SYMMETRY RESTORATION VIA QUANTUM QUENCH

WORK WITH BIDYUT DEY, IIT - K.

9/3 BANGALORE

MOTIVATION

QUANTUM QUENCH

- How does a quantum system thermalize, after being driven out of equilibrium?

- A set-up to investigate non-equilibrium physics is via a quantum quench,

$$S \to S + \int \lambda(t/\delta t)\mathcal{O}_{\Delta}.$$ (Polkovnikov, Sengupta, Silva, Vengalattore, 2011)

- Pumping energy into the system results in thermalization, Holography : BH formation. (Janik, Peschanski 2006)

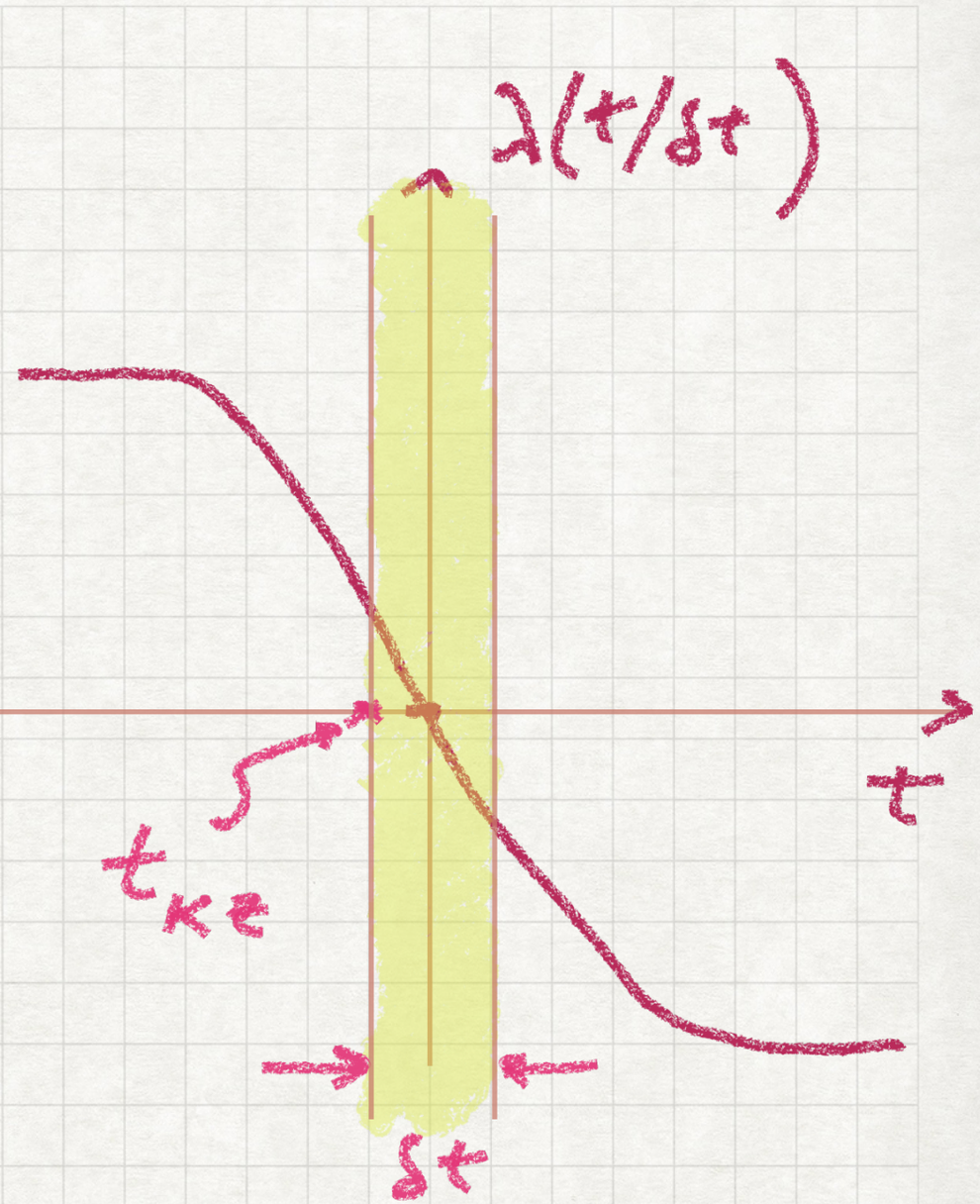
- Integrable systems generically settle down to generalized Gibbs ensembles.

- Most analytical studies are limited to sudden quenches, and exactly solvable smooth quench profiles.

MOTIVATION

QUANTUM QUENCH

- For smooth quenches, the non-equilibrium dynamics is characterized by a Kibble-Zurek scaling. (Kibble 1976, Zurek 1985)
- As system approaches criticality breakdown of adiabaticity gives rise to a time-scale : $E_g(t) = E_g^{(0)}(t) + E_g^{(1)}(t) + \ldots$
- $E_g^{(0)}(t_{KZ}) \sim E_g^{(1)}(t_{KZ})$, and the systems freezes at this scale : a decoupling of scales.
- $\langle \mathcal{O}_\Delta \rangle \sim t_{KZ}^{-\Delta}$: seems to hold for a wide range of situations!



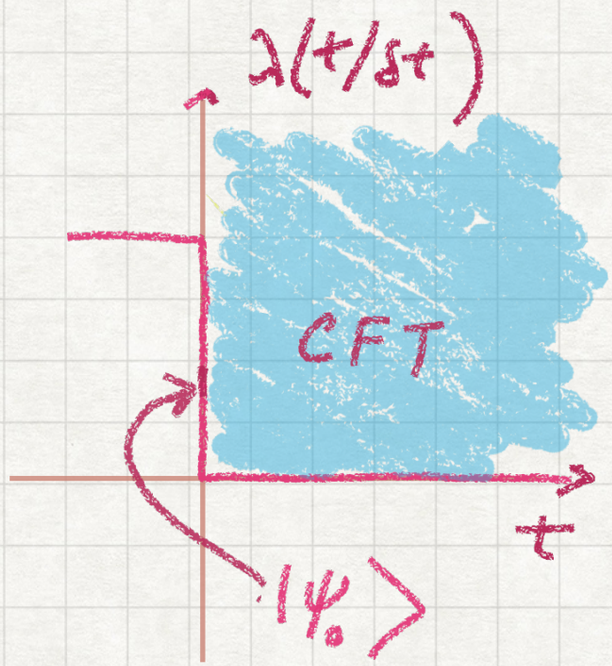
MOTIVATION

QUANTUM QUENCH

- For sudden critical quenches too, universal dynamics have been discovered.

$$|\psi_0\rangle = e^{-\tau_0 H_{CFT}} |\mathscr{B}\rangle\rangle$$ (Cardy, Calabrese 2008)

- $\langle \mathcal{O}_\Delta(t) \rangle \sim e^{-\pi \Delta t/(2\tau_0)}$, follows from a calculation in the CFT boundary state.

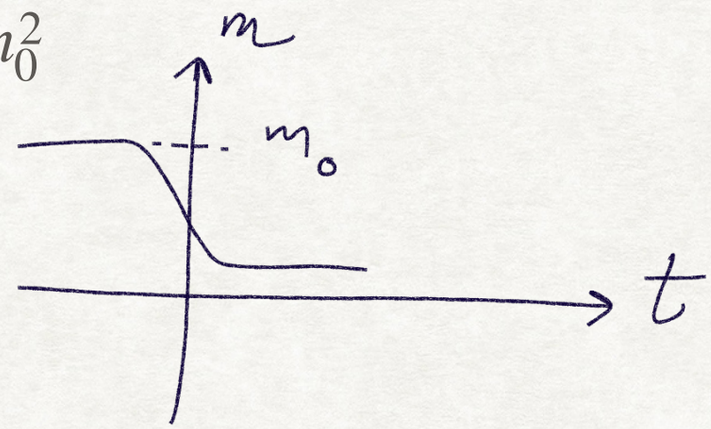


- However for, KZ : $\Lambda > E_g^{(0)} > \delta t^{-1}$
- For, $\Lambda > \delta t^{-1} > E_g^{(0)}$ too there is a scaling, and this is dictated by conformal perturbation about the UV CFT.
- KZ has been shown for theories with quantum critical points, free field theories, holographic theories.

KZ IN FREE SCALAR FIELD THEORIES

- $S = -\int dt \int d^{d-1}x \left((\partial\phi)^2 + m^2(t) \right)$, use mode expansion :

  $\phi(x,t) = \int \frac{d^{d-1}k}{(2\pi)^{\frac{d-1}{2}}} \left(a_k u_k(t) + a_k^\dagger u_k^*(t) \right)$, where the modes have right asymptotic behaviour : $\lim_{t\to-\infty} u_k(t) = \frac{1}{\sqrt{2\sqrt{k^2+m_0^2}}} e^{-i\sqrt{k^2+m_0^2}\,t + ikx}$ and then one looks at observables like : ${}_{in}\langle 0|\phi^2(t)|0\rangle_{in}$



- When exact solution is available then one directly computes, else one can make the ansatz (inspired from equilibrium) :

$u_k(t) = \frac{1}{\sqrt{2\,\Omega_k(t)}}\, e^{i\vec{k}\cdot\vec{x} - i\int_{dt'}^{t} \Omega_k(t')}$

$i\,{}_{in}\langle 0|\phi^2|0\rangle_{in} = \int \frac{d^{d-1}k}{\sqrt{k^2+m_0^2}}\, \frac{1}{\Omega_k(t)}$

KZ IN FREE SCALAR FIELD THEORIES

$$\Omega_k^2 = \omega_k^2 - \frac{1}{2}\frac{\ddot{\Omega}_k}{\Omega_k} + \frac{3}{4}\frac{\dot{\Omega}_k^2}{\Omega_k^2};$$

$$\omega_k = k^2 + m^2(t);$$

$$\Omega_k(t) = \omega_k - \frac{1}{2}\left(\frac{\ddot{\omega}_k}{\omega_k} - \frac{3}{2}\frac{\dot{\omega}_k^2}{\omega_k^2}\right) + \cdots$$

$$\omega_k > \dot{\omega}_k > \ddot{\omega}_k \cdots$$

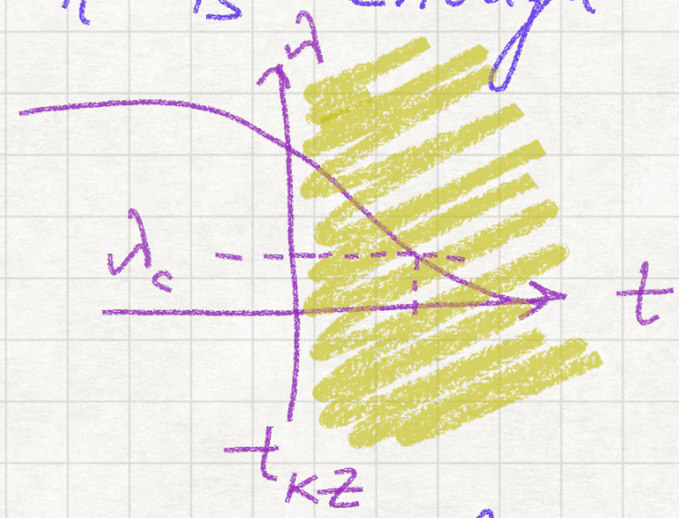
ADIABATICITY BREAKS WHEN $:\rightarrow \omega_k \sim \frac{\ddot{\omega}_k}{\omega_k} - \frac{3}{2}\frac{\dot{\omega}_k^2}{\omega_k^2} \Rightarrow t_{KZ}$

(Das, Myers, Galante 2014)

MOTIVATION

BEMUSEMENTS & HINDSIGHTS

- To derive KZ it is enough to be in the "diabatic" zone.



- In SYK model, main dynamics is due to "near" conformal mode

(Bagrets, Altland, Kamenev 2016)

- $\int d^d x \sqrt{g}\, \mathcal{O} \approx \int d^d x\, \lambda(t)\, \mathcal{O}$: A quench can be thought of as coupling the theory to gravity.

OUTLINE

- Gross-Neveu model : Equilibrium properties.
- Dynamical gap equation.
- Quench results in symmetry restoration.
- Kibble-Zurek scaling.
- A *double-scaling* limit (!)
- Conclusions via generalizations.

GROSS-NEVEU MODEL

(Gross-Neveu, March 1974)

- $$S = \int d^2x \left\{ \bar{\psi}_j \, i\not{\partial} \, \psi_j + \frac{g^2}{2N} \left(\bar{\psi}_j \psi_j \right)^2 \right\}$$

j: 1 to N

$$\gamma^0 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; ; \; \gamma^1 \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \; ; \; \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Discrete chiral symmetry

$$\psi_j \longrightarrow \gamma_5 \psi_j \quad \text{and} \quad \bar{\psi}_j \longrightarrow -\bar{\psi}_j \gamma_5 .$$

GROSS-NEVEU EQUILIBRIUM

- $S = \int d^2x \left\{ \bar{\psi} i\not{\partial} \psi - \frac{N}{2g^2}\sigma^2 + \sigma\bar{\psi}\psi \right\}$

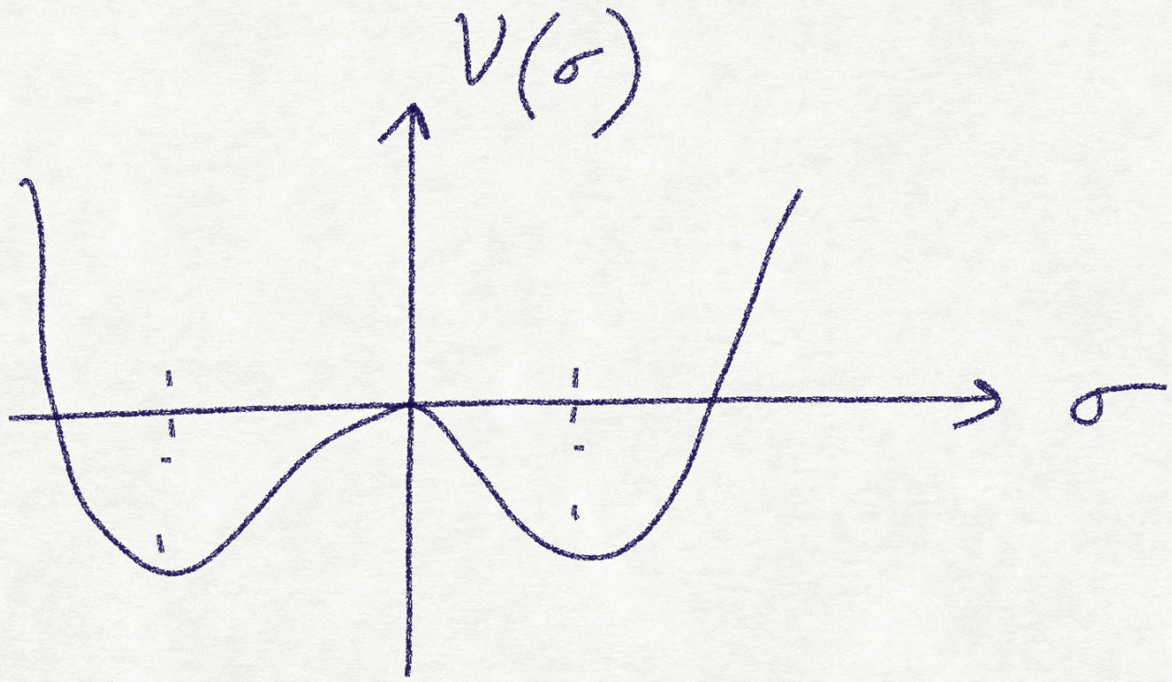
$\sigma \to -\sigma$ $\mathbb{Z}_2$ action : $\sigma = \frac{g^2}{N}\bar{\psi}\psi$

- $\mathcal{L}_{eff}(\sigma) = -N\,\mathrm{Tr}\,\log(i\not{\partial} + \sigma) - \frac{N}{2g^2}\sigma^2$

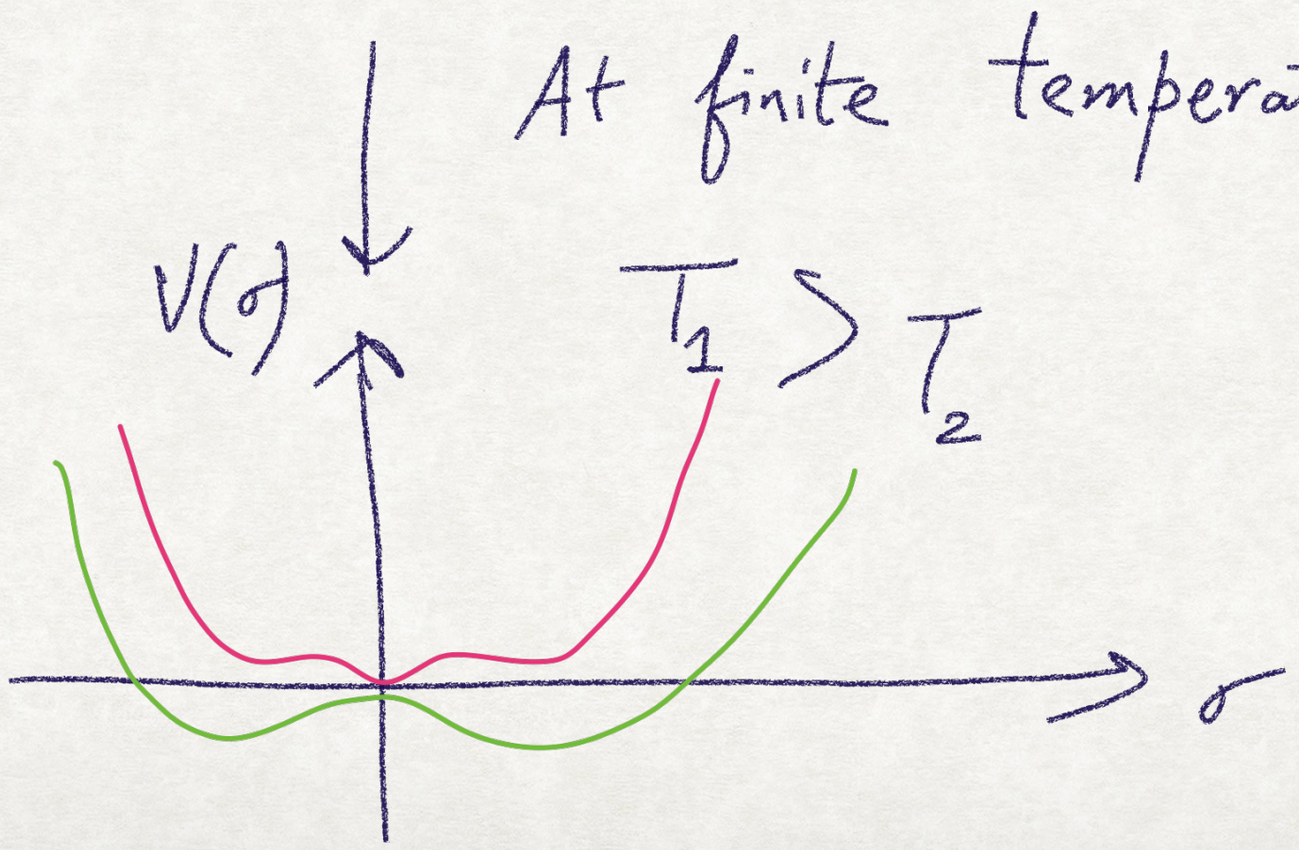
- $\frac{\sigma}{g^2} = -\mathrm{Tr}\,\frac{1}{i\not{\partial} + \sigma} \Rightarrow \boxed{\sigma \sim \Lambda \exp(-\pi/g^2)}$

- Theory is also asymptotically free!

GROSS-NEVEU EQUILIBRIUM



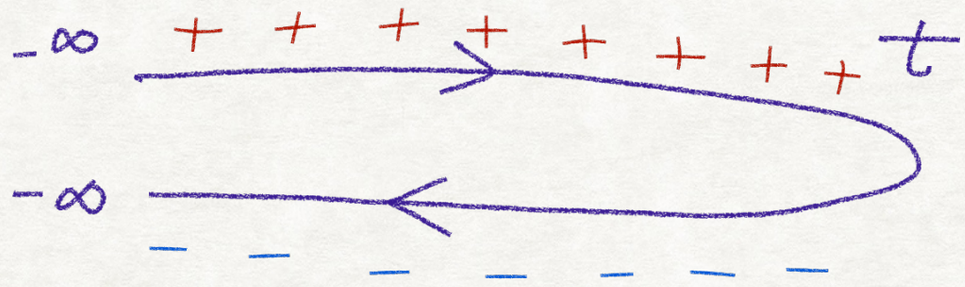
SYMMETRY GETS RESTORED



Aug
Harrington, Yildiz 1974

DYNAMICAL SADDLE

- We smoothly quench $g \longrightarrow g(t)$

- 

Schwinger - Keldysh Contour

$$\int \mathcal{D}\bar{\psi}_{\pm}\,\mathcal{D}\psi_{\pm}\,\mathcal{D}\sigma_{\pm}\; e^{iS_+ - iS_-}$$

- $$S_{eff} = -N\,Tr\,\log\begin{pmatrix} i\not{\partial} + \sigma_+ & 0 \\ 0 & -i\not{\partial} - \sigma_- \end{pmatrix} - \frac{N}{2g^2}\int \sigma_+^2 - \sigma_-^2$$

$$\Rightarrow \frac{\sigma_{\pm}(t)}{g^2(t)} = -Tr\left[\frac{1}{i\not{\partial} + \sigma_{\pm}(t)}\right]$$

DYNAMICAL SADDLE

$$\frac{\sigma_{\pm}(t)}{g^2(t)} = -\,Tr\left[\frac{1}{i\not{\partial} + \sigma_{\pm}(t)}\right]$$

Better depend on time

Fermion propagaton with a time-dependent mass!

$$= -\int_{-\infty}^{\infty} dk \left\langle \bar{\chi}_k(t)\ \chi_k(t) \right\rangle$$

- THERMAL QUENCH :

$$R\cdot H\cdot S = -\int_{-\infty}^{\infty} dk \left\langle \bar{\chi}_k(t)\ \chi_k(t) \right\rangle_{\beta}$$

FERMION PROPAGATOR WITH TIME DEPENDENT MASS

$$\left\{ i\gamma^0 \partial_0 + i\gamma^1 \partial_1 + \sigma(t) \right\} \chi(x,t) = 0$$

$$\chi(x,t) \longleftarrow \int dk\, \chi_k(t)\, e^{ikx}$$

$$\chi_k(t) = B_k\, u_k(t) + D_k^{\dagger}\, v_k(t)$$

$$u_k(t) = \frac{e^{ikx}}{\sqrt{2\pi}} \begin{pmatrix} h_k^{I}(t) \\ -h_k^{II}(t) \end{pmatrix} \quad ; \quad v_k(t) = \frac{e^{-ikx}}{\sqrt{2\pi}} \begin{pmatrix} h_{-k}^{II\,*}(t) \\ h_{-k}^{I\,*}(t) \end{pmatrix}$$

$$\dot{h}_k^{I} - ik\, h_k^{I} + i\sigma\, h_k^{II} = 0$$

$$\dot{h}_k^{II} + ik\, h_k^{II} + i\sigma\, h_k^{I} = 0$$

$$\left| h_k^{I} \right|^2 + \left| h_k^{II} \right|^2 = 1$$

ANSATZ & SOLUTION

$$\dot{h}_k^{I} - ik\, h_k^{I} + i\sigma\, h_k^{II} = 0$$

$$\dot{h}_k^{II} + ik\, h_k^{II} + i\sigma\, h_k^{I} = 0$$

$$\left|h_k^{I}\right|^2 + \left|h_k^{II}\right|^2 = 1$$

$$h_k^{I}(t) = \sqrt{\frac{\omega_k(t)-k}{2\omega_k(t)}}\; e^{-i\int^t \omega_k(t')\,dt'}$$

$$h_k^{II}(t) = -\sqrt{\frac{\omega_k(t)+k}{2\omega_k(t)}}\; e^{-i\int^t \omega_k(t')\,dt'}$$

ANSATZ

$$\int dk \left\langle \bar{\chi}_k(t)\, \chi_k(t) \right\rangle = \int_0^\infty \frac{dk}{\pi}\, A_k(t)$$

$$\frac{\sqrt{|\omega_k(t)|^2 - k^2}}{|\omega_k(t)|}$$

DYNAMICAL SADDLE SOLUTION

In $\sigma > \dot{\sigma} > \ddot{\sigma}$ (derivative expansion)

$$\omega_k = \sqrt{\sigma^2+k^2} + \ddot{\sigma}\,\frac{\sigma k\left(k-\sqrt{\sigma^2+k^2}\right)}{4\left(\sigma^2+k^2\right)^{3/2}\left(2k\left(\sqrt{k^2+\sigma^2}-k\right)-\sigma^2\right)}$$

$$+\ \dot{\sigma}^2\,\frac{k\sigma^2\left(6\sqrt{\sigma^2+k^2}-5k\right)}{8\left(2k\left(\sqrt{k^2+\sigma^2}-k\right)-\sigma^2\right)\left(k^2+\sigma^2\right)^{5/2}}.$$

DYNAMICAL SADDLE SOLUTION

$$\frac{\sigma(t)}{g^2(t)} = \int_0^\infty \frac{dk}{\pi} \frac{\sqrt{|\omega_k(t)|^2 - k^2}}{|\omega_k(t)|}$$

WITH A UV CUT-OFF THIS INTEGRAL CAN BE DONE!

DYNAMICAL SADDLE SOLUTION

The order parameter σ satisfies the ODE :-

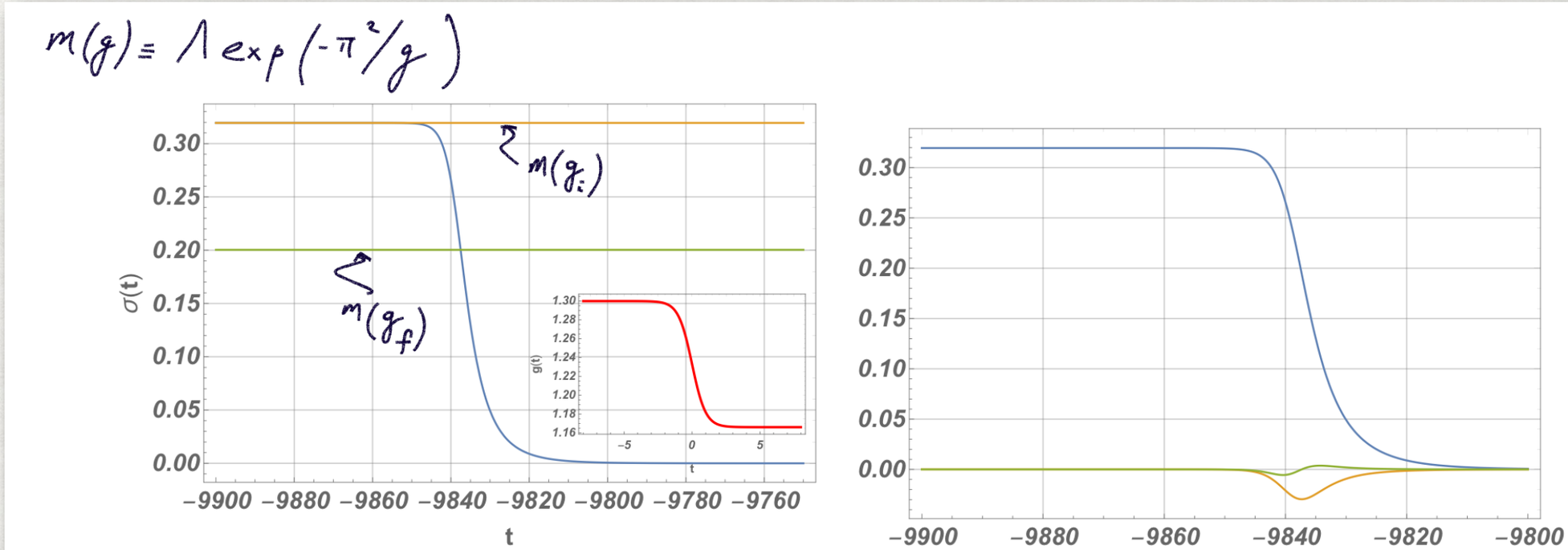
$$\frac{24\pi\sigma^4}{g^2} = 6(4\sigma^4 - \dot{\sigma}^2 + \sigma\ddot{\sigma})\log\left(\Lambda + \sqrt{\Lambda^2+\sigma^2}\right) + \frac{3\sigma^2(\dot{\sigma}^2+\sigma\ddot{\sigma})}{\Lambda^2+\sigma^2} + \frac{2\Lambda\sigma^2(2\dot{\sigma}^2+\sigma\ddot{\sigma})}{(\Lambda^2+\sigma^2)^{3/2}}$$
$$+ \Lambda\frac{5\dot{\sigma}^2 - 8\sigma\ddot{\sigma}}{\sqrt{\Lambda^2+\sigma^2}} - 3(\dot{\sigma}^2 - \sigma\ddot{\sigma})\log\left(\Lambda^2+\sigma^2\right) - 3\sigma^4\dot{\sigma}^2\left(\frac{\Lambda}{(\Lambda^2+\sigma^2)^{5/2}} + \frac{1}{(\Lambda^2+\sigma^2)^2}\right)$$
$$- 3\sigma\ddot{\sigma}(1+4\log\sigma) + 12\dot{\sigma}^2\log\sigma - 24\sigma^4\log\sigma. \qquad (3.4)$$

1

- In equilibrium this reduces to:

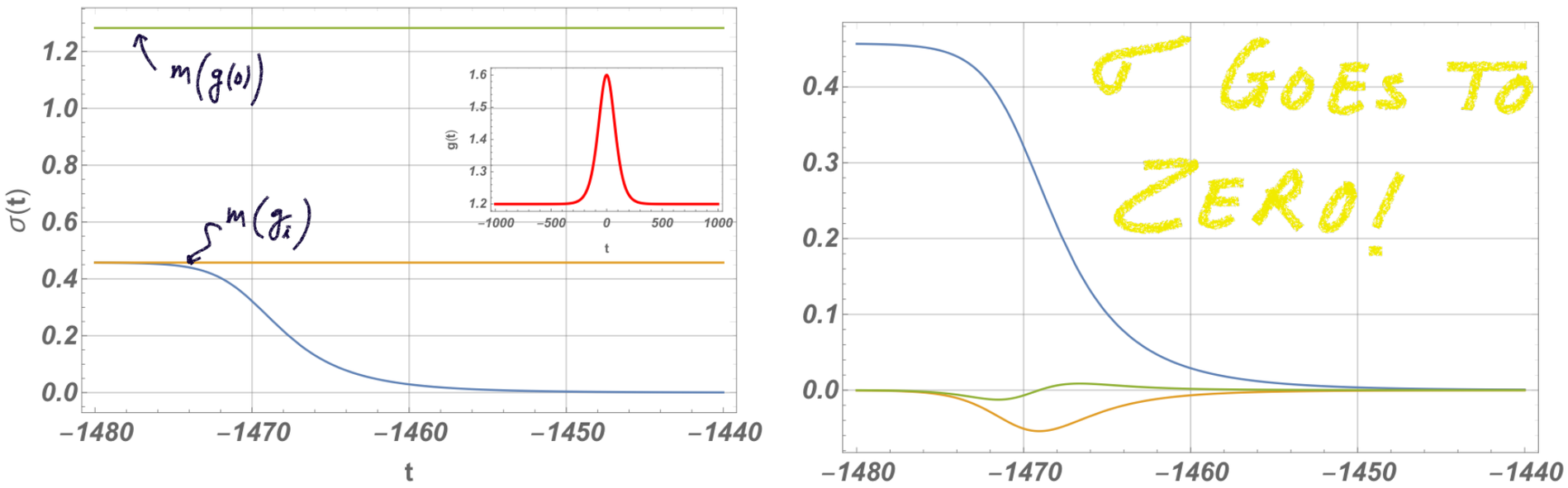
$$\frac{\pi}{g^2} = \log\left(\Lambda + \sqrt{\Lambda^2+\sigma^2}\right) \Longrightarrow \sigma \sim \Lambda e^{-\pi/g^2}$$

- We solve eq(1) numerically with equilibrium boundary conditions for different $g(t)$ profiles.



(a) Tanh quench

(b) $\sigma(t)$ and its derivatives



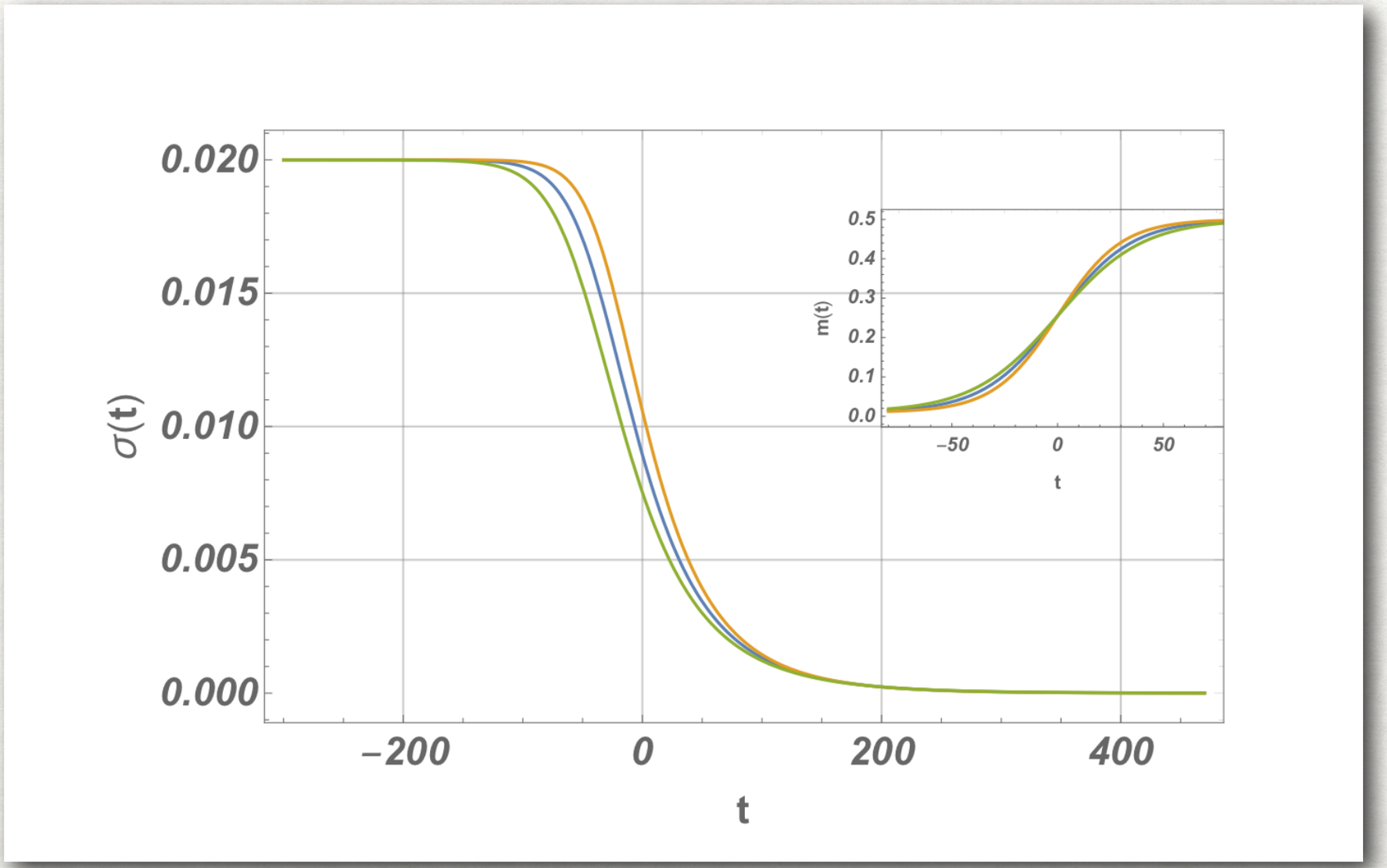
(a) Sech squared quench

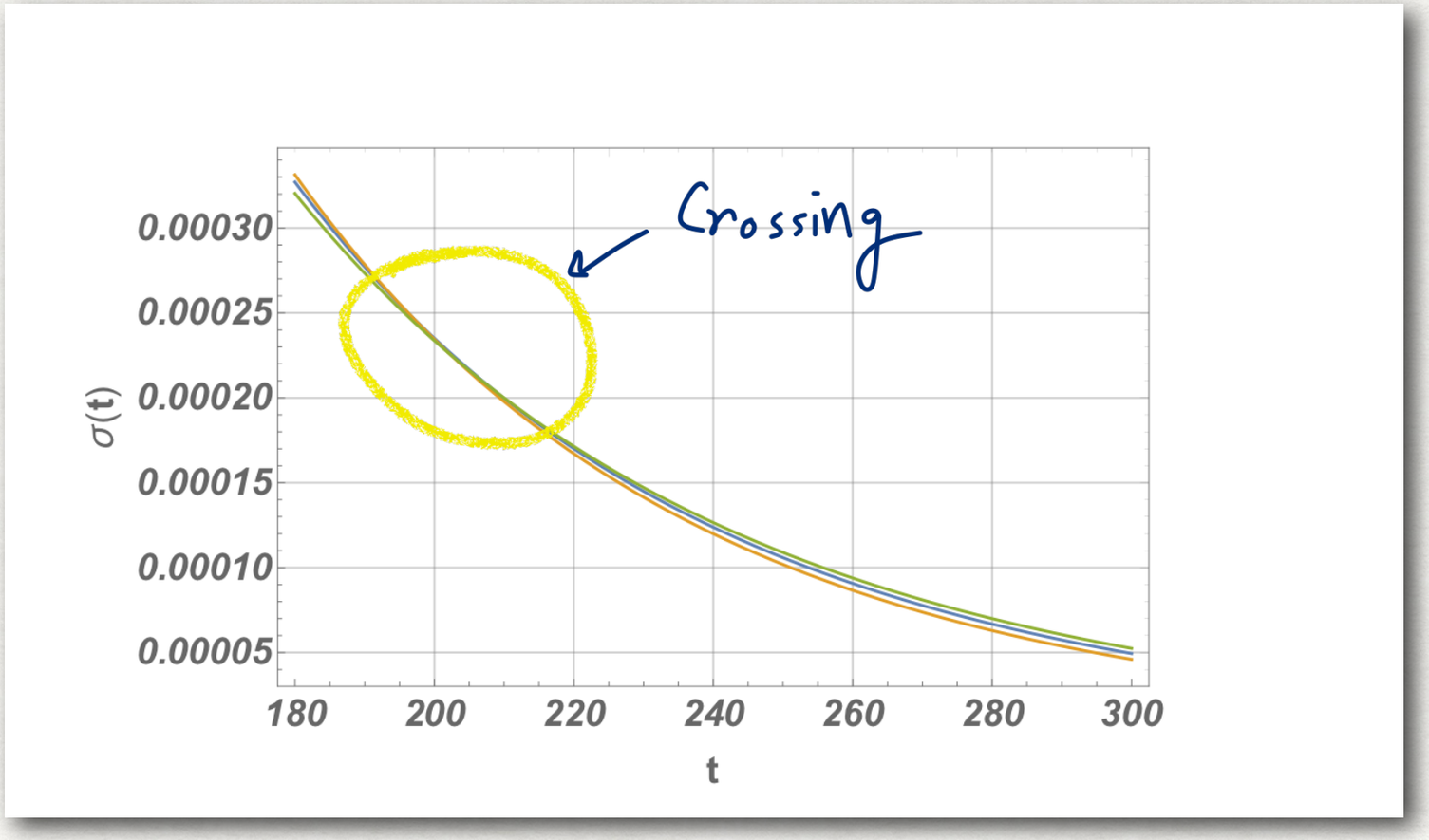
(b) $\sigma(t)$ and its derivatives

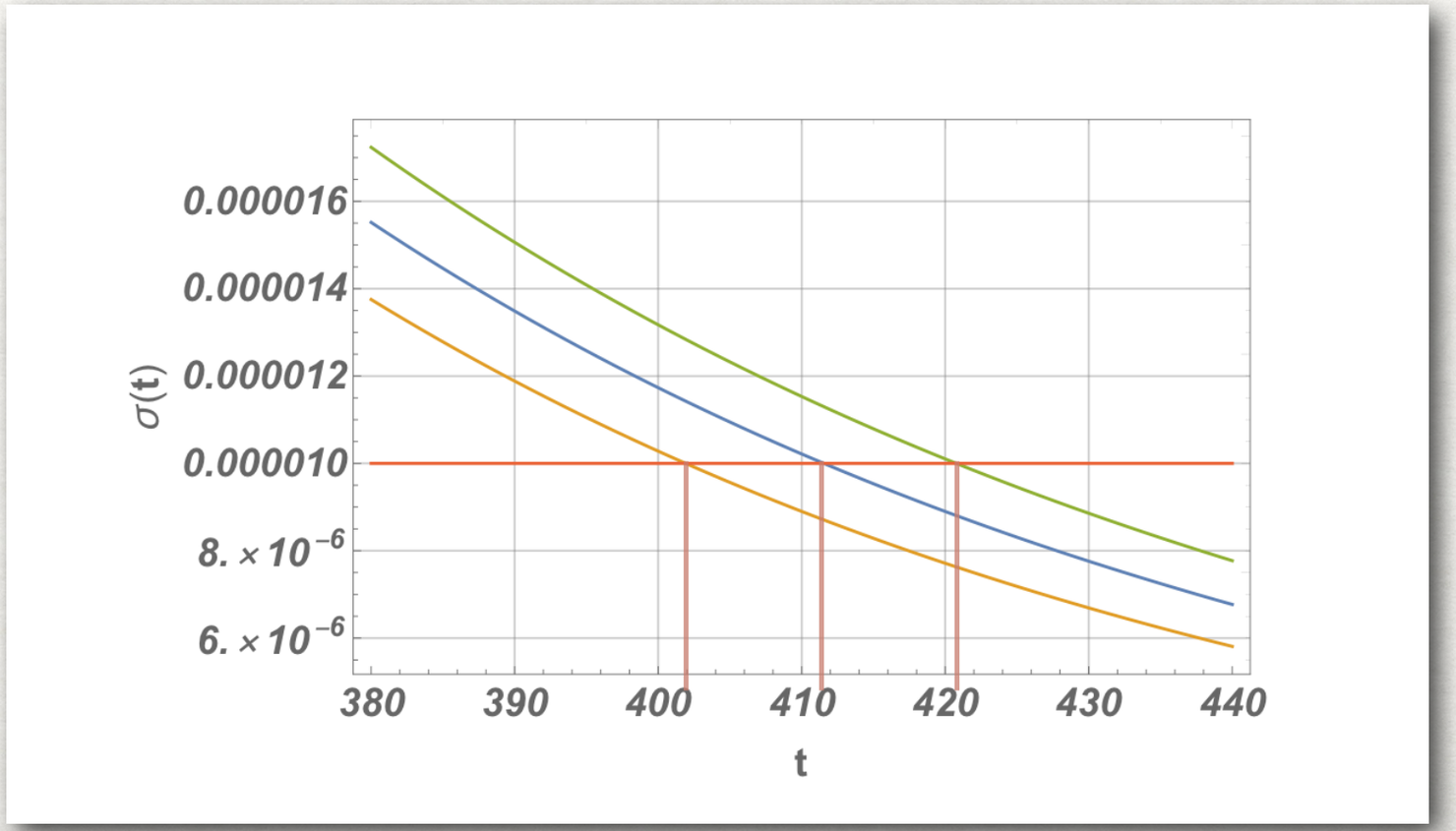
KIBBLE-ZUREK SCALING (?)

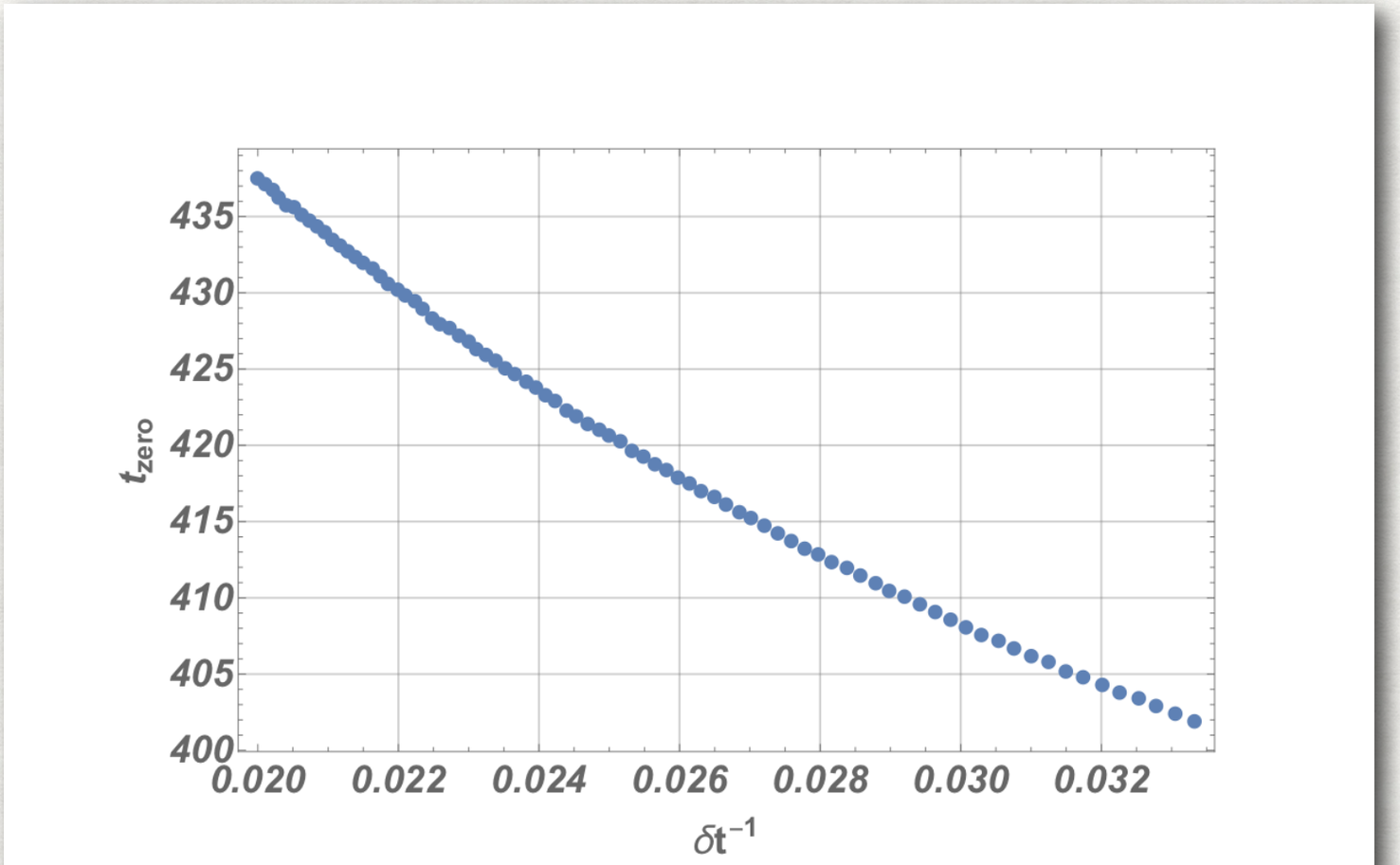
- Since the KZ scale emerges from energy considerations, it is better to think of the quench in terms of $m(t) = \Lambda e^{-\pi/g(t)^2}$.

- We want to analyze the time of zero σ for different quench rates.




Crossing
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
σ(t)
180
200
220
240
260
280
300
t


0.000016
0.000014
0.000012
0.000010
8. × 10^-6
6. × 10^-6
σ(t)
380
390
400
410
420
430
440
t


435
430
425
420
415
410
405
400
t_{zero}
0.020
0.022
0.024
0.026
0.028
0.030
0.032
δt^{-1}

A POWER LAW ?!?!?!

KIBBLE-ZUREK SCALE

- I_N $\Lambda/\sigma \gg 1$ LIMIT

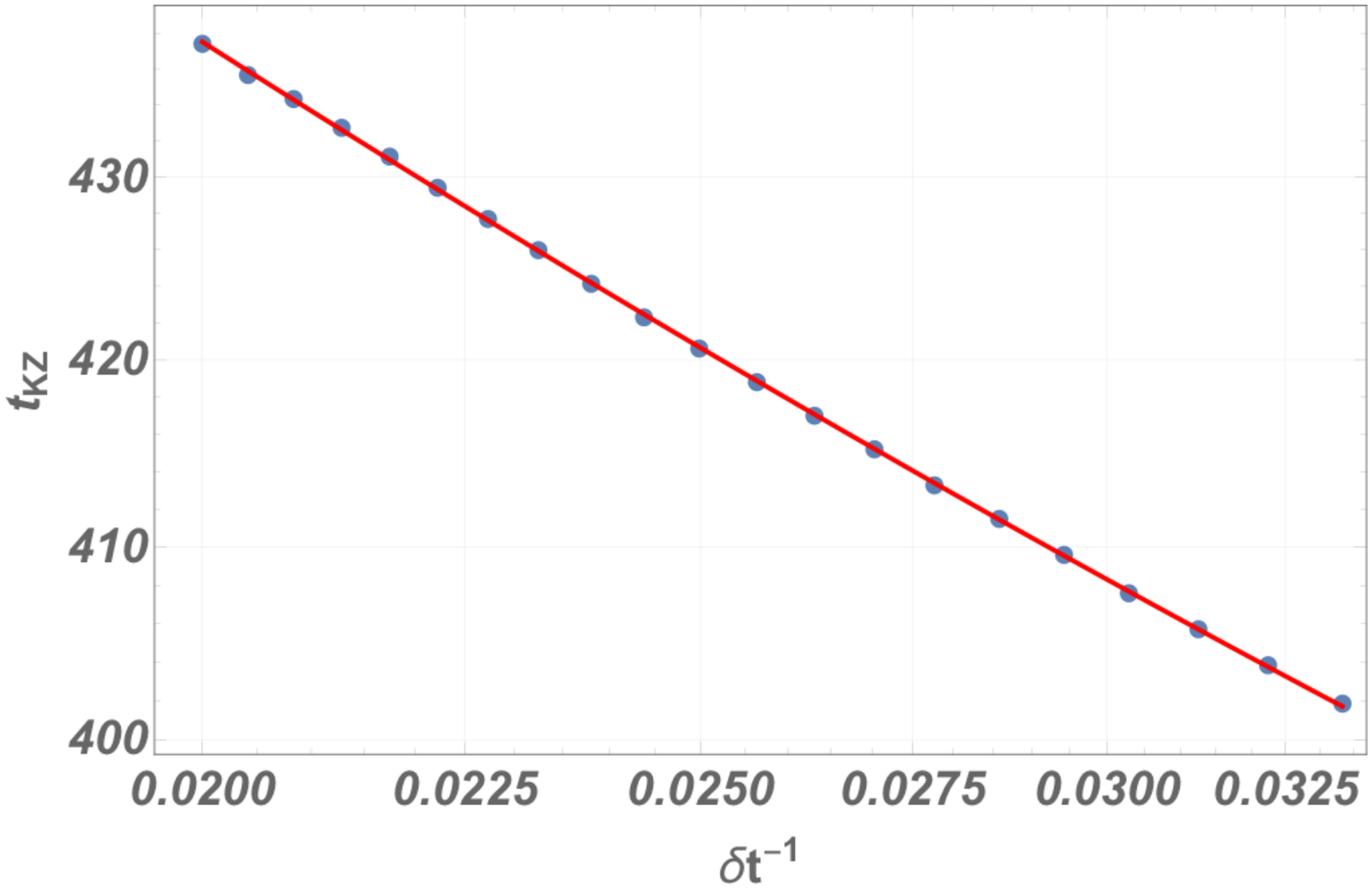
$$\frac{\pi}{g^2} = \log\frac{\Lambda}{\sigma}\left(1 - \frac{\dot\sigma^2}{2\sigma^4} + \frac{\ddot\sigma}{2\sigma^3}\right)$$

- WITH $m = 2\Lambda e^{-\pi/g^2}$

INVERT: $\sigma = m\left(1 + \frac{\pi}{2m^2g^2}\left(\frac{\ddot m}{m} - \frac{\dot m^2}{m^2}\right) + \cdots\right)$

WITH DERIVATIVE EXPANSION

$$\text{FIT:}\quad t_{kz} \sim \sqrt{\delta t}\; W\left(4\delta t^2\right)^{1/4}$$



CONCLUSIONS

- We have seen that the Gross-Neveu model restores chiral symmetry via smooth quantum quenches.

- We have identified the restoration time with the Kibble-Zurek time in the problem.

A DOUBLE SCALING

- In the $\frac{1}{\sigma} \gg 1$ limit in terms of $\sigma = \Lambda \exp(-y)$

$$\ddot{y} - 2\Lambda^2 e^{-2y}\left(\frac{\pi - yg}{yg^2}\right) = 0$$

- Now scale $y \to \infty$ keeping σ fixed.

LIOUVILLE HEURESTICS

with $\sigma = \Lambda e^{-y}$, consider the double scaling

$\Lambda, y \to \infty$ with σ fixed.

$$\ddot{y} + 2\Lambda^2 e^{-2y} = 0$$

$$\phi = -2y$$

$$\tau = it$$

$$S = \int d\tau \left(\frac{\dot{\phi}^2}{8} + \Lambda^2 e^{2\phi} \right)$$

THIS IS LIOUVILLE WITH SPATIAL FROZEN.

– Moore, Seiberg

Liouville wavefunctional:

$$-2\partial_\phi^2\Psi_k + \Lambda^2 e^\phi \Psi_k = 2k^2\Psi_k.$$

$$\Psi_k[\phi(\sigma)] = \frac{2}{\Gamma(2ik)} K_{2ik}\left(\sqrt{2}\sigma\right)$$

NORM PEAKS AT $\sigma = 0$!

ALSO DESCRIBES SYK DYNAMICS

-Bagrets, Altland, Kamenev (2016)

QUESTIONS...

SOLVE NUMERICALLY

$$\Omega_k^2 = \omega_k^2 - \frac{1}{2}\frac{\ddot{\Omega}_k}{\Omega_k} + \frac{3}{4}\frac{\dot{\Omega}_k^2}{\Omega_k^2}\,;$$

- Is there a fast scaling regime as well ?

$$\omega_k = k^2 + m^2(t)$$

- How does the condensate correlator decay (ETH) ? $\sim t^{-3/2}$
- Any decoupling of scales, in terms of instantons? ??
- Induced gravity, connections with Liouville? ??
- What happens when the symmetry is continuous, e.g., the Nambu and Jona-Lasinio model. similar

NAMBU & JONA-LASINIO MODEL IN 2D

(1961)

$$\mathscr{L} = \sum_{j=1}^{N} \left\{ \bar{\psi}_j i\gamma^\mu \partial_\mu \psi_j + \frac{g^2}{2N}[(\bar{\psi}_j\psi_j)^2 - (\bar{\psi}_j\gamma_5\psi_j)^2] \right\}, \quad j = 1,2,\ldots N$$

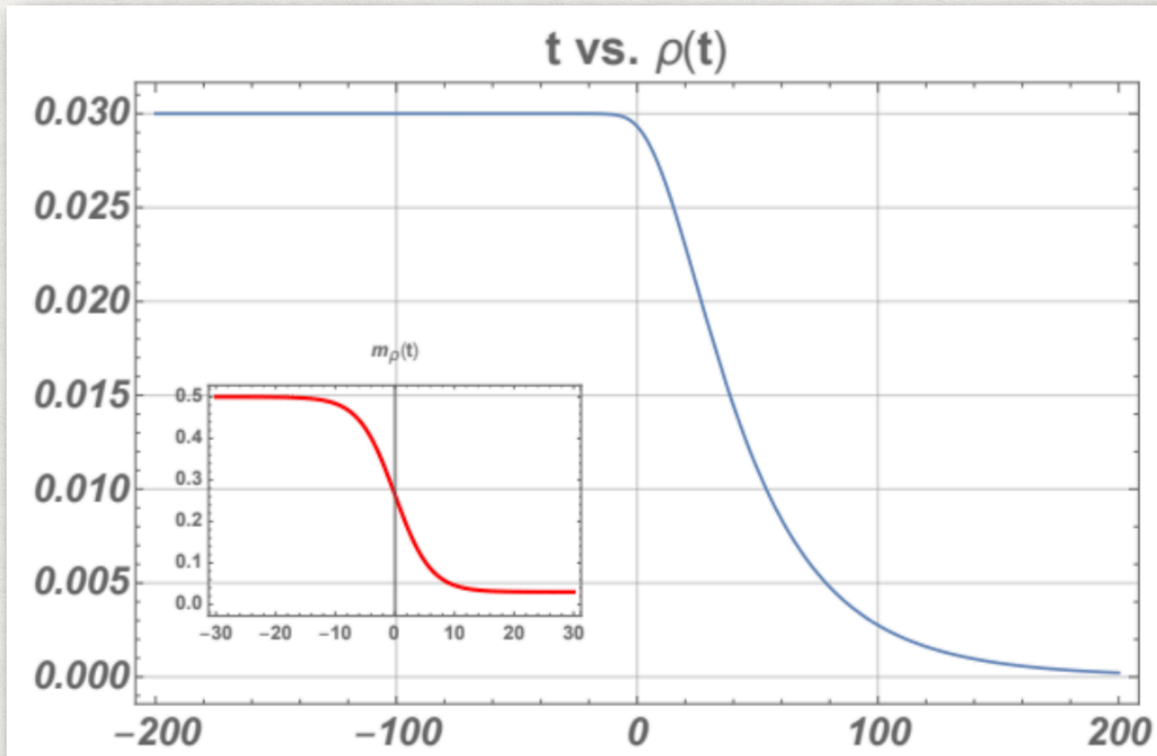
- $\psi \rightarrow e^{i\alpha\gamma_5}\psi$ $\quad$ $\mathcal{U}(1)$ SYMMETRY.

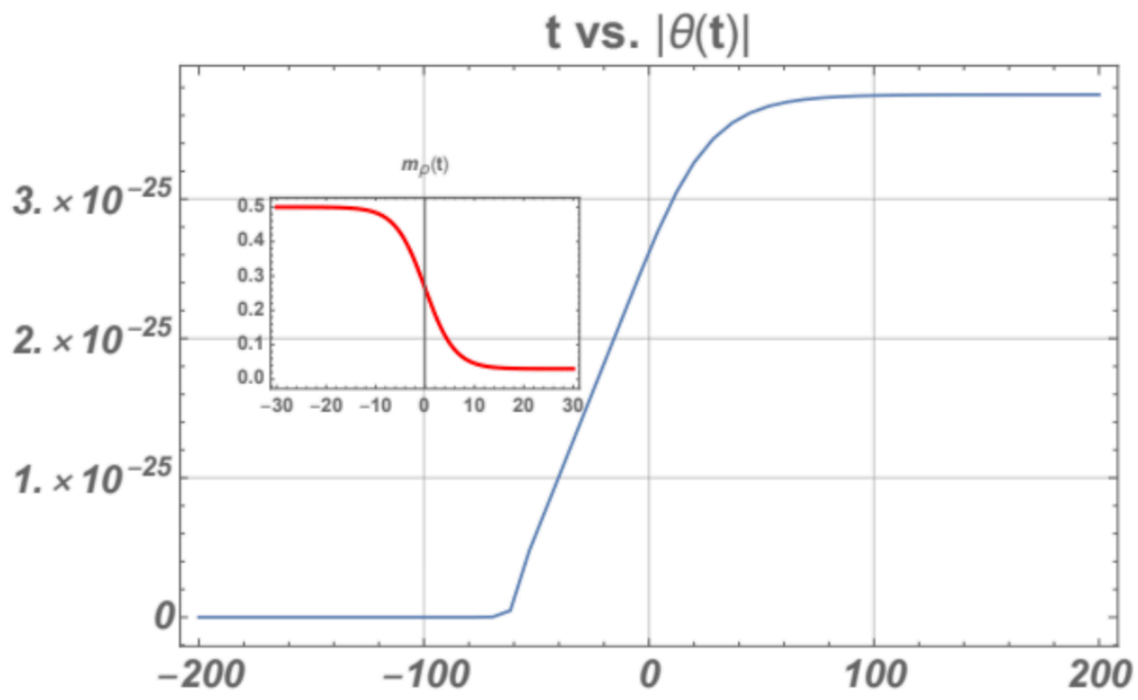
- $\sigma = \frac{g^2}{N}(\bar{\psi}\psi)$, $\pi = \frac{g^2}{N} i(\bar{\psi}\gamma_5\psi)$

- $\sigma + i\pi = \rho e^{i\theta}$ ← REMAINS MASSLESS
  GETS A VEV

QUENCH RESULTS :—

- ρ GOES TO ZERO $\Rightarrow$ SYMMETRY RESTORED





- θ GETS VEV $\Rightarrow$ CP VIOLATION ($\because$ CPT INV)

THANK YOU FOR YOUR ATTENTION.