\mathcal{M}_b,\mathcal{M}_c}\tau^{-\frac{1}{2}(x_a+x_c-x_b)}\sigma^{-\frac{1}{2}(x_b+x_c-x_a)} \\ & * (\tau-\sigma)^{-\frac{1}{2}(x_a+x_b-x_c)} \\ & exp\left[-\frac{\mathcal{M}_a r^2}{2\tau} - \frac{\mathcal{M}_b s^2}{2\sigma}\right] H\left(\frac{(r\sigma - s\tau)^2}{(\sigma-\tau)\sigma\tau}\right). \end{aligned} \tag{29}$$

Here $r = r_a - r_c$, $s = r_b - r_c$, $\tau = t_a - t_c$ and $\sigma = t_b - t_c$.

*Galilean Conformal Algebra*

The content covered in this section follows from [16]. Consider a d+1 dimensional spacetime, from the algebra, we see that $L_0$ defines the dilatation operator and $M_0^i$ defines Galilean boosts ($[L_0, M_0^i] = 0$). The eigenvalues of these operators are called the scaling dimension $\Delta$ and rapidity $\xi^i$. From the commutation relations, it can be derived that $L_n$ and $M_n^i$ lower the scaling dimension by n and $L_{-n}$ and $M_{-n}^i$ raise the scaling dimension by n. Primary operators $\mathcal{O}_p$ are defined as $[L_n, \mathcal{O}_p] = 0$ and $[M_n^i, \mathcal{O}_p] = 0$. We label primary states as $|\Delta, \xi^i\rangle$.
The action of the generators for $n \geq 0$ is,

$$\begin{aligned} &[L_n, \mathcal{O}(x,t)] = [t^{n+1}\partial_t + (n+1)t^n x^i \partial_i \\ &+ (n+1)(t^n\Delta - nt^{n-1}x_i\xi^i)]\mathcal{O}(x,t) \\ &[M_n^i, \mathcal{O}(x,t)] = [-t^{n+1}\partial_i + (n+1)t^n\xi^i]\mathcal{O}(x,t). \end{aligned} \tag{30}$$

Primaries satisfy the above relation for all $n \geq 0$ and quasi-primaries satisfy the above relation for $n = 1, 0$.

The two-point correlation function between two quasi-primary operators $\mathcal{O}_1(x_1^i, t)$ and $\mathcal{O}_2(x_2^i, t)$ with scaling dimension and rapidity $(\Delta_1, \xi_1^i)$ and $(\Delta_2, \xi_2^i)$ is given by the function $G^{(2)}(x_1^i, x_2^i, t_1, t_2)$. Translational symmetry in space and time restrict $G^{(2)} = G^{(2)}(\tau, r^i)$ where $\tau = t_1 - t_2$ and $r^i = x_1^i - x_2^i$. Invariance under Galilean Boosts ($M_0^i$) gives,

$$\begin{aligned} &M_0^i G^{(2)} = (-\tau\partial_i + \xi_i)G^{(2)} = 0, \\ &\implies G^{(2)} = C(\tau) exp\left(\frac{\xi_i r^i}{\tau}\right), \end{aligned} \tag{31}$$

$C(\tau)$ is a function of $\tau$. Invariance under dilatation implies,

$$\begin{aligned} &(\tau\partial_\tau + r_i\partial_i + \Delta)G^{(2)} = 0 \\ &\implies G^{(2)}(r_i,\tau) = C^{(2)}\tau^{-\Delta} exp\left(\frac{\xi_i r^i}{\tau}\right), \end{aligned} \tag{32}$$

where $C^{(2)}$ is an arbitrary constant, $\Delta = \Delta_1 + \Delta_2$ and $\xi^i = \xi_1^i + \xi_2^i$. $M_1^i$ and $L_1$ impose $\Delta_1 = \Delta_2$ and $\xi_1^i = \xi_2^i$. Finally, the two-point function reads,

$$G^{(2)}(r_i,\tau) = \delta_{\Delta_1,\Delta_2}\delta_{\xi_1^i,\xi_2^i} C^{(2)}\tau^{-\Delta} exp\left(\frac{\xi_i r^i}{\tau}\right). \tag{33}$$

By a similar analysis, the three-point function can be fixed up to a constant $C^{(3)}$

$$\begin{aligned} G^{(3)}(r_i,s_i,\tau,\sigma) =& C^{(3)}\tau^{-(\Delta_1-\Delta_2+\Delta_3)}\sigma^{-(\Delta_2+\Delta_3-\Delta_1)} \\ & * (\tau-\sigma)^{-(\Delta_1+\Delta_2-\Delta_3)} \\ & exp\left(\frac{(\xi_1^i - \xi_2^i + \xi_3^i)r_i}{\tau} + \frac{(\xi_2^i - \xi_1^i + \xi_3^i)s_i}{\sigma}\right. \\ & \left. + \frac{(\xi_1^i - \xi_3^i + \xi_2^i)(r_i - s_i)}{(\tau-\sigma)}\right). \end{aligned} \tag{34}$$

Both the three and two-point functions in 2D can be derived from the corresponding correlation functions in 2D relativistic CFT, by performing the non-relativistic scaling [1].

*Conformal Carroll Algebra*

The content covered in this section follows from [4]. In this section, we derive the two-point correlator $G(u,z,\bar{z},u',z',\bar{z}')$ of primary fields in 3D Carroll CFT. The action of the generators on primaries with scaling dimension $(h,\bar{h})$ is given as

$$\begin{aligned} &L_n\phi_{h,\bar{h}}(u,z,\bar{z}) = \left[z^{n+1}\partial_z + (n+1)z^n\left(h + \frac{1}{2}u\partial_u\right)\right]\phi_{h,\bar{h}}, \\ &M_{r,s}\phi_{h,\bar{h}}(u,z,\bar{z}) = [z^r\bar{z}^s\partial_u]\,\phi_{h,\bar{h}}(u,z,\bar{z}). \end{aligned} \tag{35}$$

Combining the constraints due to Carroll time translational symmetry and Carroll boosts leads to

$$(z-z')\partial_u G(u-u', z-z', \bar{z}-\bar{z}') = 0 + \text{anti-holomorphic part} \tag{36}$$

This gives rise to two channels of Carroll correlators

$$\partial_u G = 0 \quad \text{and} \quad \partial_u G = f(u-u')\delta^2(z-z'). \tag{37}$$

On applying the restrictions from other symmetry operations, the first channel leads to the two-point correlator in 2D relativistic CFT.

In the second channel, demanding invariance under $L_{0,\pm1}, \bar{L}_{0,\pm1}$ it is seen that,

$$\begin{aligned}&(\Delta+\Delta'-2)f(u-u')+(u-u')\partial_u f(u-u')=0,\\&(\sigma+\sigma')f(u-u')=0\end{aligned} \tag{38}$$

where $\Delta = h+\bar{h}$ and $\sigma = h-\bar{h}$. Solving the above equations determines the correlation function up to a constant

$$G(u,z,\bar{z},u',z',\bar{z}') = \frac{C\delta^2(z-z')}{(u-u')^{\Delta+\Delta'-2}}\delta_{\sigma+\sigma',0}. \tag{39}$$