Assignment 8: Applications of Integration, Pappus Theorem

1. **(D)** Find the area of the region in the first quadrant bounded on the left by the Y-axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $x = (y-1)^2$, and above right by the line $x = 3 - y$.

2. **(D)** Sketch the graph of $r = 1 + \sin\theta$. Find the area of the region that is inside the circle $r = 3\sin\theta$ and also inside $r = 1 + \sin\theta$.

5. **(D)** The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the X-axis to generate a solid. Find the volume of the solid.

6. **(D)** The region in the first quadrant bounded by the parabola $y = x^2$, the Y-axis and the line $y = 1$ is revolved about the line $x = 2$ to generate a solid. Find the volume of the solid.

8. **(D)** Find the area of the surface generated by revolving the curve $y = x^3$, $0 \leq x \leq 1/2$, about the X-axis.

10. **(D)** A regular hexagon is inscribed in the circle $x^2 + (y-2)^2 = 1$ and is rotated about the X-axis. Find the volume and the surface area of the solid so formed.

Assignment 8 - Solutions

1. Area= $\int\limits_0^1 (1+\sqrt{x}-\frac{x^2}{4})dx + \int\limits_1^2 (3-x-\frac{x^2}{4})dx = \frac{5}{2}$.

2. Area= $2.\frac{1}{2}[\int\limits_0^{\frac{\pi}{6}} (3\sin\theta)^2 d\theta + \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1+\sin\theta)^2 d\theta] = \frac{5\pi}{4}$.

5. Use the washer method: The outer radius , $r_2(x) = -x+3$ and the inner radius $r_1(x) = x^2+1$.

   $V = \int\limits_{-2}^{1} \pi((-x+3)^2 - (x^2+1)^2)dx = 117\frac{\pi}{3}$.

6. By the shell method: Observe that shell radius $=2-x$ and the shell height $=1-x^2$.

   The volume $V = \int\limits_0^1 2\pi(2-x)(1-x^2)dx = \frac{13\pi}{6}$.

   The volume can also be computed by the washer method.

8. $S = \int\limits_0^{\frac{1}{2}} 2\pi y ds$.

   Since $y = x^3$, $ds = \sqrt{dx^2+dy^2} \Rightarrow ds = \sqrt{1+9x^4}dx$. Therefore $S = \frac{61\pi}{1728}$.

10 . From the figure, the area of the hexagon is $\frac{3\sqrt{3}}{2}$ and the perimeter is 6. $\rho = 2$ since the centroid is $(0, 2)$.

By Pappus theorem, $V = 6\sqrt{3}\pi$ and $S = 24\pi$.

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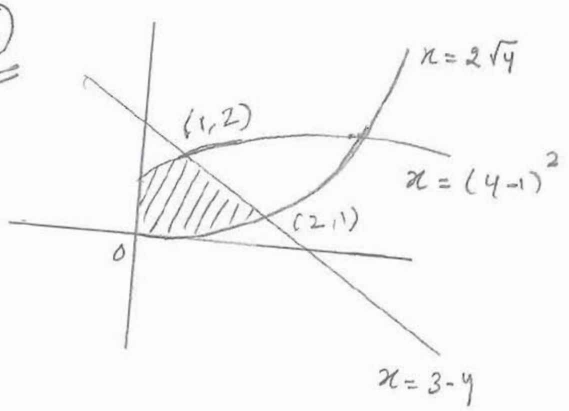

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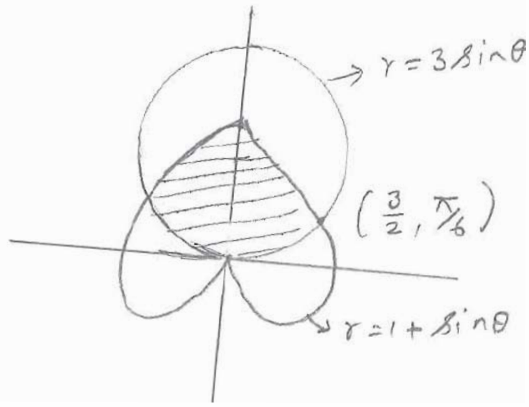

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